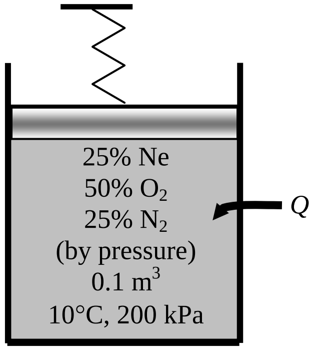
13-96 A mixture of gases is placed in a spring-loaded piston-cylinder device. The device is now heated until the pressure rises to a specified value. The total work and heat transfer for this process are to be determined.

Properties The molar masses of Ne, O_2, and N_2 are 20.18, 32.0, 28.0 kg/kmol, respectively and the gas constants are 0.4119, 0.2598, and 0.2968 kJ/kg·K, respectively (Table A-1). The constant-volume specific heats are 0.6179, 0.658, and 0.743 kJ/kg·K, respectively (Table A-2a).



Analysis The total pressure is 200 kPa and the partial pressures are

$$P_{\text{Ne}} = y_{\text{Ne}} P_m = (0.25)(200\ \text{kPa}) = 50\ \text{kPa}$$

$$P_{\text{O2}} = y_{\text{O2}} P_m = (0.50)(200\ \text{kPa}) = 100\ \text{kPa}$$

$$P_{\text{N2}} = y_{\text{N2}} P_m = (0.25)(200\ \text{kPa}) = 50\ \text{kPa}$$

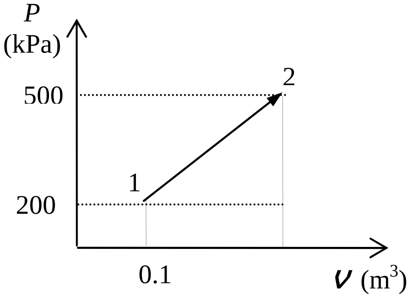
The mass of each constituent for a volume of 0.1 m^3 and a temperature of 10°C are

$$m_{\text{Ne}} = \frac{P_{\text{Ne}} \mathbf{V}_m}{R_{\text{Ne}} T} = \frac{(50\ \text{kPa})(0.1\ \text{m}^3)}{(0.4119\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(283\ \text{K})} = 0.04289\ \text{kg}$$

$$m_{\text{O2}} = \frac{P_{\text{O2}} \mathbf{V}_m}{R_{\text{O2}} T} = \frac{(100\ \text{kPa})(0.1\ \text{m}^3)}{(0.2598\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(283\ \text{K})} = 0.1360\ \text{kg}$$

$$m_{\text{N2}} = \frac{P_{\text{N2}} \mathbf{V}_m}{R_{\text{N2}} T} = \frac{(50\ \text{kPa})(0.1\ \text{m}^3)}{(0.2968\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(283\ \text{K})} = 0.05953\ \text{kg}$$

$$m_{\text{total}} = 0.04289 + 0.1360 + 0.05953 = 0.2384\ \text{kg}$$



The mass fractions are

$$\text{mf}_{\text{Ne}} = \frac{m_{\text{Ne}}}{m_m} = \frac{0.04289\ \text{kg}}{0.2384\ \text{kg}} = 0.1799$$

$$\text{mf}_{\text{O2}} = \frac{m_{\text{O2}}}{m_m} = \frac{0.1360\ \text{kg}}{0.2384\ \text{kg}} = 0.5705$$

$$\text{mf}_{\text{N2}} = \frac{m_{\text{N2}}}{m_m} = \frac{0.05953\ \text{kg}}{0.2384\ \text{kg}} = 0.2497$$

The constant-volume specific heat of the mixture is determined from

$$\begin{aligned} c_v &= \text{mf}_{\text{Ne}} c_{v,\text{Ne}} + \text{mf}_{\text{O2}} c_{v,\text{O2}} + \text{mf}_{\text{N2}} c_{v,\text{N2}} \\ &= 0.1799 \times 0.6179 + 0.5705 \times 0.658 + 0.2497 \times 0.743 = 0.672\ \text{kJ/kg}\cdot\text{K} \end{aligned}$$

The moles are

$$N_{\text{Ne}} = \frac{m_{\text{Ne}}}{M_{\text{Ne}}} = \frac{0.04289\ \text{kg}}{20.18\ \text{kg/kmol}} = 0.002126\ \text{kmol}$$

$$N_{\text{O2}} = \frac{m_{\text{O2}}}{M_{\text{O2}}} = \frac{0.1360\ \text{kg}}{32\ \text{kg/kmol}} = 0.00425\ \text{kmol}$$

$$N_{\text{N2}} = \frac{m_{\text{N2}}}{M_{\text{N2}}} = \frac{0.05953\ \text{kg}}{28\ \text{kg/kmol}} = 0.002126\ \text{kmol}$$

$$N_{\text{m}} = N_{\text{Ne}} + N_{\text{O2}} + N_{\text{N2}} = 0.008502\ \text{kmol}$$

Then the apparent molecular weight of the mixture becomes

$$M_m = \frac{m_m}{N_m} = \frac{0.2384\ \text{kg}}{0.008502\ \text{kmol}} = 28.04\ \text{kg/kmol}$$

The apparent gas constant of the mixture is

$$R = \frac{R_u}{M_m} = \frac{8.314\ \text{kJ/kmol}\cdot\text{K}}{28.05\ \text{kg/kmol}} = 0.2964\ \text{kJ/kg}\cdot\text{K}$$

The mass contained in the system is

$$m = \frac{P_1 \mathbf{V}_1}{R T_1} = \frac{(200\ \text{kPa})(0.1\ \text{m}^3)}{(0.2964\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(283\ \text{K})} = 0.2384\ \text{kg}$$

Noting that the pressure changes linearly with volume, the final volume is determined by linear interpolation to be

$$\frac{500-200}{1000-200}=\frac{\mathcal{V}_2-0.1}{1.0-0.1}\longrightarrow \mathcal{V}_2=0.4375\ \text{m}^3$$

The final temperature is

$$T_2=\frac{P_2\mathcal{V}_2}{mR}=\frac{(500\ \text{kPa})(0.4375\ \text{m}^3)}{(0.2384\ \text{kg})(0.2964\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})}=3096\ \text{K}$$

The work done during this process is

$$W_{\text{out}}=\frac{P_1+P_2}{2}(\mathcal{V}_2-\mathcal{V}_1)=\frac{(500+200)\ \text{kPa}}{2}(0.4375-0.1)\ \text{m}^3=\mathbf{118\ kJ}$$

An energy balance on the system gives

$$Q_{\text{in}}=W_{\text{out}}+mc_{\mathcal{v}}(T_2-T_1)=118+(0.2384\ \text{kg})(0.672\ \text{kJ/kg}\cdot\text{K})(3096-283)\ \text{K}=\mathbf{569\ kJ}$$