Gas Vapor Mixtures and HVAC

Atmospheric air normally contains some water vapor (moisture). The *dry-air* contains no water. Although the amount of water vapor in the air is small, it plays a major role in human comfort and thus in air-conditioning applications.

The temperature of air in air-conditioning applications ranges from about -10 to 50°C. In that range both dry air and atmospheric air (including water-vapor) can be treated as ideal gas, with negligible error. Thus the ideal-gas relation $PV = RT$ can be applied. The partial pressures of atmospheric air are:

$$P = P_a + P_v \quad (kPa)$$

Also, one can write:

$$P_a = \frac{m_a R_a T}{V_a}$$

$$P_v = \frac{m_v R_v T}{V_v}$$

Note that since air is considered as an ideal-gas, the enthalpy of air (both water-vapor and dry air) is only a function of temperature, at the temperature range of interest, i.e., -10 to 50°C.

Taking 0°C as the reference temperature; with the constant-pressure specific for dry-air (in the range of interest) $c_p = 1.005\ kJ/kg.°C$; one obtains:

$$h_{dry\ air} = c_p\, T = (1.005 \text{ kJ/kg. °C})\ \text{T} \quad \text{(kJ/kg)}$$

$$\Delta h_{dry\ air} = c_p \Delta T = \left(1.005 kJ / kg^{\circ}C\right)\Delta T$$

The enthalpy of water vapor at 0°C is 2500.9 *kJ/kg*. The average c_p value of water vapor in the temperature range 10 to 50°C can be taken to be 1.82 *kJ/kg.°C.* Then the enthalpy of water vapor can be determined from:

$$h_g = 2500.9 + 1.82\, T \quad \text{(kJ/kg)}$$

Some definitions

Specific humidity (humidity ratio) ω:

$$\omega = \frac{m_v}{m_a} \left(\text{kg of water vapor /kg of dry air}\right)$$

or

$$\omega = \frac{m_v}{m_a} = \frac{P_v V / R_v T}{P_a V / R_a T} = \frac{P_v / R_v}{P_a / R_a} = 0.622 \frac{P_v}{P_a} = 0.622 \frac{P_v}{P - P_v} \left(\text{kg of water vapor /kg of dry air}\right)$$

where P is the total pressure.

Relative humidity ϕ: The ratio of the amount of moisture that air contains over the maximum amount of moisture that air can hold (saturated air):

$$\phi = \frac{m_v}{m_g} = \frac{P_v V / R_v T}{P_g V / R_v T} = \frac{P_v}{P_g}$$

where

$P_g = P_{sat@T}$

The relative humidity can also be expressed in terms of absolute or specific humidity:

$$\phi = \frac{\omega P}{(0.622+\omega)P_g} \quad and \quad \omega = \frac{0.622\phi P_g}{P - \phi P_g}$$

The relative humidity ranges from 0 to 1 and it is a function of air temperature.

Relative humidity can change with temperature even when specific humidity remains constant.

Total enthalpy of atmospheric air is the sum of the enthalpies of dry air and the water vapor:

$$H = H_a + H_v = m_a h_a + m_v h_v$$

or,

$$h = H/m_a = h_a + m_v/m_a h_v = h_a + \omega h_v$$

The enthalpy of water-vapor in atmospheric air can be considered as saturated vapor, $h_v = h_g$;

$$h = h_a + \omega h_g$$

Dry Bulb Temperature T_{db}: the temperature measured by a thermometer placed in a mixture of air and water-vapor.

Dew Point Temperature T_{dp}: is the temperature at which condensation begins when the air is cooled at constant pressure. T_{dp} is the saturation temperature of water corresponding to the vapor pressure:

$$T_{dp} = T_{sat@\ Pv}$$

Sling Psychrometer: a rotating set of thermometers one of which measures wet bulb temperature and the other dry bulb temperature. T_{db} and T_{wb} are sufficient to fix the state of the mixture.

Adiabatic Saturation Process

Consider a long insulated channel that contains a pool of water. A steady stream of unsaturated air that has a specific humidity ω_1 (unknown) and temperature T_1 is passed through this channel.

If the channel is long enough, the air stream exits as saturated air at temperature T_2 which is called *adiabatic saturation temperature*.

The adiabatic saturation process involves no heat or work transfer. If water is added to the pool at the same rate that it evaporates (make up) to maintain a constant liquid water level, the process is steady-state steady flow.

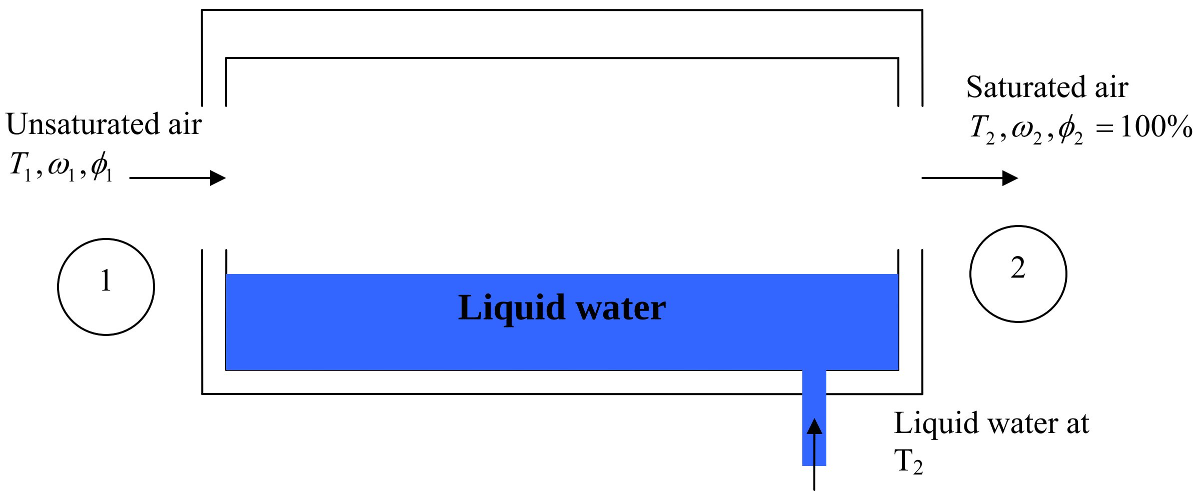


Fig. 1: Schematic of an air saturator.

Thus, one can write:

$$\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$$
$$\dot{m}_{w1} + \dot{m}_f = \dot{m}_{w2} \quad and \quad \omega_1 \dot{m}_a + \dot{m}_f = \dot{m}_a \omega_2$$

Thus,

$$\dot{m}_f = \dot{m}_a (\omega_2 - \omega_1)$$

An energy balance yields:

$$\dot{m}_a h_1 + \dot{m}_f h_{f2} = \dot{m}_a h_2$$

or

$$\dot{m}_a h_1 + \dot{m}_a (\omega_2 - \omega_1) h_{f2} = \dot{m}_a h_2$$

Dividing by dry-air mass flow rate gives:

$$h_1 + (\omega_2 - \omega_1) h_{f2} = h_2$$
$$or$$
$$\omega_1 = \frac{c_p (T_2 - T_1) + \omega_2 h_{fg2}}{h_{g1} - h_{f_2}}$$

where

$$\omega_2 = \frac{0.622 P_{g2}}{P_2 - P_{g2}}$$

since $\phi_2 = 100\%$. Note that T_2 is the wet-bulb temperature T_{wb}.

Wet Bulb Temperature T_{wb}: thermometer surrounded by a saturated wick if air/water vapor mixture is not saturated, some water in the wick evaporates and diffuses into the air and results in cooling the water in the wick. As the temperature of the water drops, heat is transferred to the water from both the air and the thermometer; the steady state temperature is the wet-bulb temperature.

The wet bulb temperature is always smaller (or at the limit) equal to the dry bulb temperature.

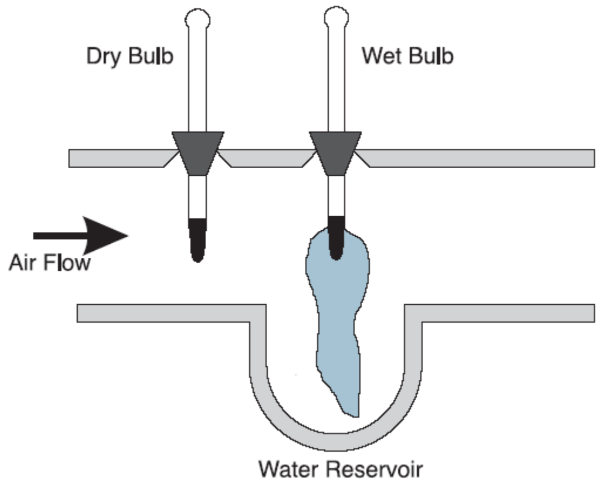


Fig.2: Dry bulb and wet bulb temperature.

Example 1

The dry- and the wet-bulb temperatures of atmospheric air at 1 atm (101.325 kPa) pressure are 25 and 15°C, respectively. Determine a) the specific humidity, and b) the relative humidity.

Analysis:

the saturation pressure of water is 1.7057 *kPa* at 15°C, and 3.1698 *kPa* at 25°C (using Table A-4).

a) Specific humidity ω_1 is determined from:

$$\omega_1 = \frac{c_p(T_2 - T_1) + \omega_2 h_{fg2}}{h_{g1} - h_{f_2}}$$

where T_2 is the wet-bulb temperature and ω_2

$$\omega_2 = \frac{0.622 P_{g2}}{P - P_{g2}} = \frac{0.622(1.7057 kPa)}{(101.325 - 1.7057) kPa} = 0.01065 kg\, H_2O / kg \text{ dry air}$$

Thus,

$$\omega_1 = \frac{1.005 kJ / kg.^\circ C (15 - 25)^\circ C + 0.01065(2465.4 kJ / kg)}{(2546.5 - 62.982) kJ / kg} = 0.00653 kg\, H_2O / kg \text{ dry air}$$

b) The relative humidity

$$\phi_1 = \frac{\omega_1 P_2}{(0.622+\omega_1)P_{g1}} = \frac{0.00653(101.325kPa)}{(0.622+0.00653)(3.1698kPa)} = 0.332 \text{ } or \text{ } 33.2\%$$

Psychrometric Chart

The state of the atmospheric air at a specified pressure is completely specified by two independent intensive properties. Psychrometric charts present the moist air properties; they are used extensively in air-conditioning applications.

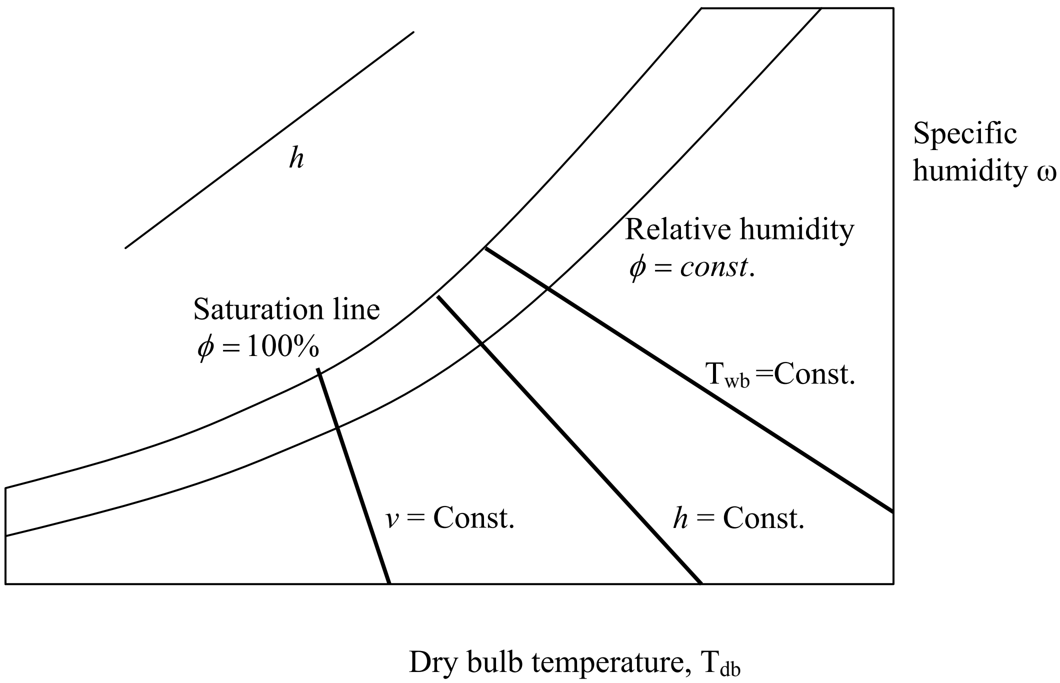


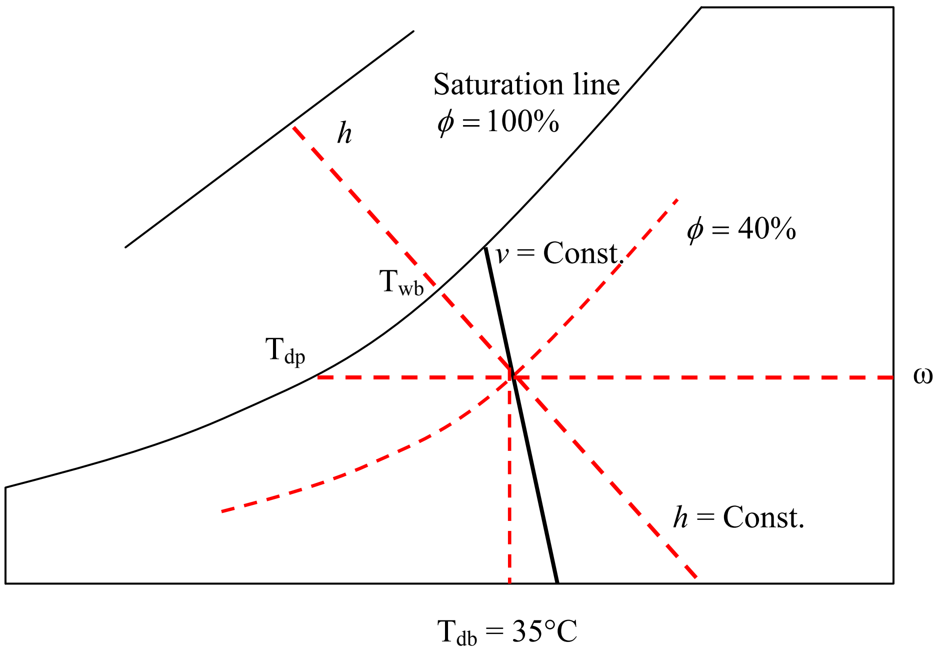
Fig. 3: Psychrometric chart.

Basic features of psychrometric chart are:

- The dry bulb temperatures are shown on the horizontal axis.
- The specific humidity ω is shown on the vertical axis.
- The curved line at the left end of the chart is the saturation line. All the saturated air states are located on this curve. Thus, it also represents the curve of relative humidity 100%. Other constant relative humidity curves have the same general shape.
- Lines of constant wet-bulb temperature have a downhill appearance to the right.
- Lines of specific volume also have downhill appearance to the right with steeper slopes.
- Lines of constant enthalpy lie very near to the constant wet-bulb temperature, thus (in some charts) lines of constant wet-bulbs are used as constant-enthalpies.
- For saturated air, the dry-bulb, wet-bulb, and dew-point temperatures are identical. Thus the dew-point temperature of atmospheric air can be determined by drawing a horizontal line to the saturated curve.

Example 2

Consider air at 1 atm, 35°C and 40% relative humidity. Using psychrometric chart, determine a) the specific humidity, b) the enthalpy, c) the wet-bulb temperature, d) the dew-point temperature and e) the specific volume of the air.



Using Fig. A-31; one can read:

$\omega = 0.0142$ kg of water/kg of dry air

$h = 71.5$ kJ/kg dry air

$T_{wb} = 24°C$

$T_{dp} = 19.4°C$

$v = 0.893$ m^3 /kg dry air

Human Comfort

Human body can be viewed as a heat engine which generates waste heat that must be rejected to the environment. The rate of heat generation depends on the level of activity.

Table 1: Heat rejection for an average adult male

Activity	Heat rejection (W)
Sleeping	87
Resting or office work	115
Bowling	230
Heavy physical work	440

Depending on the type of activity, part of the rejected body heat is dissipated through *latent* heat (sweating and breathing).

The comfort of human body depends on three factors:

- Temperature: most important index of comfort, most people feel comfortable when temperature is between 22 and 27°C
- Relative humidity: it affects the amount of heat that body can dissipate through evaporation. Relative humidity is a measure of air's ability to absorb moisture. Most people prefer relative humidity of 40 to 60%.
- Air motion: it removes the warm, moist air that builds up around body and replaces it with fresh air. Most people feel comfortable at an airspeed of 15 m/min.

Other important factors in air-conditioning include: noise, filtration, air-distribution, fresh air supply, supply air temperature, etc.

HVAC Processes

Maintaining a living space or an industrial facility at the desired temperature and humidity requires some processes called air-conditioning; including:

- Simple heating: raising the air temperature.
- Simple cooling: lowering the air temperature.
- Humidifying: adding moisture.
- Dehumidifying: removing moisture.

In many applications, a combination of these processes is needed to bring the air to a desired condition.

The psychrometric charts serves as a valuable aid in visualizing the air-conditioning processes. Simple heating and cooling appear as horizontal lines on the psychrometric chart (constant specific-humidity process).

Most air-conditioning processes can be modeled as steady-flow processes; thus, one can write:

Mass balance for dry air: $\sum_{in} \dot{m}_a = \sum_{out} \dot{m}_a \quad (kg/s)$

Mass balance for water: $\sum_{in} \dot{m}_w = \sum_{out} \dot{m}_w \quad or \quad \sum_{in} \dot{m}_a \omega = \sum_{out} \dot{m}_a \omega$

Neglecting the potential and kinetic energy changes, the steady-state energy balance is:

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m}h = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \dot{m}h$$

The work term usually consists of the fan work input which is small and often negligible.

Simple Heating and Cooling (ω = Constant)

The amount of moisture in the air remains constant during this process since no moisture is added or removed to or from the air.

Notice that the relative humidity of air decreases during a heating process and increases in a cooling process. This is because the relative humidity is the ratio of the moisture capacity of the air and it increases with increasing the air temperature.

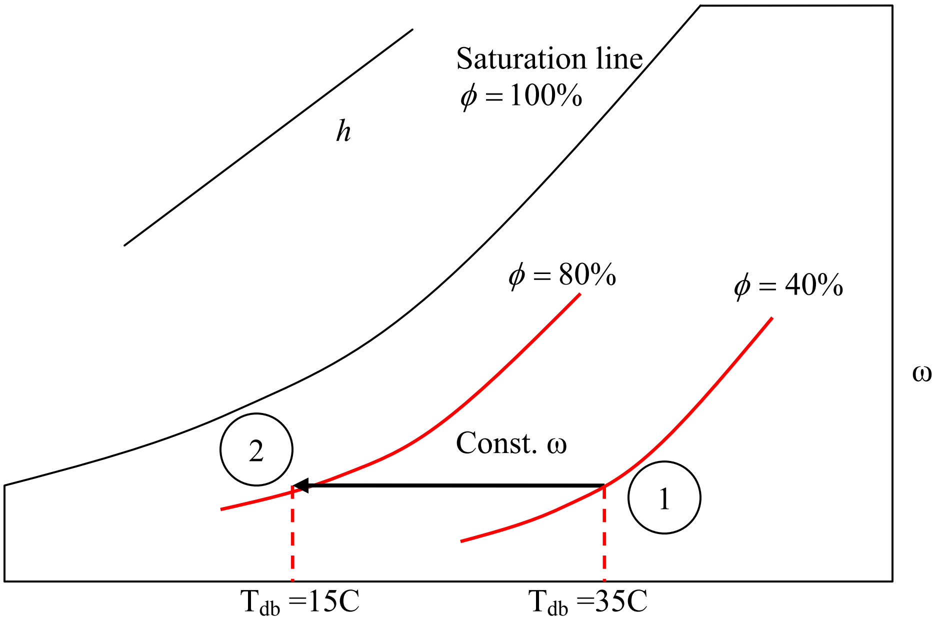


Fig.4: A simple cooling process.

The conservation of mass reduces to: $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a \quad \omega_1 = \omega_2$

Neglecting any fan work, the conservation of energy yields: $\dot{Q} = \dot{m}_a (h_2 - h_1) \quad or \quad q = (h_2 - h_1)$

where h_1 and h_2 are enthalpies per unit mass of dry air at the inlet and exit of the heating/cooling section, respectively.

Heating with Humidification

To maintain comfortable relative humidity, simple heating is typically accompanied with humidification, Fig. 5. That is accomplished by passing the air through a humidifying section. If steam is used in the humidifier, we will have additional heating; thus $T_3 > T_2$. If water is sprayed in the humidifier section, part of the latent heat of vaporization comes from the air which results in the cooling of the air; thus $T_3 < T_2$.

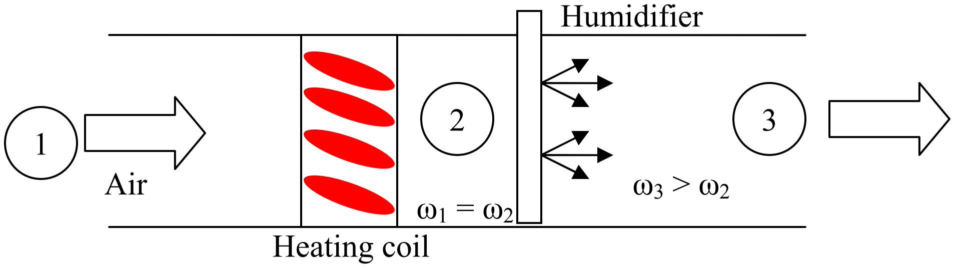


Fig. 5: Heating with humidification.

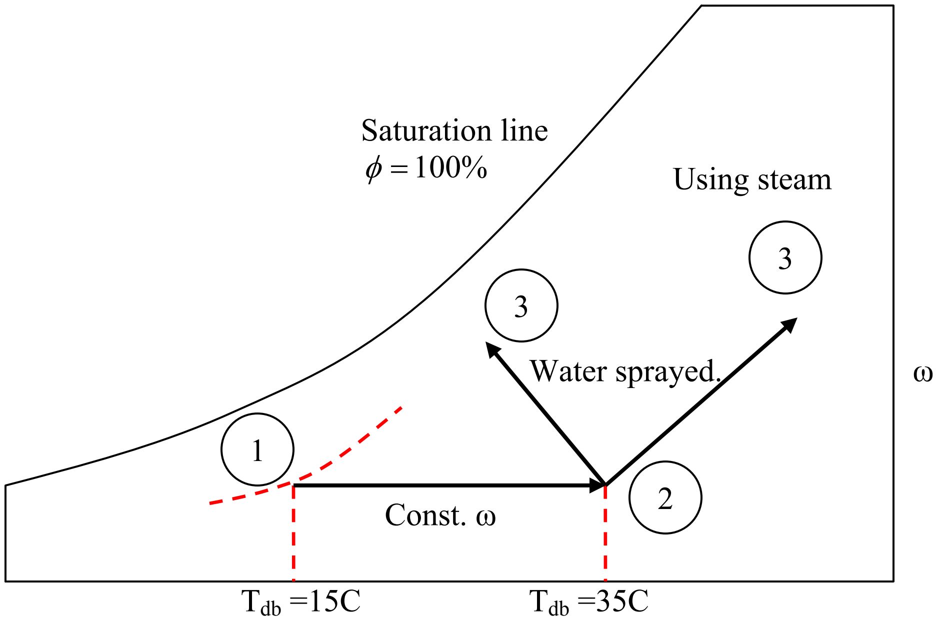


Fig. 6: Heating with humidification, using water spray or steam.

Cooling with Dehumidification

To remove some moisture from the air, it should be cooled below its dew-point temperature. Passing through cooling coil, air temperature decreases and its relative humidity increases at constant specific humidity until air temperature reaches its dew-point temperature. Any further cooling results in condensation of part of the moisture in the air. Note that air remains saturated during the entire condensation process.

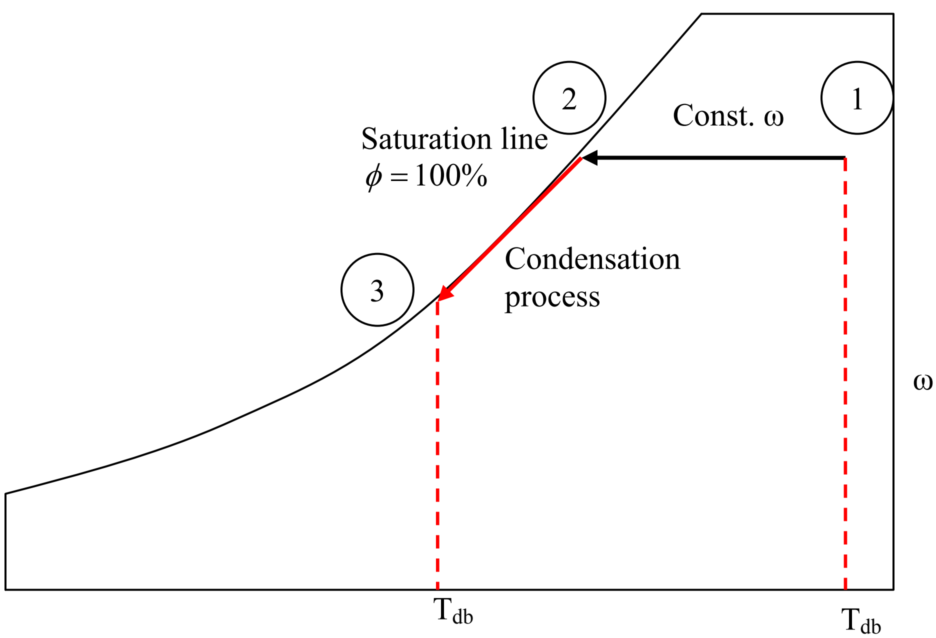


Fig. 7: Cooling with dehumidification.

Evaporative Cooling

Evaporative cooling is based on a simple principle: as water evaporates, the latent heat of vaporization is absorbed from the water body and the surrounding air. As a result, both water and air are cooled.

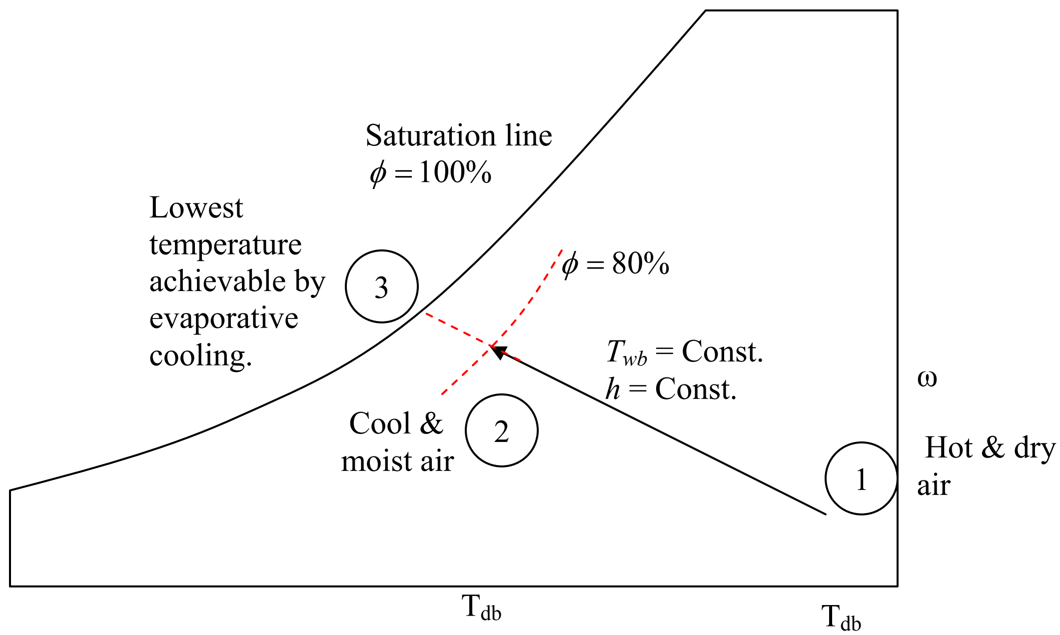


Fig. 8: Evaporative cooling.

The evaporative cooling is essentially identical to the adiabatic saturation process. Thus the evaporative cooling process follows a line of constant wet-bulb temperature on the psychrometric chart.

Adiabatic Mixing

Mixing processes normally involve no work interactions; thus, one can write (for adiabatic mixing of two streams):

Mass balance (dry air) $\quad \dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$

Mass balance (water - vapor) $\quad \omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 \dot{m}_{a3}$

Energy balance $\quad \dot{m}_{a1} h_1 + \dot{m}_{a2} h_2 = \dot{m}_{a3} h_3$

Eliminating $\dot{m}_{a3}$, one obtains:

$$\frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{\omega_2 - \omega_3}{\omega_3 - \omega_1} = \frac{h_2 - h_3}{h_3 - h_1}$$

When two streams at two different states are mixed adiabatically, the state of mixture lies on the straight line connecting states 1 and 2, on the psychrometric chart.

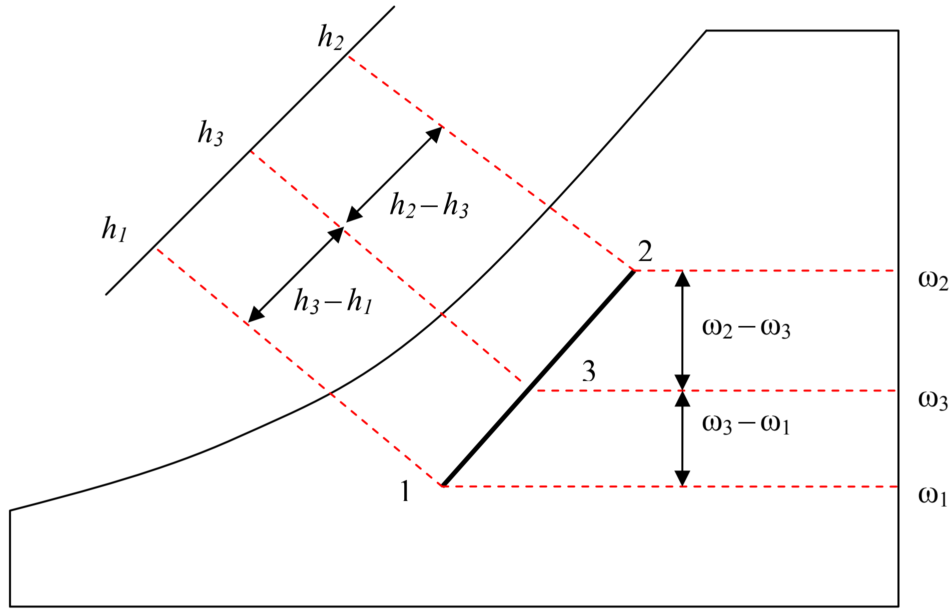


Fig. 9: Adiabatic mixing of two streams.

Example 3

Saturated air at 14°C at a rate of 50 m^3/min is mixed adiabatically with the outside air at 32°C and 60% relative humidity at a rate of 20 m^3/min. Assuming that the mixing occurs at 1 atm, determine:

a) The specific humidity

b) The relative humidity

c) The dry-bulb, and

d) The wet-bulb of the mixture.

Analysis:

Using the psychrometric chart, the properties of each stream can be determined:

h_1 = 39.4 kJ/kg dry air

ω_1 = 0.010 kg water/kg dry air

v_1 = 0.826 m^3/kg dry air

and

h_2 = 79.0 kJ/kg dry air

ω_2 = 0.0182 kg water/kg dry air

v_2 = 0.889 m^3/kg dry air

The mass flow rates of dry air in each stream are:

$$\dot{m}_{a1} = \frac{\dot{V}_1}{v_1} = \frac{50m^3/\min}{0.826m^3/kg\ dry\ air} = 60.5\ kg/\min$$

$$\dot{m}_{a2} = \frac{\dot{V}_2}{v_2} = \frac{20m^3/\min}{0.889m^3/kg\ dry\ air} = 22.5\ kg/\min$$

From the mass balance of dry air

$$\dot{m}_{a3} = \dot{m}_{a1} + \dot{m}_{a2} = 83kg/\min$$

The specific heat and the enthalpy of the mixture can be determined from:

$$\frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{\omega_2 - \omega_3}{\omega_3 - \omega_1} = \frac{h_2 - h_3}{h_3 - h_1}$$

$$\frac{60.5}{22.5} = \frac{0.0182 - \omega_3}{\omega_3 - 0.010} = \frac{79.0 - h_3}{h_3 - 39.4}$$

which yield

$$\omega_3 = 0.0122\ kg\ H_2O/kg\ dry\ air$$

$$h_3 = 50.1\ kJ/kg\ dry\ air$$

Using these two properties, we can fix the state of the mixture on the psychrometric chart. Other properties of the mixture can be easily read from the chart:

$$T_3 = 19.0^\circ C$$

$$\phi_3 = 89\%$$

$$v_3 = 0.844m^3/kg\ dry\ air$$

The volume flow rate of the mixture is determined from:

$$\dot{V}_3 = \dot{m}_{a3} v_3 = 70.1m^3/\min$$

Note that the volume flow rate of the mixture is approximately equal to the sum of the volume flow rate of the two streams.