13-46 The volumetric analysis of a mixture of gases is given. The volumetric and mass flow rates are to be determined using three methods.

Properties The molar masses of O_2, N_2, CO_2, and CH_4 are 32.0, 28.0, 44.0, and 16.0 kg/kmol, respectively (Table A-1).

Analysis (*a*) We consider 100 kmol of this mixture. Noting that volume fractions are equal to the mole fractions, mass of each component are

$$\begin{aligned}
m_{O2} &= N_{O2}M_{O2} = (30\text{ kmol})(32\text{ kg/kmol}) = 960\text{ kg}\\
m_{N2} &= N_{N2}M_{N2} = (40\text{ kmol})(28\text{ kg/kmol}) = 1120\text{ kg}\\
m_{CO2} &= N_{CO2}M_{CO2} = (10\text{ kmol})(44\text{ kg/kmol}) = 440\text{ kg}\\
m_{CH4} &= N_{CH4}M_{CH4} = (20\text{ kmol})(16\text{ kg/kmol}) = 320\text{ kg}
\end{aligned}$$

30% O_2
40% N_2
10% CO_2
20% CH_4
(by volume)

The total mass is

$$\begin{aligned}
m_m &= m_{O2} + m_{N2} + m_{CO2} + m_{CH4}\\
&= 960 + 1120 + 440 + 320 = 2840\text{ kg}
\end{aligned}$$

The apparent molecular weight of the mixture is

$$M_m = \frac{m_m}{N_m} = \frac{2840\text{ kg}}{100\text{ kmol}} = 28.40\text{ kg/kmol}$$

Mixture
8 MPa, 15°C

The apparent gas constant of the mixture is

$$R = \frac{R_u}{M_m} = \frac{8.314\text{ kJ/kmol}\cdot\text{K}}{28.40\text{ kg/kmol}} = 0.2927\text{ kJ/kg}\cdot\text{K}$$

The specific volume of the mixture is

$$v = \frac{RT}{P} = \frac{(0.2927\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(288\text{ K})}{8000\text{ kPa}} = 0.01054\text{ m}^3/\text{kg}$$

The volume flow rate is

$$\dot{V} = AV = \frac{\pi D^2}{4}V = \frac{\pi(0.02\text{ m})^2}{4}(3\text{ m/s}) = \mathbf{0.0009425\ m^3/s}$$

and the mass flow rate is

$$\dot{m} = \frac{\dot{V}}{v} = \frac{0.0009425\text{ m}^3/\text{s}}{0.01054\text{ m}^3/\text{kg}} = \mathbf{0.08942\ kg/s}$$

(*b*) To use the Amagat's law for this real gas mixture, we first need the mole fractions and the Z of each component at the mixture temperature and pressure. The compressibility factors are obtained using Fig. A-15 to be

$$\left.\begin{aligned}
T_{R,O2} &= \frac{T_m}{T_{cr,O2}} = \frac{288\text{ K}}{154.8\text{ K}} = 1.860\\
P_{R,O2} &= \frac{P_m}{P_{cr,O2}} = \frac{8\text{ MPa}}{5.08\text{ MPa}} = 1.575
\end{aligned}\right\} Z_{O2} = 0.95$$

$$\left.\begin{aligned}
T_{R,N2} &= \frac{288\text{ K}}{126.2\text{ K}} = 2.282\\
P_{R,N2} &= \frac{8\text{ MPa}}{3.39\text{ MPa}} = 2.360
\end{aligned}\right\} Z_{N2} = 0.99$$

$$\left.\begin{aligned}
T_{R,CO2} &= \frac{288\text{ K}}{304.2\text{ K}} = 0.947\\
P_{R,CO2} &= \frac{8\text{ MPa}}{7.39\text{ MPa}} = 1.083
\end{aligned}\right\} Z_{CO2} = 0.199$$

$$\left.\begin{aligned}
T_{R,CH4} &= \frac{288\text{ K}}{191.1\text{ K}} = 1.507\\
P_{R,CH4} &= \frac{8\text{ MPa}}{4.64\text{ MPa}} = 1.724
\end{aligned}\right\} Z_{CH4} = 0.85$$

and

$$\begin{aligned}
Z_m &= \sum y_i Z_i = y_{O2}Z_{O2} + y_{O2}Z_{O2} + y_{CO2}Z_{CO2} + y_{CH4}Z_{CH4}\\
&= (0.30)(0.95) + (0.40)(0.99) + (0.10)(0.199) + (0.20)(0.85) = 0.8709
\end{aligned}$$

Then,

$$v = \frac{Z_m RT}{P} = \frac{(0.8709)(0.2927\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(288\ \text{K})}{8000\ \text{kPa}} = 0.009178\ \text{m}^3/\text{kg}$$

$$\dot{V} = \mathbf{0.0009425\ m^3/s}$$

$$\dot{m} = \frac{\dot{V}}{v} = \frac{0.0009425\ \text{m}^3/\text{s}}{0.009178\ \text{m}^3/\text{kg}} = \mathbf{0.10269\ kg/s}$$

(*c*) To use Kay's rule, we need to determine the pseudo-critical temperature and pseudo-critical pressure of the mixture using the critical point properties of mixture gases.

$$T'_{\text{cr},m} = \sum y_i T_{\text{cr},i} = y_{\text{O2}} T_{\text{cr,O2}} + y_{\text{N2}} T_{\text{cr,N2}} + y_{\text{CO2}} T_{\text{cr,CO2}} + y_{\text{CH4}} T_{\text{cr,CH4}}$$
$$= (0.30)(154.8\ \text{K}) + (0.40)(126.2\ \text{K}) + (0.10)(304.2\ \text{K}) + (0.20)(191.1\ \text{K}) = 165.6\ \text{K}$$

$$P'_{\text{cr},m} = \sum y_i P_{\text{cr},i} = y_{\text{O2}} P_{\text{cr,O2}} + + y_{\text{N2}} P_{\text{cr,N2}} + y_{\text{CO2}} P_{\text{cr,CO2}} + y_{\text{CH4}} P_{\text{cr,CH4}}$$
$$= (0.30)(5.08\ \text{MPa}) + (0.40)(3.39\ \text{MPa}) + (0.10)(7.39\ \text{MPa}) + (0.20)(4.64\ \text{MPa}) = 4.547\ \text{MPa}$$

and

$$\left.\begin{aligned} T_R &= \frac{T_m}{T'_{\text{cr},m}} = \frac{288\ \text{K}}{165.6\ \text{K}} = 1.739 \\ P_R &= \frac{P_m}{P'_{\text{cr},m}} = \frac{8\ \text{MPa}}{4.547\ \text{MPa}} = 1.759 \end{aligned}\right\} Z_m = 0.92 \qquad \text{(Fig. A-15)}$$

Then,

$$v = \frac{Z_m RT}{P} = \frac{(0.92)(0.2927\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(288\ \text{K})}{8000\ \text{kPa}} = 0.009694\ \text{m}^3/\text{kg}$$

$$\dot{V} = \mathbf{0.0009425\ m^3/s}$$

$$\dot{m} = \frac{\dot{V}}{v} = \frac{0.0009425\ \text{m}^3/\text{s}}{0.09694\ \text{m}^3/\text{kg}} = \mathbf{0.009723\ kg/s}$$