13-60 The volume fractions of components of a gas mixture during the expansion process of the ideal Otto cycle are given. The thermal efficiency of this cycle is to be determined.

Assumptions All gases will be modeled as ideal gases with constant specific heats.

Properties The molar masses of N_2, O_2, H_2O, and CO_2 are 28.0, 32.0, 18.0, and 44.0 kg/kmol, respectively (Table A-1). The constant-pressure specific heats of these gases at room temperature are 1.039, 0.918, 1.8723, and 0.846 kJ/kg·K, respectively. The air properties at room temperature are c_p = 1.005 kJ/kg·K, c_v = 0.718 kJ/kg·K, k = 1.4 (Table A-2a).

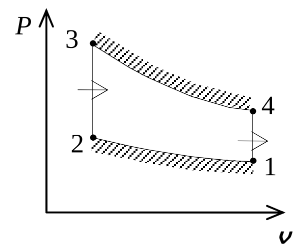
Analysis We consider 100 kmol of this mixture. Noting that volume fractions are equal to the mole fractions, mass of each component are

$$m_{\text{N2}} = N_{\text{N2}} M_{\text{N2}} = (30\text{ kmol})(28\text{ kg/kmol}) = 840\text{ kg}$$
$$m_{\text{O2}} = N_{\text{O2}} M_{\text{O2}} = (10\text{ kmol})(32\text{ kg/kmol}) = 320\text{ kg}$$
$$m_{\text{H2O}} = N_{\text{H2O}} M_{\text{H2O}} = (35\text{ kmol})(18\text{ kg/kmol}) = 630\text{ kg}$$
$$m_{\text{CO2}} = N_{\text{CO2}} M_{\text{CO2}} = (25\text{ kmol})(44\text{ kg/kmol}) = 1100\text{ kg}$$

30% N_2
10% O_2
35% H_2O
25% CO_2
(by volume)

The total mass is

$$\begin{aligned} m_m &= m_{\text{N2}} + m_{\text{O2}} + m_{\text{H2O}} + m_{\text{CO2}} \\ &= 840 + 320 + 630 + 1100 \\ &= 2890\text{ kg} \end{aligned}$$



Then the mass fractions are

$$\text{mf}_{\text{N2}} = \frac{m_{\text{N2}}}{m_m} = \frac{840\text{ kg}}{2890\text{ kg}} = 0.2907$$
$$\text{mf}_{\text{O2}} = \frac{m_{\text{O2}}}{m_m} = \frac{320\text{ kg}}{2890\text{ kg}} = 0.1107$$
$$\text{mf}_{\text{H2O}} = \frac{m_{\text{H2O}}}{m_m} = \frac{630\text{ kg}}{2890\text{ kg}} = 0.2180$$
$$\text{mf}_{\text{CO2}} = \frac{m_{\text{CO2}}}{m_m} = \frac{1100\text{ kg}}{2890\text{ kg}} = 0.3806$$

The constant-pressure specific heat of the mixture is determined from

$$\begin{aligned} c_p &= \text{mf}_{\text{N2}} c_{p,\text{N2}} + \text{mf}_{\text{O2}} c_{p,\text{O2}} + \text{mf}_{\text{H2O}} c_{p,\text{H2O}} + \text{mf}_{\text{CO2}} c_{p,\text{CO2}} \\ &= 0.2907 \times 1.039 + 0.1107 \times 0.918 + 0.2180 \times 1.8723 + 0.3806 \times 0.846 \\ &= 1.134\text{ kJ/kg}\cdot\text{K} \end{aligned}$$

The apparent molecular weight of the mixture is

$$M_m = \frac{m_m}{N_m} = \frac{2890\text{ kg}}{100\text{ kmol}} = 28.90\text{ kg/kmol}$$

The apparent gas constant of the mixture is

$$R = \frac{R_u}{M_m} = \frac{8.314\text{ kJ/kmol}\cdot\text{K}}{28.90\text{ kg/kmol}} = 0.2877\text{ kJ/kg}\cdot\text{K}$$

Then the constant-volume specific heat is

$$c_v = c_p - R = 1.134 - 0.2877 = 0.846\text{ kJ/kg}\cdot\text{K}$$

The specific heat ratio is

$$k = \frac{c_p}{c_v} = \frac{1.134}{0.846} = 1.340$$

The average of the air properties at room temperature and combustion gas properties are

$$c_{p,\text{avg}} = 0.5(1.134 + 1.005) = 1.070 \text{ kJ/kg}\cdot\text{K}$$
$$c_{v,\text{avg}} = 0.5(0.846 + 0.718) = 0.782 \text{ kJ/kg}\cdot\text{K}$$
$$k_{\text{avg}} = 0.5(1.34 + 1.4) = 1.37$$

These average properties will be used for heat addition and rejection processes. For compression, the air properties at room temperature and during expansion, the mixture properties will be used. During the compression process,

$$T_2 = T_1 r^{k-1} = (288\text{ K})(8)^{0.4} = 662\text{ K}$$

During the heat addition process,

$$q_{\text{in}} = c_{v,\text{avg}}(T_3 - T_2) = (0.782 \text{ kJ/kg}\cdot\text{K})(1373 - 662)\text{ K} = 556 \text{ kJ/kg}$$

During the expansion process,

$$T_4 = T_3\left(\frac{1}{r}\right)^{k-1} = (1373\text{ K})\left(\frac{1}{8}\right)^{0.37} = 636.1\text{ K}$$

During the heat rejection process,

$$q_{\text{out}} = c_{v,\text{avg}}(T_4 - T_1) = (0.782 \text{ kJ/kg}\cdot\text{K})(636.1 - 288)\text{ K} = 272.2 \text{ kJ/kg}$$

The thermal efficiency of the cycle is then

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{272.2 \text{ kJ/kg}}{556 \text{ kJ/kg}} = \mathbf{0.511}$$