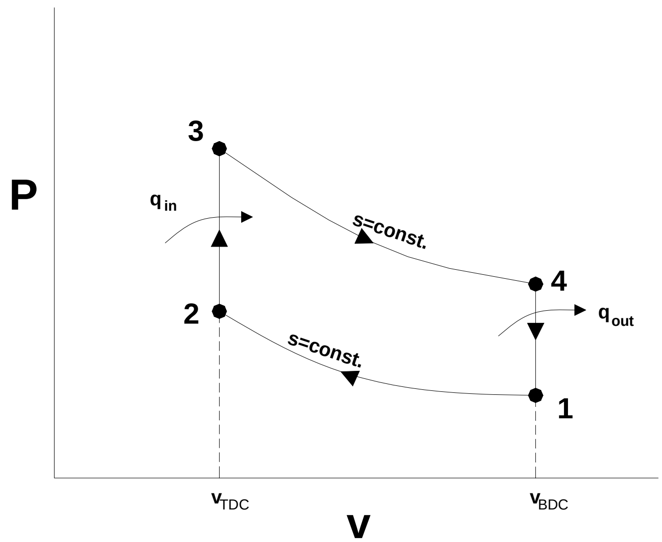
**ENSC 461 Tutorial, Week#4 – IC Engines**

The compression ratio in an air-standard Otto cycle is 10. At the beginning of the compression stroke the pressure is 0.1 MPa and the temperature is 15°C. The heat transfer to the air per cycle is 1800 kJ/kg. Determine:

a) The pressure and temperature at the end of each process of the cycle,
b) the net work output,
c) the thermal efficiency,
d) the mean effective pressure,
e) and the irreversibility if this cycle was executed with a heat source temperature of 3500 K and a heat sink temperature of 250 K

**Step 1: Draw a diagram to represent the system**

A process diagram is drawn to visualize the processes occurring during the cycle.




**Step 2: Write out what is required to solve for**

a) The pressure and temperature at the end of each process of the cycle
b) the net work output
c) the thermal efficiency
d) the mean effective pressure
e) the cycle irreversibility if this cycle was executed with a heat source temperature of 3500 K and a heat sink temperature of 250 K

**Step 3: Property table**

| | T [K] | P [kPa] | v [m³/kg] |
|---|---|---|---|
| 1 | 288 | 100 | |
| 2 | | | |
| 3 | | | $v_2$ |
| 4 | | | $v_1$ |

**Step 4: Assumptions**

1) Δke, Δpe ≈ 0
2) cold-air-standard assumption are applicable

**Step 5: Solve**

**Part a)**

$P_2$ and $T_2$ will be determined first. Referring to the process diagram, state 1 to 2 is an isentropic compression process. Therefore the ideal gas relations for isentropic processes can be used. The temperature ratio of the two states is related to the specific volume ratio through k as shown in Eq1.

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} \qquad \text{(Eq1)}$$

Noting that the ratio $v_1/v_2$ (equivalent to $V_{max}/V_{min}$) is the compression ratio, r, and the value of k for air is 1.4, the temperature at state 2 can be determined.

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (288)(10)^{1.4-1} = 723.4[K] \qquad \mathbf{T_2}$$

Again, since the process from state 1 to 2 is isentropic, the ideal gas relation relating the specific volume and pressure ratios through k can be used as shown in Eq2.

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{k} \rightarrow P_2 = P_1\left(\frac{v_1}{v_2}\right)^{k} \qquad \text{(Eq2)}$$

Noting that $v_1/v_2$ is equal to the compression ratio, the pressure at state 2 can be determined as shown below.

$$P_2 = P_1\left(\frac{v_1}{v_2}\right)^k = 100[kPa](10)^{1.4} = 2511.9[kPa] \qquad \mathbf{P_2}$$

Performing an energy balance for the constant volume heat addition process (2 → 3), Eq3 is obtained.

$$q_{in} = u_3 - u_2 \qquad \text{(Eq3)}$$

For an ideal gas the internal energy is a function of temperature only. Using the assumption of constant specific heats evaluated at room temperature, the change in internal energy can be determined using Eq4.

$$u_3 - u_2 = c_v(T_3 - T_2) \qquad \text{(Eq4)}$$

The problem statement gives the value of $q_{in}$ as 1800 kJ/kg. Substituting Eq4 into Eq3 along with the known value of $q_{in}$, the temperature at state 3 can be determined.

$$q_{in} = c_v(T_3 - T_2) \rightarrow T_3 = \frac{q_{in}}{c_v} + T_2 = \frac{1800\left[\frac{kJ}{kg}\right]}{0.718\left[\frac{kJ}{kg \bullet K}\right]} + 723.4K = 3230.4[K] \qquad \mathbf{T_3}$$

Since the process from 2 to 3 is executed over a constant volume, the ideal gas law can be applied separately to both state 3 and state 2 and combined as shown below in Eq5.

$$v_2 = \frac{T_2R}{P_2} = \frac{T_3R}{P_3} = v_3 \rightarrow P_3 = P_2\left(\frac{T_3}{T_2}\right) \qquad \text{(Eq5)}$$

Substituting the known values into Eq5, the pressure at state 3 can be solved for as shown below.

$$P_3 = P_2\left(\frac{T_3}{T_2}\right) = (2511.9[kPa])\left(\frac{3230.4K}{723.4K}\right) = 11216.6[kPa] \qquad \mathbf{P_3}$$

Since the process from 3 to 4 is isentropic, the temperature at state 4 can be determined using the ideal gas relation relating the temperature and specific volume ratios through k as shown in Eq6.

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} \quad \text{(Eq6)}$$

Noting that $v_3/v_4$ is the inverse of the compression ratio, the temperature at state 4 can be determined.

$$T_4 = (3230.4[K])\left(\frac{1}{10}\right)^{1.4-1} = 1286[K] \quad \mathbf{T_4}$$

The pressure can be determined from the isentropic relation for an ideal gas, which relates the pressure and the specific volume ratios through k as shown in Eq7.

$$P_4 = P_3\left(\frac{v_3}{v_4}\right)^{k} \quad \text{(Eq7)}$$

Noting again that $v_3/v_4$ is the inverse of the compression ratio, the pressure at state 4 can be determined.

$$P_4 = P_3\left(\frac{v_3}{v_4}\right)^{k} = 11216.6[kPa]\left(\frac{1}{10}\right)^{1.4} = 445.6[kPa] \quad \mathbf{P_4}$$

**Part b)**

An overall energy balance on the cycle can be used to find an expression for the net work output as shown in Eq8.

$$q_{in} + w_{in} = q_{out} + w_{out} \rightarrow w_{net} = w_{out} - w_{in} = q_{in} - q_{out} \quad \text{(Eq8)}$$

The value of $q_{in}$ is given in the problem statement so the problem reduces to finding the value of $q_{out}$. Performing an energy balance for the process from 4 to 1, $q_{out}$ can be determined from the temperature difference between state 4 and 1 as shown in Eq9.

$$q_{out} = u_4 - u_1 = c_v(T_4 - T_1) \quad \text{(Eq9)}$$

Substituting the known values into Eq9, $q_{out}$ can be determined as shown below.

$$q_{out} = c_v(T_4 - T_1) = \left(0.718\left[\frac{kJ}{kg \bullet K}\right]\right)(1286[K] - 288[K]) = 716.6\left[\frac{kJ}{kg}\right]$$

Using this result with the given $q_{in}$ = 1800 kJ/kg and Eq8, the net work output can be determined as shown below.

$$w_{net} = q_{in} - q_{out} = (1800 - 716.6)\left[\frac{kJ}{kg}\right] = 1083.4\left[\frac{kJ}{kg}\right]$$

**Answer (b)**

**Part c)**

To calculate the thermal efficiency the general expression for efficiency (benefit/cost) can be used.

$$\eta_{th} = \frac{benefit}{\cos t} = \frac{w_{net}}{q_{in}} = \frac{1083.4[kJ]}{1800[kJ]} = 60.2\%$$

**Answer (c)**

The Otto cycle thermal efficiency can also be determined using the equation that makes use of the compression ratio.

$$\eta_{th,Otto} = 1 - \frac{1}{r^{k-1}} = 1 - \frac{1}{10^{0.4}} = 60.2\%$$

**Answer (c)**

**Part d)**

The mean effective pressure (MEP) can be determined using Eq10.

$$MEP = \frac{w_{net}}{v_1 - v_2} \qquad \text{(Eq10)}$$

The value of $w_{net}$ was determined in part b) but the values $v_1$ and $v_2$ are unknown. $v_1$ can be determined by applying the ideal gas law to state 1 as shown below.

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(0.287\left[\frac{kJ}{kg \bullet K}\right]\right)(288[K])}{100[kPa]} = 0.827\left[\frac{m^3}{kg}\right]$$

$v_2$ is related $v_1$ through the compression ration, r, and can be determined as shown below.

$$\frac{v_1}{v_2} = r \rightarrow v_2 = \frac{v_1}{r} = \frac{0.827\left[\frac{m^3}{kg}\right]}{10} = 0.0827\left[\frac{m^3}{kg}\right]$$

Substituting these results into Eq10, the value of the MEP can be determined as shown below.

$$MEP = \frac{w_{net}}{v_1 - v_2} = \frac{1083.4\left[\frac{kJ}{kg}\right]}{(0.827 - 0.827)\left[\frac{m^3}{kg}\right]} = 1456.4[kPa]$$

**Answer (d)**

**Part e)**

The irreversibility of the cycle (exergy destroyed) if the source and sink temperatures were 3500 K and 250 K respectively, can be determined from application of Eq11.

$$x_{destroyed} = T_0 s_{gen} \qquad \text{(Eq11)}$$

The entropy generated during this cycle can be determined by performing an entropy balance over each process as shown in Eq12 - 15.

Since the process from 1 to 2 is isentropic with no heat transfer and occurs in a closed system there will be no entropy generated.

$$s_{gen,1\rightarrow 2} = \Delta s_{sys} + s_{out} - s_{in} = 0 \qquad \text{(Eq12)}$$

Since the process from 2 to 3 occurs over constant volume with heat transfer into the system, there will be entropy generated as shown in Eq13.

$$s_{gen,2\rightarrow 3} = \Delta s_{sys} + s_{out} - s_{in} = (s_3 - s_2) - \frac{q_{in}}{T_{source}} \qquad \text{(Eq13)}$$

Since the process from 3 to 4 is isentropic with no heat transfer and occurs in a closed system there will be no entropy generated.

$$s_{gen,3\rightarrow 4} = \Delta s_{sys} + s_{out} - s_{in} = 0 \qquad \text{(Eq14)}$$

Since the process from 4 to 1 occurs over constant volume with heat transfer out of the system, there will be entropy generated as shown in Eq15.

$$s_{gen,4\to1} = \Delta s_{sys} + s_{out} - s_{in} = (s_1 - s_4) + \frac{q_{out}}{T_{\sin k}} \qquad \text{(Eq15)}$$

The total entropy generated will be the sum of the entropy generated during each process as shown in Eq16.

$$s_{gen} = \left(\frac{q_{out}}{T_{\sin k}} - \frac{q_{in}}{T_{source}}\right) + (s_1 - s_4) + (s_3 - s_2) \qquad \text{(Eq16)}$$

Since the compression and expansion processes are modeled as isentropic $s_4 = s_3$ and $s_2 = s_1$. Therefore Eq16 reduces to Eq17.

$$s_{gen} = \left(\frac{q_{out}}{T_{\sin k}} - \frac{q_{in}}{T_{source}}\right) \qquad \text{(Eq17)}$$

The total entropy generated during the cycle is determined by substituting all of the known parameters into Eq17 as shown below.

$$s_{gen} = \left(\frac{716.6\left[\frac{kJ}{kg}\right]}{250[K]} - \frac{1800\left[\frac{kJ}{kg}\right]}{3500[K]}\right) = 2.352\left[\frac{kJ}{kg \bullet K}\right]$$

Substituting this result into Eq11, the irreversibility of cycle is determined as shown below.

$$x_{destroyed} = T_0 s_{gen} = (298[K])\left(2.352\left[\frac{kJ}{kg \bullet K}\right]\right) = 700.93\left[\frac{kJ}{kg}\right]$$

**Answer (e)**

**Step 5: Concluding Remarks & Discussion**

The pressures and temperatures at the end of each process are summarized in the table below.

| | **T [K]** | **P [kPa]** |
|---|---|---|
| 1 | 288 | 100 |
| 2 | 723.4 | 2511.9 |

| | | |
|---|---|---|
| 3 | 3230.4 | 11216.6 |
| 4 | 1286 | 445.6 |

The net work output was found to be 1083.4 kJ/kg. The thermal efficiency of the cycle was found to be 60.2%. The MEP was determined to be 1456.4 kPa. The irreversibility of the cycle if the source and sink temperatures were 3500 K and 250 K would be 700.93 kJ/kg.