8-129 A system consisting of a compressor, a storage tank, and a turbine as shown in the figure is considered. The change in the exergy of the air in the tank and the work required to compress the air as the tank was being filled are to be determined.

Assumptions **1** Changes in the kinetic and potential energies are negligible. **4** Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are R = 0.287 kPa·m^3/kg·K, c_p = 1.005 kJ/kg·K, c_v = 0.718 kJ/kg·K, k = 1.4 (Table A-2a).

Analysis The initial mass of air in the tank is

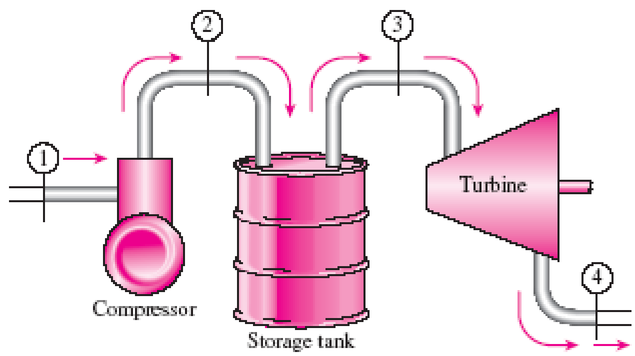
$$m_{\text{initial}} = \frac{P_{\text{initial}}\boldsymbol{V}}{RT_{\text{initial}}} = \frac{(100\text{ kPa})(5\times10^5\text{ m}^3)}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{ K})} = 0.5946\times10^6\text{ kg}$$

and the final mass in the tank is

$$m_{\text{final}} = \frac{P_{\text{final}}\boldsymbol{V}}{RT_{\text{final}}} = \frac{(600\text{ kPa})(5\times10^5\text{ m}^3)}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\text{ K})} = 3.568\times10^6\text{ kg}$$

Since the compressor operates as an isentropic device,

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$



The conservation of mass applied to the tank gives

$$\frac{dm}{dt} = \dot{m}_{\text{in}}$$

while the first law gives

$$\dot{Q} = \frac{d(mu)}{dt} - h\frac{dm}{dt}$$

Employing the ideal gas equation of state and using constant specific heats, expands this result to

$$\dot{Q} = \frac{\boldsymbol{V}c_v}{R}\frac{dP}{dt} - c_pT_2\frac{\boldsymbol{V}}{RT}\frac{dP}{dt}$$

Using the temperature relation across the compressor and multiplying by dt puts this result in the form

$$\dot{Q}dt = \frac{\boldsymbol{V}c_v}{R}dP - c_pT_1\left(\frac{P}{P_1}\right)^{(k-1)/k}\frac{\boldsymbol{V}}{RT}dP$$

When this integrated, it yields (i and f stand for initial and final states)

$$Q = \frac{\boldsymbol{V}c_v}{R}(P_f - P_i) - \frac{k}{2k-1}\frac{c_p\boldsymbol{V}}{R}\left[P_f\left(\frac{P_f}{P_i}\right)^{(k-1)/k} - P_i\right]$$

$$= \frac{(5\times10^5)(0.718)}{0.287}(600-100) - \frac{1.4}{2(1.4)-1}\frac{(1.005)(5\times10^5)}{0.287}\left[600\left(\frac{600}{100}\right)^{0.4/1.4} - 100\right]$$

$$= -6.017\times10^8\text{ kJ}$$

The negative result show that heat is transferred from the tank. Applying the first law to the tank and compressor gives

$$(\dot{Q} - \dot{W}_{\text{out}})dt = d(mu) - h_1dm$$

which integrates to

$$Q - W_{\text{out}} = (m_fu_f - m_iu_i) - h_1(m_f - m_i)$$

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Upon rearrangement,

$$
\begin{aligned}
W_{\text{out}} &= Q + (c_p - c_v)T(m_f - m_i) \\
&= -6.017\times10^8 + (1.005 - 0.718)(293)[(3.568 - 0.5946)\times10^6] \\
&= \mathbf{-3.516\times10^8\ kJ}
\end{aligned}
$$

The negative sign shows that work is done on the compressor. When the combined first and second laws is reduced to fit the compressor and tank system and the mass balance incorporated, the result is

$$
\dot{W}_{\text{rev}} = \dot{Q}\left(1 - \frac{T_0}{T_R}\right) - \frac{d(U - T_0 S)}{dt} + (h - T_0 s)\frac{dm}{dt}
$$

which when integrated over the process becomes

$$
\begin{aligned}
W_{\text{rev}} &= Q\left(1 - \frac{T_0}{T_R}\right) + m_i\left[(u_i - h_1) - T_0(s_i - s_1)\right] - m_f\left[(u_f - h_1) - T_0(s_f - s_1)\right] \\
&= -6.017\times10^8\left(1 - \frac{293}{293}\right) + 0.5946\times10^6\left[(0.718 - 1.005)293\right] \\
&\quad - 3.568\times10^6\left[(0.718 - 1.005)293\right]\left[(0.718 - 1.005)293 + 293(0.287)\ln\frac{600}{100}\right] \\
&= \mathbf{-2.876\times10^8\ kJ}
\end{aligned}
$$

This is the exergy change of the air stored in the tank.

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