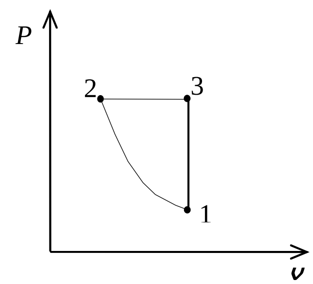
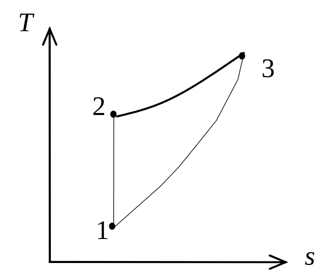
9-24 An air-standard cycle executed in a piston-cylinder system is composed of three specified processes. The cycle is to be sketcehed on the *P*-v and *T*-*s* diagrams; the heat and work interactions and the thermal efficiency of the cycle are to be determined; and an expression for thermal efficiency as functions of compression ratio and specific heat ratio is to be obtained.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** Air is an ideal gas with constant specific heats.

Properties The properties of air are given as R = 0.3 kJ/kg·K and c_v = 0.3 kJ/kg·K.

Analysis (a) The *P*-v and *T*-*s* diagrams of the cycle are shown in the figures.





(b) Noting that

$$c_p = c_v + R = 0.7 + 0.3 = 1.0\ \text{kJ/kg}\cdot\text{K}$$

$$k = \frac{c_p}{c_v} = \frac{1.0}{0.7} = 1.429$$

Process 1-2: Isentropic compression

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = T_1 r^{k-1} = (293\ \text{K})(5)^{0.429} = 584.4\ \text{K}$$

$$w_{1-2,\text{in}} = c_v(T_2 - T_1) = (0.7\ \text{kJ/kg}\cdot\text{K})(584.4 - 293)\ \text{K} = \mathbf{204.0\ kJ/kg}$$

$$q_{1-2} = \mathbf{0}$$

From ideal gas relation,

$$\frac{T_3}{T_2} = \frac{v_3}{v_2} = \frac{v_1}{v_2} = r \longrightarrow T_3 = (584.4)(5) = 2922$$

Process 2-3: Constant pressure heat addition

$$w_{2-3,\text{out}} = \int_2^3 P\,dv = P_2(v_3 - v_2) = R(T_3 - T_2)$$

$$= (0.3\ \text{kJ/kg}\cdot\text{K})(2922 - 584.4)\ \text{K} = \mathbf{701.3\ kJ/kg}$$

$$q_{2-3,\text{in}} = w_{2-3,out} + \Delta u_{2-3} = \Delta h_{2-3}$$

$$= c_p(T_3 - T_2) = (1\ \text{kJ/kg}\cdot\text{K})(2922 - 584.4)\ \text{K} = \mathbf{2338\ kJ/kg}$$

Process 3-1: Constant volume heat rejection

$$q_{3-1,\text{out}} = \Delta u_{1-3} = c_v(T_3 - T_1) = (0.7\ \text{kJ/kg}\cdot\text{K})(2922 - 293)\ \text{K} = \mathbf{1840.3\ kJ/kg}$$

$$w_{3-1} = \mathbf{0}$$

(c) Net work is

$$w_{\text{net}} = w_{2-3,\text{out}} - w_{1-2,\text{in}} = 701.3 - 204.0 = 497.3\ \text{kJ/kg}\cdot\text{K}$$

The thermal efficiency is then

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{497.3\ \text{kJ}}{2338\ \text{kJ}} = 0.213 = \mathbf{21.3\%}$$

(d) The expression for the cycle thermal efficiency is obtained as follows:

$$
\begin{aligned}
\eta_{\text{th}} &= \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{w_{2-3,\text{out}} - w_{1-2,\text{in}}}{q_{\text{in}}} \\
&= \frac{R(T_3 - T_2) - c_v(T_2 - T_1)}{c_p(T_3 - T_2)} \\
&= \frac{R}{c_p} - \frac{c_v(T_1 r^{k-1} - T_1)}{c_p(rT_1 r^{k-1} - T_1 r^{k-1})} \\
&= \frac{R}{c_p} - \frac{c_v T_1 r^{k-1}\left(1 - \dfrac{T_1}{T_1 r^{k-1}}\right)}{c_p T_1 r^{k-1}(r-1)} \\
&= \frac{R}{c_p} - \frac{1}{k(r-1)}\left(1 - \frac{T_1}{T_1 r^{k-1}}\right) \\
&= \frac{R}{c_p} - \frac{1}{k(r-1)}\left(1 - \frac{1}{r^{k-1}}\right) \\
&= \left(1 - \frac{1}{k}\right) - \frac{1}{k(r-1)}\left(1 - \frac{1}{r^{k-1}}\right)
\end{aligned}
$$

since

$$
\frac{R}{c_p} = \frac{c_p - c_v}{c_p} = 1 - \frac{c_v}{c_p} = 1 - \frac{1}{k}
$$