13-76 Two mass streams of two different ideal gases are mixed in a steady-flow chamber while receiving energy by heat transfer from the surroundings. Expressions for the final temperature and the exit volume flow rate are to be obtained and two special cases are to be evaluated.

Assumptions Kinetic and potential energy changes are negligible.

Analysis (a) Mass and Energy Balances for the mixing process:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

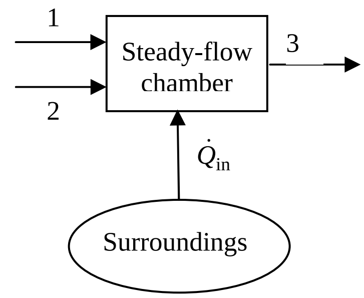
$$\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{Q}_{in} = \dot{m}_3 h_3$$

$$h = C_P T$$

$$\dot{m}_1 C_{P,1} T_1 + \dot{m}_2 C_{P,2} T_2 + \dot{Q}_{in} = \dot{m}_3 C_{P,m} T_3$$

$$C_{P,m} = \frac{\dot{m}_1}{\dot{m}_3} C_{P,1} + \frac{\dot{m}_2}{\dot{m}_3} C_{P,2}$$

$$T_3 = \frac{\dot{m}_1 C_{P,1}}{\dot{m}_3 C_{P,m}} T_1 + \frac{\dot{m}_2 C_{P,2}}{\dot{m}_3 C_{P,m}} T_2 + \frac{\dot{Q}_{in}}{\dot{m}_3 C_{P,m}}$$



(b) The expression for the exit volume flow rate is obtained as follows:

$$\dot{V}_3 = \dot{m}_3 v_3 = \dot{m}_3 \frac{R_3 T_3}{P_3}$$

$$\dot{V}_3 = \frac{\dot{m}_3 R_3}{P_3}\left[\frac{\dot{m}_1 C_{P,1}}{\dot{m}_3 C_{P,m}} T_1 + \frac{\dot{m}_2 C_{P,2}}{\dot{m}_3 C_{P,m}} T_2 + \frac{\dot{Q}_{in}}{\dot{m}_3 C_{P,m}}\right]$$

$$\dot{V}_3 = \frac{C_{P,1} R_3}{C_{P,m} R_1}\frac{\dot{m}_1 R_1 T_1}{P_3} + \frac{C_{P,2} R_3}{C_{P,m} R_2}\frac{\dot{m}_2 R_2 T_2}{P_3} + \frac{R_3 \dot{Q}_{in}}{P_3 C_{P,m}}$$

$$P_3 = P_1 = P_2$$

$$\dot{V}_3 = \frac{C_{P,1} R_3}{C_{P,m} R_1}\dot{V}_1 + \frac{C_{P,2} R_3}{C_{P,m} R_2}\dot{V}_2 + \frac{R_3 \dot{Q}_{in}}{P_3 C_{P,m}}$$

$$R = \frac{R_u}{M}, \quad \frac{R_3}{R_1} = \frac{R_u}{M_3}\frac{M_1}{R_u} = \frac{M_1}{M_3}, \quad \frac{R_3}{R_2} = \frac{M_2}{M_3}$$

$$\dot{V}_3 = \frac{C_{P,1} M_1}{C_{P,m} M_3}\dot{V}_1 + \frac{C_{P,2} M_2}{C_{P,m} M_3}\dot{V}_2 + \frac{R_u \dot{Q}_{in}}{P_3 M_3 C_{P,m}}$$

The mixture molar mass M_3 is found as follows:

$$M_3 = \sum y_i M_i, \quad y_i = \frac{m_{fi}/M_i}{\sum m_{fi}/M_i}, \quad m_{fi} = \frac{\dot{m}_i}{\sum \dot{m}_i}$$

(c) For adiabatic mixing $\dot{Q}_{in}$ is zero, and the mixture volume flow rate becomes

$$\dot{V}_3 = \frac{C_{P,1} M_1}{C_{P,m} M_3}\dot{V}_1 + \frac{C_{P,2} M_2}{C_{P,m} M_3}\dot{V}_2$$

(d) When adiabatically mixing the same two ideal gases, the mixture volume flow rate becomes

$$M_3 = M_1 = M_2$$

$$C_{P,3} = C_{P,1} = C_{P,2}$$

$$\dot{V}_3 = \dot{V}_1 + \dot{V}_2$$