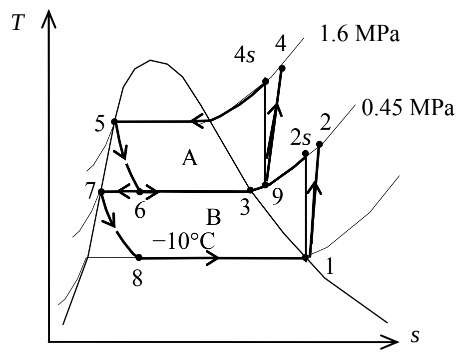
**11-66** A two-stage cascade refrigeration cycle with a flash chamber with refrigerant-134a as the working fluid is considered. The mass flow rate of the refrigerant through the high-pressure compressor, the rate of refrigeration, the COP are to be determined. Also, the rate of refrigeration and the COP are to be determined if this refrigerator operated on a single-stage vapor-compression cycle under similar conditions.

***Assumptions*** **1** Steady operating conditions exist. **2** Kinetic and potential energy changes are negligible.

***Analysis*** (a) From the refrigerant-134a tables (Tables A-11 through A-13)

$$h_1 = h_{g@-10°C} = 244.51 \text{ kJ/kg}$$
$$s_1 = s_{g@-10°C} = 0.9377 \text{ kJ/kg.K}$$




$$\left.\begin{array}{l} P_2 = 450 \text{ kPa} \\ s_2 = s_1 \end{array}\right\} h_{2s} = 261.07 \text{ kJ/kg}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1}$$

$$0.86 = \frac{261.07 - 244.51}{h_2 - 244.51} \longrightarrow h_2 = 263.76 \text{ kJ/kg}$$

$$h_3 = h_{g@\,450 \text{ kPa}} = 257.53 \text{ kJ/kg}$$
$$h_5 = h_{f@1600 \text{ kPa}} = 135.93 \text{ kJ/kg}$$
$$h_6 = h_5 = 135.93 \text{ kJ/kg}$$
$$h_7 = h_{f@\,450 \text{ kPa}} = 68.81 \text{ kJ/kg}$$
$$h_8 = h_7 = 68.81 \text{ kJ/kg}$$

$$\left.\begin{array}{l} h_6 = 135.93 \text{ kJ/kg} \\ P_6 = 450 \text{ kPa} \end{array}\right\} x_6 = 0.3557$$

The mass flow rate of the refrigerant through the high pressure compressor is determined from a mass balance on the flash chamber

$$\dot{m} = \frac{\dot{m}_7}{1 - x_6} = \frac{0.11 \text{ kg/s}}{1 - 0.3557} = \mathbf{0.1707\ kg/s}$$

Also,

$$\dot{m}_3 = \dot{m} - \dot{m}_7 = 0.1707 - 0.11 = 0.06072 \text{ kg/s}$$

(*b*) The enthalpy at state 9 is determined from an energy balance on the mixing chamber:

$$\dot{m}h_9 = \dot{m}_7 h_2 + \dot{m}_3 h_3$$

$$(0.1707 \text{ kg/s})h_9 = (0.11 \text{ kg/s})(263.76 \text{ kJ/kg}) + (0.06072 \text{ kg/s})(257.53 \text{ kJ/kg}) \longrightarrow h_9 = 261.54 \text{ kJ/kg}$$

Then,

$$\left.\begin{array}{l} P_9 = 450 \text{ kPa} \\ h_9 = 261.54 \text{ kJ/kg} \end{array}\right\} s_9 = 0.9393 \text{ kJ/kg}$$

$$\left.\begin{array}{l} P_4 = 1600 \text{ kPa} \\ s_4 = s_9 \end{array}\right\} h_{4s} = 288.41 \text{ kJ/kg}$$

$$\eta_C = \frac{h_{4s} - h_9}{h_4 - h_9}$$

$$0.86 = \frac{288.41 - 261.54}{h_4 - 261.54} \longrightarrow h_4 = 292.78 \text{ kJ/kg}$$

The rate of heat removal from the refrigerated space is

$$\dot{Q}_L = \dot{m}_7(h_1 - h_8) = (0.11\text{ kg/s})(244.51 - 68.81)\text{kJ/kg} = \mathbf{19.33\ kW}$$

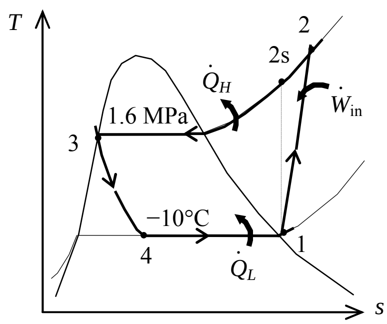
(*c*) The power input and the COP are

$$\dot{W}_{\text{in}} = \dot{m}_7(h_2 - h_1) + \dot{m}(h_4 - h_9)$$
$$= (0.11\text{ kg/s})(263.76 - 244.51)\text{kJ/kg} + (0.1707\text{ kg/s})(292.78 - 261.54)\text{kJ/kg} = 7.45\text{ kW}$$

$$\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{\text{in}}} = \frac{19.33}{7.45} = \mathbf{2.59}$$

(*d*) If this refrigerator operated on a single-stage cycle between the same pressure limits, we would have

$$h_1 = h_{g@-10°\text{C}} = 244.51\text{ kJ/kg}$$
$$s_1 = s_{g@-10°\text{C}} = 0.9377\text{ kJ/kg.K}$$




$$\left.\begin{array}{l} P_2 = 1600\text{ kPa} \\ s_2 = s_1 \end{array}\right\} h_{2s} = 287.85\text{ kJ/kg}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1}$$

$$0.86 = \frac{287.85 - 244.51}{h_2 - 244.51} \longrightarrow h_2 = 294.90\text{ kJ/kg}$$

$$h_3 = h_{f@1600\text{ kPa}} = 135.93\text{ kJ/kg}$$
$$h_4 = h_3 = 135.93\text{ kJ/kg}$$

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = (0.1707\text{ kg/s})(244.51 - 135.93)\text{kJ/kg} = \mathbf{18.54\ kW}$$

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1) = (0.1707\text{ kg/s})(294.90 - 244.51)\text{kJ/kg} = 8.60\text{ kW}$$

$$\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{\text{in}}} = \frac{18.54}{8.60} = \mathbf{2.16}$$

***Discussion*** The cooling load decreases by 4.1% while the COP decreases by 16.6% when the cycle operates on the single-stage vapor-compression cycle.