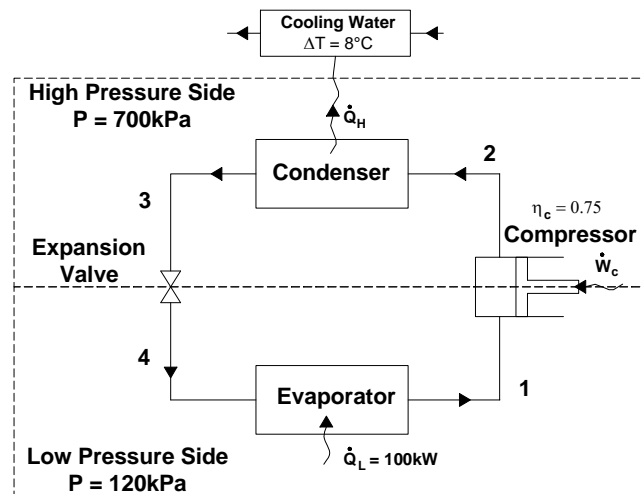


ENSC 461 Tutorial, Week#6 – Refrigeration Cycle

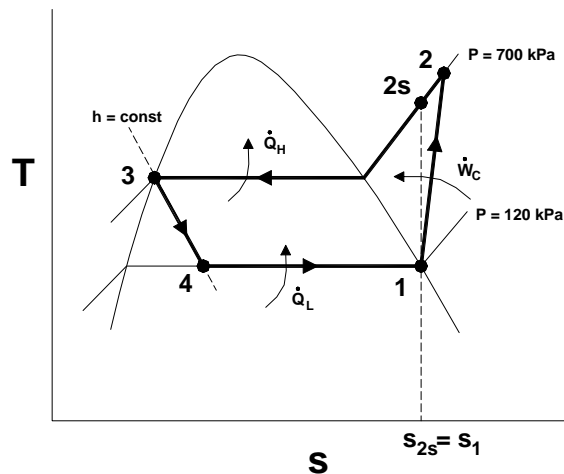
A large refrigeration plant is to be maintained at -15°C , and it requires refrigeration at a rate of 100kW . The condenser of the plant is to be cooled by liquid water, which experiences a temperature rise of 8°C as it flows over the coils of the condenser. The plant uses refrigerant-134a between the pressure limits of 120 and 700kPa . Assuming the compressor has an isentropic efficiency of 75 percent, determine:

- The mass flow rate of the refrigerant,
- the power input to the compressor,
- the mass flow rate of the cooling water, and
- the rate of exergy destruction associated with the compression process (assume $T_0 = 25^{\circ}\text{C}$)

Step 1: Draw a diagram to represent the system



To better visualize what is happening during the cycle we can draw a T-s process diagram.



Step 2: Write out what is required to solve for

- The mass flow rate of the refrigerant,
- the power input to the compressor,
- the mass flow rate of the cooling water, and
- the rate of exergy destruction associated with the compression process (assume $T_0 = 25^\circ\text{C}$)

Step 3: Property table

	P [kPa]	h [kJ/kg]	s [kJ/kg*K]
1 (sat vap)	120		
2	700		
2s	700		
3 (sat liq)	700		
4	120		

Step 4: Assumptions

Assumptions:

- $\Delta ke, \Delta pe \cong 0$
- SSSF
- throttling process is adiabatic
- state 1 is a saturated vapor @ $P = 120\text{kPa}$
- state 3 is a saturated liquid @ $P = 700\text{kPa}$

Step 5: Solve**Part a)**

The mass flow rate of the refrigerant, \dot{m}_{refrig} , can be determined by looking at a device in the cycle that the energy transfer (work or heat transfer), in or out, is known. The heat transfer into the evaporator is given, $\dot{Q}_L = 100$ kW, so \dot{m}_{refrig} can be determined from an energy balance on the evaporator as shown in Eq1.

$$\dot{m}_{refrig} = \frac{\dot{Q}_L}{(h_1 - h_4)} \quad (\text{Eq1})$$

Before the mass flow of refrigerant can be determined, the enthalpies at state 1 and 4 must be determined. Since state 1 is a saturated vapor at a pressure of 120kPa, the enthalpy and entropy can be determined using Table A-12.

State 1 → Table A-12 @ P = 0.12 MPa

$$\rightarrow h_1 = h_{g@P=120\text{kPa}} = 233.86 \text{ kJ/kg}$$

$$\rightarrow s_1 = s_{g@P=120\text{kPa}} = 0.9354 \text{ kJ/kg}\cdot\text{K}$$

Performing an energy balance on the throttling valve using the assumptions that Δke , $\Delta pe \cong 0$ and the process is adiabatic gives $h_4 = h_3$. So h_4 will be known if h_3 is known. State 3 is a saturated liquid at a pressure of 700kPa, therefore h_3 can be determined using Table A-12.

State 3 → Table A-12 @ P = 0.7 MPa

$$h_3 = h_4 = h_{f@P=700\text{kPa}} = 86.78 \text{ kJ/kg}$$

Substituting these values into Eq1 the value of \dot{m}_{refrig} can be determined.

$$\rightarrow \dot{m}_{refrig} = \frac{\dot{Q}_L}{(h_1 - h_4)} = \frac{100 \left[\frac{\text{kJ}}{\text{s}} \right]}{(233.86 - 86.78) \left[\frac{\text{kJ}}{\text{kg}} \right]}$$

$$= \mathbf{0.68 \text{ kg/s}} \quad \mathbf{\text{Answer a)}}$$

Part b)

The power input to the compressor, \dot{W}_c , can be found by performing an energy balance on the compressor as shown in Eq2.

$$\dot{W}_c = \dot{m}_{refrig} (h_2 - h_1) \quad (\text{Eq2})$$

\dot{m}_{refrig} was calculated in part a) but $(h_2 - h_1)$ must still be determined. The problem statement provides an isentropic efficiency for the compressor. Rearranging the definition of the isentropic efficiency, as shown in Eq3, an expression for $(h_2 - h_1)$ in terms of known quantities \dot{m}_{refrig} and h_1 , and h_{2s} , which can be determined, is found.

$$\eta_c = \frac{(h_{2s} - h_1)}{(h_2 - h_1)} \rightarrow (h_2 - h_1) = \frac{(h_{2s} - h_1)}{\eta_c} \quad (\text{Eq3})$$

For state 2s, $s_2 = s_1 = 0.9354 \text{ kJ/kg}\cdot\text{K}$. Using this information with Table A-12, it is found that state 2s is outside of the vapor dome and into the superheated region i.e. $s_2 > s_{g@P=700\text{kPa}}$. Looking in Table A-13 @ P = 0.7 MPa, it is found that s_2 lies between the entropies with corresponding temperatures of 30°C and 40°C. Using $s_2 = 0.9354 \text{ kJ/kg}\cdot\text{K}$ to interpolate

between the entropies at 30°C and 40°C, we can find the enthalpy at the state 2s.

$$\rightarrow \frac{h_{2s} - h_{s=0.9197}}{h_{s=0.9539} - h_{s=0.9197}} = \frac{0.9354 - 0.9197}{0.9539 - 0.9197} \rightarrow h_{2s} = h_{s=0.9197} + 0.459(h_{s=0.9539} - h_{s=0.9197})$$

$$\rightarrow h_{2s} = 265.37 + (0.459)(275.93 - 265.37) = 270.22 \left[\frac{\text{kJ}}{\text{kg}} \right]$$

Note: h_{2s} is what the enthalpy at state 2 would be if the compression process from 1→2 was isentropic i.e. $s_2 = s_1$. Substituting Eq3 into Eq2, the value of \dot{W}_c can be determined.

$$\rightarrow \dot{W}_c = \dot{m}_{\text{refrig}} \frac{(h_{2s} - h_1)}{\eta_c} = (0.68) \left[\frac{\text{kg}}{\text{s}} \right] \frac{(270.22 - 233.86) \left[\frac{\text{kJ}}{\text{kg}} \right]}{0.75}$$

$$= \mathbf{32.96 \text{ kJ/s} \quad \text{Answer b)}$$

Part c)

Part c) asks for the mass flow rate of the cooling water, \dot{m}_{cw} , used in the condenser. The rate of heat transfer out of the condenser must be equal to the rate of heat transfer into the cooling water. The rate of heat transfer out of the condenser, \dot{Q}_H , can be determined by performing an energy balance on the condenser as shown in Eq4.

$$\dot{Q}_H = \dot{m}_{\text{refrig}} (h_2 - h_3) \quad (\text{Eq4})$$

The rate of heat transfer into the cooling water can be expressed as a function of the temperature rise the cooling water experiences using Eq5.

$$\dot{Q}_H = \dot{m}_{cw} c \Delta T \quad (\text{Eq5})$$

Combining Eq4 and Eq5, and rearranging to isolate for the mass flow rate of the cooling water Eq6 is obtained.

$$\dot{Q}_c = \dot{m}_{\text{refrig}} (h_2 - h_3) = \dot{m}_{cw} c \Delta T \rightarrow \dot{m}_{cw} = \frac{\dot{m}_{\text{refrig}} (h_2 - h_3)}{c \Delta T} \quad (\text{Eq6})$$

h_3 and \dot{m}_{refrig} have been previously determined, so only h_2 , c , and ΔT need to be found. Rearranging Eq3, the enthalpy at state 2, h_2 , can be found.

$$\rightarrow h_2 = \frac{(h_{2s} - h_1)}{\eta_c} + h_1 = \frac{(270.22 - 233.86)}{0.75} + 233.86 = 282.34 [kJ/kg]$$

From Table A-3, c for water is given as 4.18 kJ/kg*°C. The problem statement gives the temperature rise in the cooling water as 8°C i.e. $\Delta T = 8^\circ C$. Substituting the values of h_2 , c , ΔT , h_3 and \dot{m}_{refrig} into Eq6 we can determine the mass flow rate of the cooling water.

$$\rightarrow \dot{m}_{cw} = \frac{\dot{m}_{refrig} (h_2 - h_3)}{c \Delta T} = \frac{(0.68) \left[\frac{kg}{s} \right] (282.34 - 86.78) \left[\frac{kJ}{kg} \right]}{(4.18) \left[\frac{kJ}{kg \cdot ^\circ C} \right] (8) [^\circ C]}$$

= 3.98 kg/s Answer c)

Part d)

The rate of exergy destruction can be expressed in terms of the surroundings temperature, T_0 , and the rate of entropy generation, \dot{S}_{GEN} , as shown in Eq7.

$$\dot{X}_{destroyed} = T_0 \dot{S}_{GEN} \tag{Eq7}$$

We can determine the entropy generation term, \dot{S}_{GEN} , by applying an entropy balance on the compressor as shown in Eq8.

$$\dot{S}_{in} - \dot{S}_{out} + \dot{S}_{gen} = \Delta \dot{S}_{system} \tag{Eq8}$$

Since there is no heat transfer in or out of the compressor the only mechanism for entropy transfer into and out of the compressor is by the mass flow. Therefore, \dot{S}_{in} and \dot{S}_{out} are equal to $\dot{m}_{refrig} s_1$ and $\dot{m}_{refrig} s_2$ respectively. Using the steady state assumption, the $\Delta \dot{S}_{system} = 0$ term is zero. Applying the above information to Eq8, Eq9 is obtained.

$$\dot{S}_{GEN} = \dot{m}_{refrig} (s_2 - s_1) \tag{Eq9}$$

Substituting Eq9 into Eq7, Eq10 is obtained.

$$\dot{X}_{destroyed} = T_0 \dot{m}_{refrig} (s_2 - s_1) \quad (\text{Eq10})$$

s_2 can be determined from Table A-13 @ $P = 0.7$ MPa using $h_2 = 282.34$ kJ/kg to interpolate between the entropies corresponding to temperatures of 40°C and 50°C.

$$\rightarrow \frac{s_2 - s_{h=275.93}}{s_{h=286.35} - s_{h=275.93}} = \frac{282.34 - 275.93}{286.35 - 275.93} \rightarrow s_2 = s_{h=275.93} + 0.615(s_{h=286.35} - s_{h=275.93})$$

$$\rightarrow s_2 = 0.9539 + (0.615)(0.9867 - 0.9539) = 0.9741 \left[\frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right]$$

Substituting the known variables, assuming $T_0 = 25^\circ\text{C}$, into Eq10, the rate of exergy destruction during the compression process can be determined.

$$\rightarrow \dot{X}_{destroyed} = (273 + 25)[\text{K}](0.68) \left[\frac{\text{kg}}{\text{s}} \right] (0.9741 - 0.9354) \left[\frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right]$$

$$= \mathbf{7.84 \text{ kJ/s} \quad \text{Answer d)}$$

Step 5: Concluding Remarks & Discussion

The mass flow rate of the refrigerant was found to be 0.68 kg/s. The power input to the compressor was found to be 32.96 kJ/s. The mass flow rate of the cooling water was found to be 3.98 kg/s. The rate of exergy destruction associated with the compression process was found to be 7.84 kJ/s assuming a dead state temperature, T_0 , of 25°C.