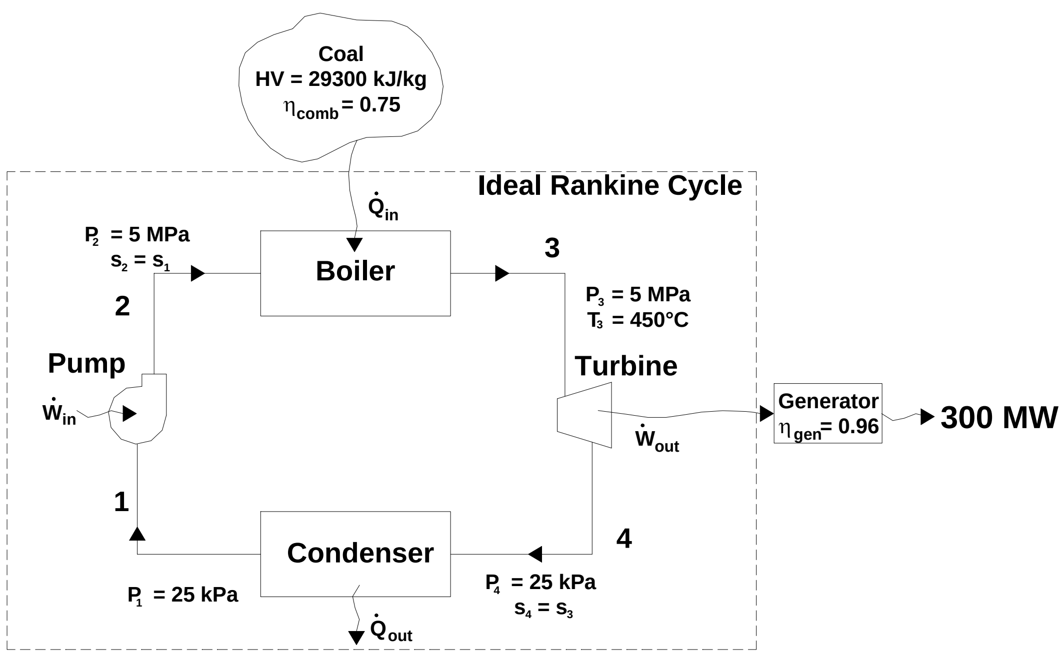
ENSC 461 Tutorial, Week#10 - Rankine Cycle

Consider a coal-fired steam power plant that produces 300MW of electric power. The power plant operates on a simple *ideal* Rankine cycle with turbine inlet conditions of 5 MPa and 450°C and a condenser pressure of 25 kPa. The coal used has a heating value (energy released when the fuel is burned) of 29,300 kJ/kg. Assuming that 75 percent of this energy is transferred to the steam in the boiler and that the electric generator has an efficiency of 96 percent, determine:

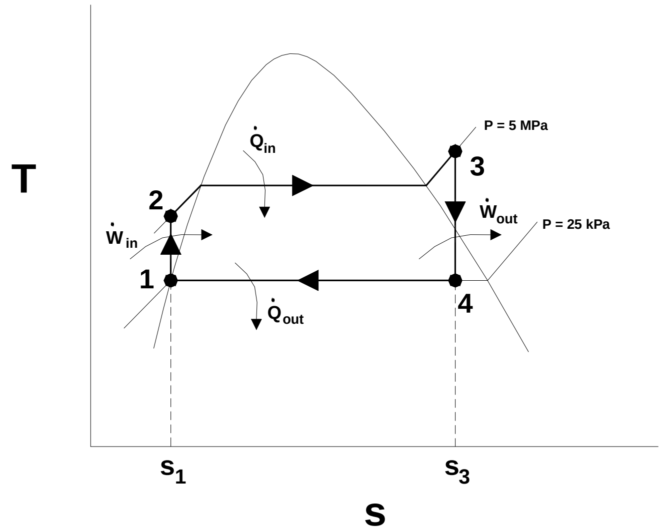
a) the overall plant efficiency (the ratio of net electric power output to the energy input as fuel)
b) the required rate of coal supply

Step 1: Draw a diagram to represent the system

The dashed line shown in the diagram below encloses the ideal Rankine Cycle. The problem states that this is an ideal Rankine Cycle so it can be assumed that there is isentropic compression ($s_2 = s_1$) and expansion ($s_4 = s_3$). It is also standard to model boilers and condensers as constant pressure devices. Therefore $P_3 = P_2 = 5$ MPa & $P_4 = P_1 = 25$ kPa. Also for an ideal Rankine Cycle, it is common to assume that the liquid at location 1 is saturated.



In order to understand the cycle better, it is advised to draw a process diagram - in this case a T-s diagram.



Step 2: Write out what is required to solve for

a) the overall plant efficiency
b) the required rate of coal supply

Step 3: Prepare a property table

Preparing a property table becomes increasingly important when solving cycle-based problems.

	Property				
Location	**T [°C]**	**P [kPa]**	**h [kJ/kg]**	**s [kJ/kg*K]**	**v [m3/kg]**
1		25			
2		5000		s_1	v_1
3	450	5000			
4		25		s_3	

Step 4: Assumptions

Assumptions:

1) Steady operating conditions
2) Δke, $\Delta pe \approx 0$
3) Isentropic compression (1→ 2) / isentropic expansion (3→ 4)
4) State 1 is a saturated liquid & incompressible substance
5) Boiler & condenser are constant pressure devices

Step 5: Solve

Part a)

The problem requires finding the overall plant efficiency for converting the chemical energy stored in the coal into electricity. The overall efficiency will be based on:

1) How efficiently the energy stored in the coal can be converted into a heat input to the boiler (η_{comb} = 0.75) as expressed in Eq1.

$$\dot{Q}_{in} = \eta_{comb}\,\dot{E}_{coal} \qquad \text{(Eq1)}$$

2) How efficiently the ideal Rankine Cycle operates in converting the heat input to the boiler into net work at the turbine shaft ($\eta_{th} = \dot{W}_{net,out}/\dot{Q}_{in}$) as expressed in Eq2.

$$\dot{W}_{net,out} = \eta_{th}\,\dot{Q}_{in} \qquad \text{(Eq2)}$$

3) How efficiently a generator can convert the net work output of the turbine into electricity (η_{gen} = 0.96) as expressed in Eq3.

$$\dot{E}_{elec} = \eta_{gen}\,\dot{W}_{net,out} \qquad \text{(Eq3)}$$

Substituting Eq1 into Eq2 we obtain Eq4.

$$\dot{W}_{net,out} = \eta_{comb}\eta_{th}\,\dot{E}_{coal} \qquad \text{(Eq4)}$$

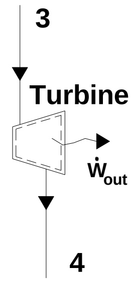
Substituting Eq4 into Eq3 we obtain Eq5, which is an expression for the ratio of net electric power output to the energy input as fuel in terms of the different efficiencies found in the plant.

$$\dot{E}_{elec} = \eta_{comb}\eta_{th}\eta_{gen}\,\dot{E}_{coal} \rightarrow \frac{\dot{E}_{elec}}{\dot{E}_{coal}} = \eta_{comb}\eta_{th}\eta_{gen} = \eta_{overall} \qquad \text{(Eq5)}$$

Since the values η_{comb} and η_{gen} are given in the problem statement, the problem is reduced to finding the thermal efficiency of the cycle as expressed in Eq6.

$$\eta_{th} = \frac{Benefit}{Cost} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{out} - \dot{W}_{in}}{\dot{Q}_{in}} \qquad \text{(Eq6)}$$

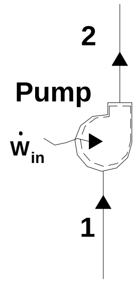
To find the rate of work output (turbine 3 $\rightarrow$ 4), a control volume is constructed that encloses the steam in the turbine as shown below.



Writing an energy balance on the turbine from state 3 → 4 using the assumptions that Δke, Δpe ≅ 0 and steady operating conditions exist Eq7 is obtained.

$$\dot{W}_{out} = \dot{m}_{H2O}\left(h_3 - h_4\right) \qquad \text{(Eq7)}$$

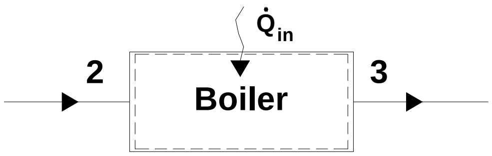
To find the rate of work input (pump 1 → 2), a control volume is constructed that encloses the water in the pump as shown below.



Writing an energy balance on the pump from state 1 → 2 using the assumptions that Δke, Δpe ≅ 0 and steady operating conditions exist Eq8 is obtained.

$$\dot{W}_{in} = \dot{m}_{H2O}\left(h_2 - h_1\right) \qquad \text{(Eq8)}$$

To find the rate of heat input (boiler 2 → 3), a control volume is constructed that encloses the water/steam in the boiler as shown below.



Writing an energy balance on the boiler from state 2 → 3 using the assumptions that Δke, Δpe ≅ 0 and steady operating conditions exist Eq9 is obtained.

$$\dot{Q}_{in} = \dot{m}_{H2O}(h_3 - h_2) \qquad \text{(Eq9)}$$

Substituting Eq7, Eq8, and Eq9 into Eq6, Eq10 is obtained.

$$\eta_{th} = \frac{\dot{W}_{out} - \dot{W}_{in}}{\dot{Q}_{in}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)} = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)} = 1 - \frac{q_{out}}{q_{in}} \qquad \text{(Eq10)}$$

Note: We could have obtained Eq10 from Eq6 and written the energy balances for the condenser and boiler directly.

Eq10 shows that the enthalpy at each location in the system must be determined in order to calculate the thermal efficiency. Starting at location 1, since the H_2O is a saturated liquid @ P = 25 kPa the properties can be determined using Table A-5.

<u>From Table A-5 @ P = 25 kPa</u>
$h_1 = h_{f@P=25kPa}$ = 271.93 [kJ/kg]
$s_1 = s_{f@P=25kPa}$ = 0.8931 [kJ/kg*K]
$v_1 = v_{f@P=25kPa}$ = 0.001020 [m^3/kg]

To find the enthalpy at location 2, Eq 6-53 (from text) can be used to express the reversible work in as shown in Eq11. The liquid H_2O can be assumed to be an incompressible substance ($v_1 = v_2 = v$)

$$\dot{W}_{in} = \dot{m}_{H2O}\, v(P_2 - P_1) \qquad \text{(Eq11)}$$

Combining Eq11 with Eq8, and isolating for h_2, Eq12 is obtained.

$$\dot{m}_{H2O}(h_2 - h_1) = \dot{m}_{H2O}\, v(P_2 - P_1) \rightarrow h_2 = v(P_2 - P_1) + h_1 \qquad \text{(Eq12)}$$

Substituting the known values into Eq12, h_2 can be determined.

$$\rightarrow h_2 = \left(0.001020\left[\frac{m^3}{kg}\right](5000 - 25)\left[\frac{kN}{m^2}\right]\right) + 271.93\left[\frac{kJ}{kg}\right] = 277\left[\frac{kJ}{kg}\right]$$

The temperature (450°C) and pressure (5 MPa) at location 3 are known, so the properties can be determined from the steam tables. Looking in Table A-5 @ P = 5 MPa, it is observed that the corresponding saturated temperature is 263.99°C. Since the temperature at location 3 is greater than the saturated temperature, the steam must be superheated. Using Table A-6 @ P = 5 MPa & T = 450°C the properties at location 3 can be determined.

Table A-6 @ P = 5 MPa, T = 450°C
h_3 = 3316.2 [kJ/kg]
s_3 = 6.8186 [kJ/kg*K]

The entropy at location 4 can be determined by using the assumption that the expansion of the steam by the turbine from state 3→4 is isentropic.

→ $s_4 = s_3$ = 6.8186 kJ/kg*K

Looking in Table A-5 using s_4 =6.8186 [kJ/kg*K] with the known pressure P_4 = 25 kPa, it is observed that s_4 is in between the saturated liquid value, s_f, and the saturated vapor value, s_g. The quality of the saturated liquid-vapor mixture at state 4 can be determined as shown below.

$$\rightarrow x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.8186 - 0.8931}{6.9383} = 0.854$$

The enthalpy at state 4 can be determined by interpolating using the quality between the saturated liquid and vapor states @ P=25 kPa in Table A-5.

Table A-5 @ P = 25 kPa
$\rightarrow h_4 = h_{f@P=25kPa} + x h_{fg@P=25kPa}$ = 271.93 +0.854(2346.3) = 2275.74 [kJ/kg]

The thermal efficiency can be calculated by substituting the values of h_1, h_2, h_3 and h_4 into Eq10.

$$\rightarrow \eta_{th} = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)} = 1 - \frac{(2275.74 - 271.93)}{(3316.2 - 277)} = 1 - \frac{2003.47}{3039.2} = 0.341$$

The overall plant efficiency can be obtained by substituting the value of the thermal efficiency into Eq5

$\eta_{overall} = \eta_{comb}\eta_{th}\eta_{gen} = (0.75)(0.341)(0.96)$ **= 24.6%** **Answer a)**

Part b)

Part b) involves finding the required rate of coal supply. The problem states that coal has a heating value of 29300 kJ/kg_{coal} and that 75 percent of the energy from combusting the coal is transferred to the boiler. Making use of the fact that the overall efficiency is the ratio of net electric power output, $\dot{E}_{elec}$, to the energy input as fuel, $\dot{E}_{coal}$, as expressed in Eq5, the energy input as fuel can be expressed in terms of the heating value of coal and the rate of coal supply as shown in Eq13.

$$\dot{E}_{coal} = HV_{coal}\,\dot{m}_{coal} \qquad \text{(Eq13)}$$

Substituting Eq13 into Eq5, an expression for the rate of coal supply is obtained as shown in Eq14.

$$\dot{m}_{coal} = \frac{\dot{E}_{electric}}{HV_{coal}\,\eta_{overall}} \qquad \text{(Eq14)}$$

The rate of coal supply can be determined by substituting in the net electric power output (300MW), the heating value of coal (29300 kJ/kg_{coal}), and the overall efficiency (0.246) into Eq14.

$$\dot{m}_{coal} = \frac{300000\left[\frac{kJ}{s}\right]}{29300\left[\frac{kJ}{kg_{coal}}\right](0.246)}$$ **= 41.62 kg_{coal}/s** **Answer b)**

Step 5: Concluding Statement and Remarks

The overall plant efficiency was found to be 24.6%. The required rate of coal supply was found to be 41.62 kg_{coal}/s.