13-59 The mass fractions of components of a gas mixture are given. This mixture is compressed in a reversible, isothermal, steady-flow compressor. The work and heat transfer for this compression per unit mass of the mixture are to be determined.

Assumptions All gases will be modeled as ideal gases with constant specific heats.

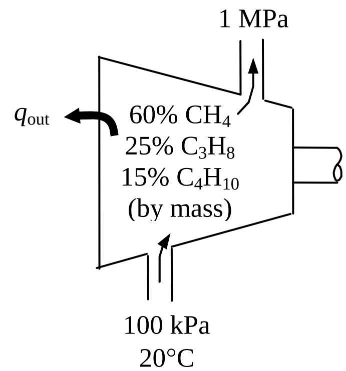
Properties The molar masses of CH_4, C_3H_8, and C_4H_{10} are 16.0, 44.0, and 58.0 kg/kmol, respectively (Table A-1).

Analysis The mole numbers of each component are

$$N_{CH4} = \frac{m_{CH4}}{M_{CH4}} = \frac{60 \text{ kg}}{16 \text{ kg/kmol}} = 3.75 \text{ kmol}$$

$$N_{C3H8} = \frac{m_{C3H8}}{M_{C3H8}} = \frac{25 \text{ kg}}{44 \text{ kg/kmol}} = 0.5682 \text{ kmol}$$

$$N_{C4H10} = \frac{m_{C4H10}}{M_{C4H10}} = \frac{15 \text{ kg}}{58 \text{ kg/kmol}} = 0.2586 \text{ kmol}$$



The mole number of the mixture is

$$N_m = N_{CH4} + N_{C3H8} + N_{C4H10}$$
$$= 3.75 + 0.5682 + 0.2586 = 4.5768 \text{ kmol}$$

The apparent molecular weight of the mixture is

$$M_m = \frac{m_m}{N_m} = \frac{100 \text{ kg}}{4.5768 \text{ kmol}} = 21.85 \text{ kg/kmol}$$

The apparent gas constant of the mixture is

$$R = \frac{R_u}{M_m} = \frac{8.314 \text{ kJ/kmol}\cdot\text{K}}{21.85 \text{ kg/kmol}} = 0.3805 \text{ kJ/kg}\cdot\text{K}$$

For a reversible, isothermal process, the work input is

$$w_{in} = RT\ln\left(\frac{P_2}{P_1}\right) = (0.3805 \text{ kJ/kg}\cdot\text{K})(293 \text{ K})\ln\left(\frac{1000 \text{ kPa}}{100 \text{ kPa}}\right) = \mathbf{257\ kJ/kg}$$

An energy balance on the control volume gives

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{m}h_1 + \dot{W}_{in} = \dot{m}h_2 + \dot{Q}_{out}$$
$$\dot{W}_{in} - \dot{Q}_{out} = \dot{m}(h_2 - h_1)$$
$$w_{in} - q_{out} = c_p(T_2 - T_1) = 0 \quad \text{since} \quad T_2 = T_1$$
$$w_{in} = q_{out}$$

That is,

$$q_{out} = w_{in} = \mathbf{257\ kJ/kg}$$