

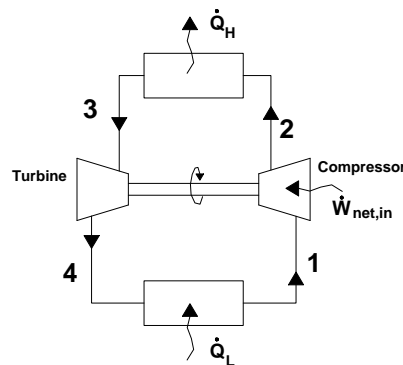
## ENSC 461 Tutorial, Week#8

### Gas Refrigeration Cycles

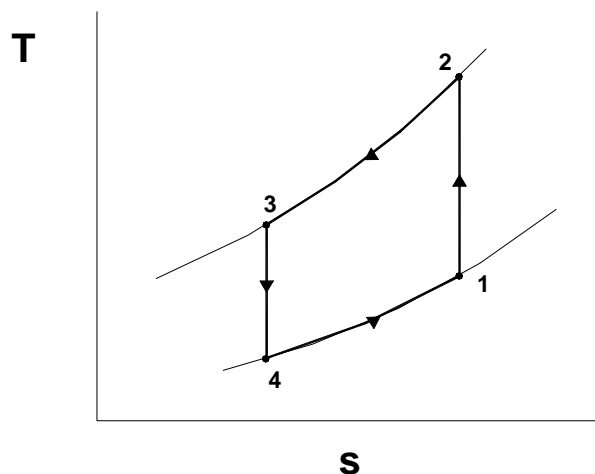
A refrigeration system using air as the working fluid, consists of an ideal Brayton cycle run in reverse with a temperature and pressure at the inlet of the compressor of  $37^\circ\text{C}$  and  $100\text{ kPa}$ . The compressor has a compression ratio of 15:1 and air must be supplied to the conditioned space at  $17^\circ\text{C}$ . The heat load on the system is  $10\text{ kW}$ .

- Calculate the required mass flow rate of air.
- Calculate the required net-work input.
- Calculate the COP for this refrigeration system.
- Calculate the cooling capacity of the system if the turbine were removed and replaced with an adiabatic expansion valve (assume states 1, 2, 3 and the mass flow rate of air remain unchanged)

#### Step 1: Draw a diagram to represent the system



Draw a T-s process diagram to better visualize the processes occurring during the cycle.



**Step 2: Write out what is required to solve for**

- a)  $\dot{m}_{air}$
- b)  $\dot{W}_{net, in}$
- c)  $COP_R$
- d) Using  $COP_R$  is process possible
- e)  $\dot{Q}_L$  if turbine is removed and replaced with an adiabatic expansion valve.

**Step 3: Prepare a property table**

	T(K)	P(kPa)
1	310	100
2		1500
3		1500
4	290	100

**Step 4: State your assumptions**

Assumptions:

- 1)  $\Delta ke, \Delta pe \approx 0$
- 2) cold-air-standard assumptions are applicable
- 3) steady operating conditions
- 4) compressor and turbine are isentropic
- 5) heat exchangers are constant pressure devices

**Step 5: Solve****Part a)**

The mass flow rate of air required can be determined by examining a component in the system where all the energy transfers and state points are known. In this case, the conditioned space (represented by the low temperature heat exchanger) fits these criteria. An energy balance on the conditioned space is performed as shown in Eq1.

$$\dot{Q}_L = \dot{m}_{air}(h_1 - h_4) \quad (\text{Eq1})$$

Isolating Eq1 for the mass flow rate of air and recognizing that for an ideal gas the enthalpy is a function of temperature only, Eq1 can be rewritten as Eq2.

$$\dot{m}_{air} = \frac{\dot{Q}_L}{(h_1 - h_4)} = \frac{\dot{Q}_L}{c_p(T_1 - T_4)} \quad (\text{Eq2})$$

The heat load on the system is given as 10 kW. The temperatures at the inlet and outlet of heat exchanger are given in the problem statement as 290K(17°C) and 310K(37°C) respectively. From Table A-2 for air at room temperature  $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$ . Substituting these values into Eq2, the required mass flow rate of air can be determined as shown below.

$$\dot{m}_{air} = \frac{\dot{Q}_L}{c_p(T_1 - T_4)} = \frac{10 \left[ \frac{\text{kJ}}{\text{s}} \right]}{1.005 \left[ \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right] (310 - 290) [\text{K}]} = 0.498 \left[ \frac{\text{kg}}{\text{s}} \right] \quad \textbf{Answer a)}$$

**Part b)**

The net work input to the compressor can be determined from an overall energy balance performed on the system as shown in Eq3. Alternatively, an energy balance could have been performed on the compressor.

$$\dot{W}_{net,in} = \dot{Q}_H - \dot{Q}_L \quad (\text{Eq3})$$

The low temperature heat transfer is given in the problem statement as 10 kW. The high temperature heat transfer is still unknown. Performing an energy balance on the high temperature heat exchanger, Eq4 is obtained.

$$\dot{Q}_H = \dot{m}_{air} (h_3 - h_2) \quad (\text{Eq4})$$

Again, recognizing that for an ideal gas the enthalpy is a function of temperature only, Eq4 can be rewritten as Eq5.

$$\dot{Q}_H = \dot{m}_{air} c_p (T_3 - T_2) \quad (\text{Eq5})$$

The mass flow rate of air was determined in part a) as 0.498 kg/s. From Table A-2 for air at room temperature  $c_p = 1.005 \text{ kJ/kg}\cdot\text{K}$ . The temperatures at the inlet and outlet of the high temperature heat exchanger must be determined.

Since the compression process is modeled as isentropic, the equation that relates the temperature ratios to the pressure ratios through k for an ideal gas undergoing an isentropic process can be used to determine the high temperature heat exchanger inlet temperature as shown in Eq6.

$$T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = 310[K] (15)^{\frac{0.4}{1.4}} = 672[K] \quad (\text{Eq6})$$

Similarly, the expansion process is also modeled as isentropic, and the equation that relates the temperature ratios to the pressure ratios through k for an ideal gas undergoing an isentropic process can be used again to determine the outlet temperature of the high temperature heat exchanger as shown in Eq7.

$$T_3 = \frac{T_4}{\left( \frac{P_4}{P_3} \right)^{\frac{k-1}{k}}} = \frac{290[K]}{\left( \frac{1}{15} \right)^{\frac{0.4}{1.4}}} = 628.7[K] \quad (\text{Eq7})$$

Substituting the known values into Eq5, the high temperature heat transfer can be determined as shown below.

$$\begin{aligned} \dot{Q}_H &= \dot{m}_{air} c_p (T_3 - T_2) = 0.498 \left[ \frac{kg}{s} \right] 1.005 \left[ \frac{kJ}{kg \cdot K} \right] (672 - 628.7)[K] \\ \rightarrow \dot{Q}_H &= 21.7[kW] \end{aligned}$$

The net work input can now be determined using Eq3 as shown below.

$$\dot{W}_{net,in} = \dot{Q}_H - \dot{Q}_L = 21.7[kW] - 10[kW] = 11.7[kW] \quad \textbf{Answer b)}$$

### Part c)

The COP for the refrigerator can be determined in terms of the benefit/cost. The benefit in this situation is the cooling provided by the system, given as 10 kW. The cost in this situation will be the net work supplied to the system which was determined in part b) as 11.7 kW. Using these definitions the COP can be calculated as shown below.

$$COP_R = \frac{\text{Benefit}}{\text{Cost}} = \frac{\dot{Q}_L}{\dot{W}_{net,in}} = \frac{10[kW]}{11.7[kW]} = 0.854 \quad \textbf{Answer c)}$$

**Part d)**

The cooling capacity of the modified system can be determined from an overall energy balance on the system as shown in Eq8. Alternatively, an energy balance could have been performed on the conditioned space.

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{net,in} \quad (\text{Eq8})$$

Since the properties of the system at location 2 and 3 are assumed unchanged, the heat rejected by the system in the high temperature heat exchanger will remain unchanged. If the turbine were replaced with an expansion valve, all of the compressor work would have to be supplied externally. The compressor work can be calculated from an energy balance over the compressor as shown in Eq9.

$$\dot{W}_{in} = \dot{m}_{air}(h_2 - h_1) \quad (\text{Eq9})$$

Recognizing that for an ideal gas the enthalpy is a function of temperature only and substituting the other known values into Eq9, the work supplied to the compressor can be determined as shown below.

$$\begin{aligned} \dot{W}_{in} &= \dot{m}_{air} c_p (T_2 - T_1) = 0.498 \left[ \frac{\text{kg}}{\text{s}} \right] 1.005 \left[ \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right] (672 - 310) [\text{K}] \\ &\rightarrow \dot{W}_{in} = 181.2 [\text{kW}] \end{aligned}$$

Substituting the known values into Eq8, the cooling capacity can be determined.

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_n = 21.7 [\text{kW}] - 181.2 [\text{kW}] = -159.5 [\text{kW}] \quad \textbf{Answer d)}$$

The refrigerator would no longer remove heat from the conditioned space but would supply heat. Therefore, the system would have no cooling capacity.

**Step 6: Concluding Remarks & Discussion**

- The mass flow rate of air is 0.498 kg/s.
- The net work supplied to the compressor is 11.7 kW.
- The COP is 0.852.
- If the turbine were replaced with an expansion valve the system would have no cooling capacity.