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Ljubljana, Slovenia

Abstracts for the Recent Trends in Graph Theory, Algebraic Combinatorics, and Graph Algorithms Conference

Bled, September 24 – September 27, 2001
Slovenia

Ljubljana, 2001

Foreword

Graph Theory meetings have a long tradition in Slovenia. A series of four "Slovenian International Conferences in Graph Theory" were held in 1985, 1991, 1995, and 1999. The fifth conference in the series is planned for 2003. In parallel, in the early 1980s graph theory groups from Ljubljana and Leoben started a series of meetings that is now called the "Leoben-Ljubljana Seminar on Discrete Mathematics". At the first meeting there were only six participants: Vladimir Batagelj, Wilfried Imrich, Bojan Mohar, Tomaž Pisanski, Norbert Seifter, and Wolfgang Woess. Since 1989 the meetings have an official program, the latest meeting "17th Leoben-Ljubljana Seminar on Discrete Mathematics" with 24 talks in three days took place in Leoben in April 2000.

The present conference "Recent Trends in Graph Theory, Algebraic Combinatorics, and Graph Algorithms" will honor Wilfried Imrich's contributions to the development of Graph Theory in Slovenia on the occasion of his 60th birthday. Together with Tomaž Pisanski he was the founder of the above mentioned LL-seminar. This act was of utmost importance for the long lasting cooperation between the graph theorists in Slovenia and Austria. Too many connections and collaborations arose in these 20 years to be mentioned here. As examples we just mention that Wilfried and Tomo are academic fathers of Sandi Klavžar and Janez Žerovnik; that Norbert Seifter closely collaborates with Dragan Marušič, Aleksander Malnič and Boris Zgrablić on automorphism groups of graphs; Wolfgang Woess and Bojan Mohar are co-authors of a nice survey on spectra of infinite graphs; and Wilfried together with Sandi Klavžar published the book "Product Graphs: Structure and Recognition". Thus the present conference can be understood as the highlight of the 20 years of the LL-meetings. It will bring together regular participants of these meetings, many colleagues that participated only occasionally, as well as many graph theorists that were not involved before.

We wish to thank the Slovenian Ministry of Education, Science and Sport for financial assistance of the conference, and the Institute of Mathematics, Physics and Mechanics for financial and technical support of the conference.

Ljubljana, September 7, 2001

Sandi Klavžar, Bojan Mohar, Tomaž Pisanski,
Norbert Seifter, Janez Žerovnik

Design of multi-stable nucleic acid sequences

Ingrid G. Abfalter

The rational design of nucleic acids molecules that can take on two or more alternative structures is an important problem in molecular biology. The structures can be represented as a certain class of vertex-labeled subcubic outerplanar graphs; the first step of the design process consists in finding all vertex labelings that obey the rules of "base pairing" in nucleic acids. The solution of this combinatorial problem implies a method for generating with a uniform distribution nucleic acid molecules that are compatible with the prescribed structures. The structures are then optimized w. r. t. energetic criteria in the second design step for which some numerical results are presented.

The number of sign graphs of an eigenvector of a tree

Türker Biyikoglu

Let G be a connected graph and let $x = (x_1, \dots, x_n)$ be a real vector. A *strong positive* (*negative*) *sign graph* of vector x is a maximal connected subgraph of G on vertices $x_i > 0$ ($x_i < 0$). We shall talk about the number of strong sign graphs of an eigenvector of a tree. We present a characterization of the maximum number of strong sign graphs of an eigenvector of a tree.

Trading crossings for handles and crosscaps

D. Archdeacon, C. Paul Bonnington, J. Širáň

Let $c_k = cr_k(G)$ denote the minimum number of edge crossings when a graph G is drawn on an orientable surface of genus k. The (orientable) *crossing sequence* $c_0, c_1, c_2, \ldots$ encodes the trade-off between adding handles and decreasing crossings. We focus on sequences of the type $c_0 > c_1 > c_2 = 0$; equivalently, we study the planar and toroidal crossing number of doubly-toroidal graphs. For every $\epsilon > 0$ we construct graphs whose orientable crossing sequence satisfies $c_1/c_0 > 5/6 - \epsilon$. In other words, we construct graphs where the addition of one handle can save roughly 1/6th of the crossings, but the addition of a second handle can save 5 times more crossings. We similarly define the *non-orientable crossing sequence* $\tilde{c}_0, \tilde{c}_1, \tilde{c}_2, \ldots$ for drawings on non-orientable surfaces. We show that for every $\tilde{c}_0 > \tilde{c}_1 > 0$ there exists a graph with non-orientable crossing sequence $\tilde{c}_0, \tilde{c}_1, 0$. We conjecture that every strictly-decreasing sequence of non-negative integers can be both an orientable crossing sequence and a non-orientable crossing sequence (with different graphs).

Median-like classes: arboreal structure and cube graphs

Boštjan Brešar

Nonbipartite generalizations of median graphs are considered first. These classes often enjoy the following properties: they are closed for gated subgraphs, gated amalgamations and Cartesian products. In addition, prime graphs of these classes (with respect to gated amalgamation and Cartesian product) usually have the property that its proper gated subgraphs are only the trivial ones, that is singletones. We prove that any graph of such a class contains a peripheral subgraph (in fact, at least two such subgraphs) which is a Cartesian product of two graphs: a gated subgraph of the graph and a prime graph minus a vertex. Hence these graphs admit a peripheral elimination procedure which is a wide generalization of pendant-vertex elimination procedure in trees and of analogous procedure in median graphs.

Concerning bipartite generalizations of median graphs we introduce a subclass of isometric subgraphs of hypercubes that has an arboreal structure. So-called tree-like partial cubes are defined as graphs that enjoy a peripheral isometric expansion procedure. That is, in each step a tree-like partial cube G is expanded with respect to isometric subgraph H of G; the resulting graph is obtained from a disjoint union of graphs G and H by connecting each vertex of H to its corresponding vertex in G. We investigate the structure of tree-like partial cubes, characterize them, and prove several generalizations of results on trees and median graphs.

Cube graph is a short name for intersection graph of maximal hypercubes of a graph. In contrast to the related concepts of line graphs and clique graphs, any graph is a cube graph of a (bipartite) graph. We prove that cube graph of a tree-like partial cube is a dismantlable graph. Moreover, we characterize chordal graphs, dually chordal graphs and related classes as cube graphs of certain subclasses of tree-like partial cubes.

Acknowledgement: A part of this talk relies on a joint work with Wilfried Imrich and Sandi Klavžar.

Fiber-complemented graphs: isometric embeddings, retractions

Marc Chastand

A subset A of a metric space (X,d) is a *prefiber* (or a *gated set*) of X if, for each $x \in X$, there exists $y \in A$ such that $d(x,z) = d(x,y) + d(y,z)$ for every $z \in A$. In a graph endowed with the structure of metric space associated with the geodesic distance, the prefibers have some properties of the fibers of a Cartesian product: for example, the above mentioned vertex y, necessarily unique, defines a *projection* of x onto A and thus we can define a projection map proj_A of X onto A. To refine this comparison between fibers and prefibers, we are led to study the graphs for which the inverse image of every vertex by any projection map onto a prefiber is a prefiber. Such graphs are called *fiber-complemented graphs.* From this property we deduce for any fiber-complemented graph G:

1. a procedure of construction of G by amalgamation or by expansion where the minimal prefibers with respect to inclusion (called *elementary prefibers*) work like building stones;

2. a theorem of canonical isometric embedding of G into a Cartesian product of graphs H (in the sense of Graham and Winkler) whose factors, called *elementary graphs*, are induced by elementary prefibers;

3. a sufficient condition to prove that G is a retract of the Cartesian product H, by adding the notion of mooring (an endomorphism φ of a graph G is a *mooring* of G onto a vertex x if, for every vertex y of G, $\{y, \varphi(y)\}$ is an edge of G with $d(x,y) = d(x,\varphi(y)) + 1$).

As a consequence, this theory gives a global approach to obtain several previous results related to median graphs, quasi-median graphs, pseudo-median graphs, weakly median graphs, bridged graphs, and to extend them to the infinite case.

Minor-minimal highly connected graphs on surfaces

Gašper Fijavž

We characterize minor-minimal 5-connected projective planar graphs and minor-minimal 6-connected toroidal graphs.

Dirac proved that every planar 5-connected graph contains an icosahedron minor. Hence, we are able to limit ourselves to 5-connected projective planar graphs, which are not planar. We split the analysis according to the face-width. In case face-width is at least 3, we find K_6 as a minor. In case face-width is at most 2, we consider two cases based on a condition similar to face-width. The set of minor-minimal 5-connected projective planar graphs contains exactly four graphs, two of them being K_6 and icosahedron.

It is well known that a toroidal graph is 6-connected if and only if it is 6-regular. Furthermore, these graphs have a particularly easy-to-describe structure. If G is 6-connected toroidal and has face-width at least 4, we describe a set of edges, whose contraction yields a smaller 6-connected toroidal graph. In case face-width equals 3, we are able to determine whether such a graph is minor-minimal or not. Altogether there exist exactly four minor-minimal 6-connected toroidal graphs.

A concept of connectedness arising in logic

Herbert Fleischner, Stefan Szeider

CNF formulas can be modelled as bipartite graphs $G(U,W)$ whose edges are partitioned into positive and negative edges. A standard path/cycle starting and ending in W is a path/cycle which contains adjacent edges of different signs whenever it passes through a vertex in U. Call $G(U,W)$ standard connected if any two vertices in W can be joined by a standard path. For example, graphs corresponding to minimally unsatisfiable CNF formulas are standard connected, because these formulas have no nontrivial autarkies. In general, however, the nonexistence of nontrivial autarkies is not sufficient for standard connectedness of the associated graph. If, on the other hand, every edge of such graph lies in some standard cycle, then the graph is standard connected; this demonstrates a certain similarity with the concept of strong connectedness in digraphs. A criterion characterising structurally such graphs in which every edge belongs to a standard cycle can be derived from a Tutte type characterisation of 1-extendable graphs.

Hypercube structure and functional equations for clones of boolean functions

Stephan Foldes

Clones, or sets of Boolean functions closed under composition, were completely classified by Post. The inclusion-ordered lattice of clones is countable, consisting of 8 infinite descending chains, plus a small (finite) list of clones not included in these chains. All clones can be described by functional equations, in accordance with a result of Ekin, Hammer, Hellerstein and this author. Particular equations for clones were proposed by Pogosyan. In a recent joint work with Pogosyan, the minimum number of variables required in an equational characterization of each clone is determined. On 4 of the 8 descending chains, this parameter grows by 1 at each step of descent, while on the other 4 chains it grows logarithmically.

In establishing some of these results, it has been advantageous to view a Boolean function as a subset T of the vertex set of a hypercube, or of a finite power set $P(S)$. In one construction related to the aforementioned logarithmic growth, the set S itself is the vertex set of a lower-dimensional cube H minus one vertex, and T is defined using the set of half-cubes (vertex sets of facets) of H.

Relevant U-path bases

Petra M. Gleiss

The perception of cyclic structures is a crucial step in the analysis of graphs. The smallest canonical set of cycles which describes the cyclic structure of a graph is the *union of all the minimum cycle bases*, called the *set of relevant cycles* $\mathcal{C}_{\mathcal{R}}$. A relevant cycle can also be defined as a cycle which is not the sum of shorter cycles.

Let $U \subseteq V$ and consider the vector space $\mathfrak{U}^*$ generated by the incidence vectors of the uv-paths – paths connecting two vertices u and $v \in U$. This construction is of interest for example in the context of chemical networks, where a subset U of all chemical species V is fed into the system from the outside or is harvested from the system. The uv-paths hence correspond to productive pathways [2].

Hartvigsen [3] introduced the $\mathfrak{U}$-space $\mathfrak{U}(G)$ as the union of $\mathfrak{U}^*$ and the cycle space $\mathfrak{C}(G)$. We will generalize the notion of relevant cycles to this extended vector space and give a polynomial time algorithm to compute *the set of relevant cycles and U-paths* $\mathcal{U}_{\mathcal{R}}$, following Vismara's approach [4].

Gleiss *et al.* [1] introduced a partitioning of $\mathcal{C}_{\mathcal{R}}$ such that each cycle in a class $\mathcal{W}$ can be expressed as a sum of other cycles in $\mathcal{W}$ and shorter cycles. We extend this partitioning to the $\mathfrak{U}$-space.

References:

[1] P. M. Gleiss, J. Leydold, and P. F. Stadler. Interchangeability of relevant cycles in graphs. *Electron. J. Combin.*, 7:R16 (16 pages), 2000.

[2] P. M. Gleiss, P. F. Stadler, A. Wagner, and D. A. Fell. Relevant cycles in chemical reaction network. *Adv. Complex Syst.*, 2001. in press.

[3] D. Hartvigsen. Minimum path bases. *J. Algorithms*, 15:125–142, 1993.

[4] P. Vismara. Union of all the minimum cycle bases of a graph. *Electron. J. Combin.*, 4:R9 (15 pages), 1997.

Type II matrices

Chris Godsil

If A and B are two matrices of the same order, their *Schur product* $A \circ B$ is the matrix whose entries are given by

$$(A \circ B)_{i,j} := A_{i,j} B_{i,j}.$$

(It is sometimes known as the "bad students" product.) The all-ones matrix J is the identity matrix for the Schur product, and we say that B is the *Schur inverse* of A if $A \circ B = J$. If A has a Schur inverse, we denote it by $A^{(-)}$. An $n \times n$ matrix A is a *type II matrix* if

$$A(A^{(-)})^T = nI.$$

(Thus the transpose of the Schur inverse equals nA^{-1}, and the bad students are occasionally right.) The simplest examples of type II matrices are the Hadamard matrices.

Type II matrices first arose in Vaughan Jones's work in knot theory, and also have applications in the theory of Von Neumann algebras. However they also occur in connection with interesting combinatorial structures. In my talk I will outline some of the theory of type II matrices, and illustrate some of the ways that they arise in combinatorics.

Can median networks help to find most parsimonious trees more efficiently?

D. Cieslik, A. Dress, K. Huber, V. Moulton

Maximum parsimony is a popular tree building technique in phylogenetic analysis. However, for large numbers of taxa it is a computationally intensive problem to find a most parsimonious tree and, moreover, depending on the data set, the number of equally good most parsimonious trees – all of which representing different evolutionary scenarios – can be very high. Thus, finding and subsequentially analyzing all most parsimonious trees can be extremely hard.

In case of *unweighted* binary data, the problem of finding a most parsimonious tree can be rephrased as the problem of finding, for a set X of n-dimensional binary vectors, a Steiner tree for X in the n-dimensional hypercube. However, it is well-known that median graphs are retracts of hypercubes, and, therefore, that every Steiner tree for X must necessarily be contained in the median closure of X. Using this fact the search space for finding a most parsimonious tree can, in some instances at least, be restricted considerably.

In this talk, we give a brief introduction to median graphs. We then present a new Divide & Conquer approach for finding Steiner trees in graphs in general, which we ultimately hope to adapt to aid in the search for most parsimonious trees.

Two of my favorite sets of problems and results

Wilfried Imrich

This talk focuses on two sets of problems and results about graphs, groups and products. The first topic is concerned with interactions between graph theory and combinatorial group theory. In particular subgroup theorems, such as Howson's theorem, the theorems of Kurosh and Grushko, a theorem of Gersten, and graphs with large girth will be discussed. The second topic pertains to planar graphs with transitive automorphisms groups, groups of graphs embedded in surfaces, and homeomorphism groups of the plane.

On constant-weight TSP tours

Scott Jones, Mark Kayll

Is it possible to label the edges of K_n with distinct positive weights so that every Hamilton cycle has the same total weight? We give a local condition characterizing the labelings that witness this question's (perhaps surprising) affirmative answer. More generally, we address the question that arises when "Hamilton cycle" is replaced by "k-factor" for positive integers k.

Lattice paths and combinatorial diophantine equations

Peter Kirschenhofer, Oliver Pfeiffer

We consider the set of integer lattice paths where in each step one can move one unit to the right or one unit upwards or both at the same time. The numbers $p_n(k)$ of these paths connecting the origin with the point (n,k) are also known as the Dellanoy numbers. It can be shown that there is a bijection between this set of paths and the set of integer lattice points $(x_1,\dots,x_n)$ in the octahedron $\sum_{i=1}^{n}|x_i|\le k$.

A second order recurrence for the numbers $p_n(k)$ can be established, which allows to continue them to polynomials $p_n(x)$, which are closely related to the special instance $P_n^{(\frac{1}{2})}(\frac{x}{2},\frac{\pi}{2})$ of the orthogonal Meixner-Pollaczek polynomials. Relying on this fact a number of results on the diophantine equation

$$p_n(x) = p_m(y)$$

have been proved in [1]. From the generating function of the $p_n(k)$ it follows that the numbers $n!p_n(k)$ may be interpretated in an "exponential model", too.

Starting from this interpretation the results from [1] can be generalized to a broad class of counting polynomials enumerating permutations with coloured cycles. From the analytical point of view this corresponds to the Meixner-Pollaczek polynomials $P_n^{(\lambda)}(x,\pi/2)$ with variable λ now.

References:

[1] P. Kirschenhofer, A. Pethő, R. F. Tichy, On analytical and diophantine properties of a family of counting polynomials, Acta Math. Sci. (Szeged) 65 (1999), 63–75.

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On my joint work with Wilfried Imrich

Sandi Klavžar

Being the most frequent co-author of Wilfried it seems appropriate to speak on this day about our joint work. In the last ten years we have written 13 joint papers, a selection is given below. Moreover, during much of this time we worked on our book on product graphs that was completed two years ago and published last year. Our main interest were and are product graphs, including, for instance, partial cubes, Hamming graphs, median graphs, and halved cubes, as well as some newly introduced classes of graphs as semi-median graphs and almost median graphs. The most frequent nontrivial word (for instance, the word "graph" is considered as trivial) in the titles of our papers is "recognizing". A selection of our bibliography follows.

1. W. Imrich, S. Klavžar, Retracts of strong products of graphs, Discrete Math. 109 (1992) 147–154.

2. W. Imrich, S. Klavžar, On the complexity of recognizing Hamming graphs and related classes of graphs, European J. Combin. 17 (1996) 209–211.

3. W. Imrich, S. Klavžar, Recognizing Hamming graphs in linear time and space, Inform. Proc. Lett. 63 (1997) 91–95.

4. W. Imrich, S. Klavžar, A convexity lemma and expansion procedures for bipartite graphs, European J. Combin. 19 (1998) 677–685.

5. J. Hagauer, W. Imrich, S. Klavžar, Recognizing median graphs in subquadratic time, Theo. Comp. Sci. 215 (1999) 123–136.

6. W. Imrich, S. Klavžar, H. M. Mulder, Median graphs and triangle-free graphs, SIAM J. Discrete Math. 12 (1999) 111–118.

7. W. Imrich, S. Klavžar, Recognizing graphs of acyclic cubical complexes, Discrete Appl. Math. 95 (1999) 321–330.

8. W. Imrich, S. Klavžar, Product Graphs: Structure and Recognition, John Wiley & Sons, New York, 2000.

Average degree and contractibility

MATTHIAS KRIESELL

An edge in a k-connected graph is called *k-contractible* if its contraction yields again a k-connected graph. A classical result in graph theory states that for $k \leq 3$, every noncomplete k-connected graph contains a k-contractible edge (TUTTE). For $k \geq 4$, a similar result is not true, but there are conditions to the girth (THOMASSEN), to the minimum degree (EGAWA), or to degree sums which ensure the existence of a k-contractible edge in a k-connected graph. The talk will be about density conditions which guarantee the existence of such an edge.

Nowhere zero 4-flow in regular matroids

HONG-JIAN LAI

Jensen and Toft (1995) conjectured that every 2-edge-connected graph without a K_5-minor has a nowhere zero 4-flow, which is equivalent to saying that every 2-edge-connected graph without a K_5-minor has a 3-cycle 2-cover. We prove that if M is a regular matroid with no coloops and if M has neither $M(K_5)$ nor $M^*(K_5)$ as a minor, then M has a 3-cycle 2-cover, thereby proving the Jensen and Toft conjecture affirmatively.

Courant's Nodal Domain Theorem for graphs

Josef Leydold

We prove two discrete analogues of Courant's Nodal Domain Theorem.

Partial cubes as subdivision graphs and as generalized Petersen graphs

Sandi Klavžar, Alenka Lipovec

Isometric subgraphs of hypercubes are known as partial cubes. The subdivision graph of a graph G is obtained from G by subdividing every edge of G. It is proved that for a connected graph G its subdivision graph is a partial cube if and only if every block of G is either a cycle or a complete graph. Regular partial cubes are also considered. In particular it is shown that $P(10, 3)$ and $P(2n, 1)$, $n \geq 2$, are the only (regular) partial cubes among the generalized Petersen graphs.

Finite contractions of graphs with polynomial growth

A. Lukács, N. Seifter

We present results on the structure of graphs with polynomial growth. For certain classes of these graphs we prove that they contain subgraphs which are similar to the lattice graph $\mathbf{L}^d$.

Trofimov characterized the automorphism groups of graphs with polynomial growth. He gave an even stronger characterization of those graphs with polynomial growth on which a group of bounded automorphisms acts transitively. He showed that these graphs are quite similar to Cayley graphs of $\mathbf{Z}^d$. We investigate these relationships in detail.

Let X be a connected, locally finite, vertex-transitive graph of quadratic growth, such that the group of bounded automorphisms $B(X)$ acts transitively on $V(X)$. Then X is spanned by a finite number of pairwise disjoint, isomorphic subgraphs which are given subdivisions of $\mathbf{L}^2$.

Let X be a locally finite, vertex-transitive, infinite graph with polynomial growth. Then there exists a quotient group of $\mathrm{Aut}(X)$ which contains a finitely generated nilpotent subgroup N which has the same growth rate as X. We show that X contains a subgraph which is finitely contractible onto the h-dimensional lattice, where h is the Hirsch number of N.

Lifting automorphisms: some recent results

Aleksander Malnič, Dragan Marušič, Primož Potočnik

Some recent results about lifting groups of graph automorphisms along regular covering projections will be presented.

On 2-arc-transitive Cayley graphs of dihedral groups

Dragan Marušič

The structure of 2-arc-transitive graphs satisfying certain additional properties will be discussed. In particular, a previously unknown family of 2-arc-transitive Cayley graphs of dihedral groups will be presented. The graphs in this family are obtained as certain 2-fold covers of the canonical double covers of the complete graphs.

Simple bounds for the Minimum Dominating Trail Set problem

Carlo Meloni

The trail cover number $TCN(G)$ of a graph G is the minimum number of trails in a trail cover of G. This parameter is related to the Minimum Dominating Trail Set (MDTS) problem that requires to find a minimum cardinality collection of pairwise edge-disjoint trails that together dominates a graph in such a way that each edge has at least one endvertex on some trail. MDTS problem is NP-hard in general graphs and it is tractable for some particular graph classes, among them there are split graphs. Simple upper bounds for $TCN(G)$ may be obtained from the study of the total interval number of G and the split graph characterization.

Keywords: graphs, trail cover, edge domination.

Valency of distance-regular antipodal graphs with diameter 4

Štefko Miklavič

A graph G on n vertices is *strongly regular* with parameters (n, k, λ, μ) when it is k-regular and any two adjacent (resp. nonadjacent) vertices have exactly λ (resp. μ) common neighbours. A graph H of diameter d is *distance-regular*, if for a pair of vertices (x, y) at distance h, the number p^h_{ij} of vertices at distance i from x and j from y depends only on integers i, j, h, and not on (x, y). A graph H of diameter d is *antipodal*, if the vertices at distance d from a given vertex are all at distance d from each other. Then "being at distance d or zero" induces an equivalence relation on the vertices of H, and the equivalence classes are called *antipodal classes.* For an antipodal graph H we define the *antipodal quotient* of H to be the graph with the antipodal classes as vertices, where two classes are adjacent if they contain adjacent vertices.

It is well known that an antipodal distance-regular graph H has diameter 4 or 5 iff its antipodal quotient G is a strongly regular graph. Let H be an antipodal distance-regular graph with diameter 4 and G its antipodal quotient. We solve an open problem by proving that if (n, k, λ, μ) are the parameters of G, then $k \leq \lfloor 2(n-1)/5 \rfloor$, unless G is the complement of triangular graphs $T(6)$ or $T(7)$, the folded Johnson graph $J(8, 4)$ or the folded halved 8-cube. This result implies that only one of a complementary pair of strongly regular graphs can be the antipodal quotient of antipodal distance-regular graph.

Construction of graphs with maximum graphical structure and tenacity

Dara Moazzami

The tenacity of a graph G, $T(G)$, is defined by

$$T(G) = \min\left\{\frac{|A| + \tau(G-A)}{\omega(G-A)}\right\},$$

where the minimum is taken over all vertex cutset A of G. We define $G-A$ to be the graph induced by the vertices of $V\backslash A$, $\tau(G-A)$ is the number of vertices in the largest component of the graph $G-A$ and $\omega(G-A)$ is the number of components of $G-A$.

In this paper, the maximum graphical structure is obtained when the number of vertices p of a connected graph G and tenacity $T(G) = T$ are given. Finally the method of constructing the sort of graphs are also presented.

Coloring-flow duality for locally planar graphs

M. Devos, L. Goddyn, B. Mohar, D. Vertigan, X. Zhu

It is a classical result of Tutte that a (2-connected) planar graph has a k-coloring if and only if its dual graph admits a nowhere-zero k-flow. For graphs on more general (orientable) surfaces, a coloring still implies existence of a nowhere-zero flow in the dual graph, but the converse is not true.

In the talk it will be shown that the duality “almost” works on general surfaces if the graph is highly locally planar. This is a recent result that involves the notion of circular chromatic number and circular flows which generalize the corresponding classical parameters. It has some interesting consequences in the theory of colorings of graphs on surfaces.

Signpost systems and their underlying graphs

Henry Martyn Mulder

Usually, when we travel and want to find our way from u to w, we look for signposts towards w and follow them until we reach w. That is, at u we follow the signpost to w, by which we first arrive at v. Then at v we look for the next signpost to w. We can denote this step in our journey by the triple (u, v, x): here u is the point where we currently are, w is the final destination, and v is the point where we first arrive after following the direction of the signpost at u.

In this talk the concept of a signpost system on a set V is introduced as a ternary relation satisfying three basic axioms. Moreover, the underlying graph of a signpost system is introduced. First results and examples are discussed. Signpost systems satisfying additional signpost axioms are studied, e.g. modular signpost systems and median signpost systems.

Groupoids and graphs

Eric Mwambene, Gert Sabidussi

Based on the modelled of Cayley graphs, general groupoid graphs are defined. With this framework, various classes of graphs are characterized.

The coloring order

Jaroslav Nešetřil

The existence of a homomorphism between graphs induces a quasiorder called coloring order. In this talk we survey some of the properties of this order.

Distance in the direct product of graphs

Iztok Peterin, Janez Žerovnik

A formula for the distance between any two vertices in the direct product of graphs is given in terms of even and odd distance in the factors. An $\mathcal{O}(n^3)$ algorithm for computing even and odd distance in a graph is outlined.

Representations of graphs and maps

Tomaž Pisanski, Arjana Žitnik

An m-dimensional representation of a graph G in the field F is a function $\rho: VG \to F^m$. A representation of a map M is a representation of its 1-skeleton. For a given ρ various edge- and face-extensions define different representations such as: affine, convex, projective, and barycentric. We generalize representations to incidence geometries or vertex colored graphs. The classical problem of drawing configurations in Euclidean plane with straight lines can be described in terms of projective representations of rank 2 incidence geometries.

On a special case of Hadwiger's Conjecture

Michael D. Plummer, Michael Stiebitz, Bjarne Toft

In 1942, H. Hadwiger conjectured that if a graph G has chromatic number χ, then G has the complete graph K_χ as a minor. Hadwiger's Conjecture has proved very difficult to attack and currently remains unsolved, even in the special case of graphs G of independence number $\alpha(G) = 2$. In the present paper we present a large number of properties possessed by a smallest counterexample to Hadwiger's Conjecture for $\alpha = 2$. Moreover, we shall prove that the conjecture is true for several infinite families of $\alpha = 2$ graphs and their inflations. (An *inflation* of a graph is obtained by replacing its vertices by complete graphs. Note that inflations of $\alpha = 2$ graphs likewise have $\alpha = 2$.)

Dually compact closed classes of graphs

Norbert Polat

A class $\mathcal{C}$ of graphs is said to be *dually compact closed* if, for every infinite $G \in \mathcal{C}$, each finite subgraph of G is contained in a finite induced subgraph of G which belongs to $\mathcal{C}$. The class of trees and more generally the one of chordal graphs are dually compact closed. We will briefly show that, due to results of Jawhari, Misane and Pouzet (1986) and independently of Pesh (1987), the class of Helly graphs is dually compact closed, and more precisely that every finite subgraph of a Helly graph G is contained in a finite isometric Helly subgraph of G.

The main part of this talk will be to settle a question of Hahn, Sands, Sauer and Woodrow by showing that the class of bridged graphs is dually compact closed. To prove this result we will use the concept of constructible graph. A (finite or infinite) graph G is *constructible* if there exists a well-ordering $\leq$ of its vertices such that, for every vertex x which is not the smallest element, there is a vertex $y < x$ which is adjacent to x and to every neighbor z of x with $z < x$. Finite graphs are constructible if and only if they are dismantlable. The case is different, however, with infinite graphs. Helly graphs and bridged graphs are particular instances of constructible graphs and, as we will see, the class of constructible graphs itself is dually compact closed. Finally we will give different sufficient conditions so that every finite subgraph of a bridged graph G is contained in a finite isometric (and thus bridged) subgraph of G.

On bounds for dominating Cartesian products

Bert L. Hartnell, Douglas F. Rall

The study of domination in Cartesian products has received most of its motivation from attempts to settle the conjecture made by V.G. Vizing in 1968. He conjectured that $\gamma(G)\gamma(H)$ is a lower bound for the domination number of the Cartesian product of any two graphs G and H. Most of the progress on settling this conjecture has been limited to verifying the conjectured lower bound if one of the two graphs has a certain structural property.

A number of authors have established bounds for dominating a Cartesian product:

1. $\gamma(G \square H) \leq \min\{\gamma(G)|H|, \gamma(H)|G|\}$. (Vizing, 1963)
2. $\gamma(G \square H) \geq \max\{\frac{|H|}{\Delta(H)+1}\gamma(G), \frac{|G|}{\Delta(G)+1}\gamma(H)\}$ (Jacobson & Kinch, 1983)
3. $\gamma(G \square H) \geq \max\{\gamma(G)\rho(H), \gamma(H)\rho(G)\}$, where ρ represents the 2-packing number of a graph. (Jacobson & Kinch, 1986)
4. $\gamma(G \square H) \geq \min\{|G|, |H|\}$. (El-Zahar & Pareek, 1991)
5. $\gamma(G \square H) \geq \frac{1}{2}\gamma(G)\gamma(H)$. (Clark & Suen, 2000)

We show how it is possible to improve some of these bounds by imposing certain conditions on both graphs. For example, we prove a new lower bound for the domination number of $T \square T$, when T is a tree. We improve Vizing's bound of 1963 in the case when one of the graphs has a dominating cover of order $k > 1$ and the other has a set D which partitions into k subsets, each dominating $V \setminus D$.

Routings with uniform load in bipartite graphs

Eric Mwambene, Gert Sabidussi

We show that certain properties of graphs having uniform geodesic routings remain valid if one drops the hypothesis that the paths of the routing be geodesics, provided one considers only bipartite graphs.

Highly arc-transitive digraphs

Norbert Seifter

An *s-arc* ($s \geq 0$) in a digraph D is a sequence of $s+1$ vertices such that for each i, $0 \leq i \leq s$, the pair (x_i, x_{i+1}) is an edge of D and $x_{i-1} \neq x_{i+1}$. A digraph D is *s-transitive* if its automorphism group acts transitively on the set of s-arcs. D is *highly arc-transitive* if it is s-transitive for all finite $s > 0$. Although the condition of being highly arc-transitive seems to be quite restrictive, these graphs are a rich source of interesting problems. In this talk we give an introduction into the theory of highly arc-transitive digraphs by presenting known results as well as open problems.

Formal spaces underlying evolutionary change

Bärbel M. R. Stadler

The search spaces in combinatorial chemistry as well as the sequence spaces underlying (molecular) evolution are conventionally thought of as graphs. Recombination, however, implies a non-graphical structure of the combinatorial search spaces. These structures, and their implications for the search process itself, hence are not very well understood in general. In this contribution we review a very general formalism from point set topology and discuss its application to combinatorial search spaces, fitness landscapes, evolutionary trajectories, and artificial chemistries. Central concepts of biology can be precisely defined within this framework. For instance, the intuitive notion of a *phenotypic character* is identified with irreducible factors of the phenotype space.

References:

[1] BMR Stadler, PF Stadler, GP Wagner, W Fontana. The topology of the possible: formal spaces underlying patterns of evolutionary change. J. Theor. Biol. 212 (2001) in press.

Minimal cycle bases

Peter F. Stadler

Minimal cycle bases (MCBs) of undirected graphs play an important role in science and engineering, for instance in structural analysis of networks, chemical structure storage and retrieval systems. We show that the MCB concept can be extended to directed circuits in strongly connected di-graphs, and that a *minimal circuit basis* can be obtained by a method similar to Horton's algorithm for the undirected case. A further extension is described in P. Gleiss' contribution. While MCBs are in general not well-behaved under graph operations, MCBs of Cartesian and strong product graphs can be obtained from MCBs of their factors.

In general, graphs do not have a unique minimal cycle basis, with the notable exception of a few very simple groups including outerplanar graphs and most Halin graphs, hence the union $\mathcal{R}$ of all minimal cycle bases is of particular interest. A partition of $\mathcal{R}$ is introduced such that each cycle in a class $\mathcal{W}$ can be expressed as a sum of other cycles in $\mathcal{W}$ and shorter cycles. It is shown that each minimum cycle basis contains the same number of representatives of a given class $\mathcal{W}$. This result can then be used to derive upper and lower bounds on the number of distinct minimum cycle bases.

References:

[1] J Leydold, PF Stadler. Minimal Cycle Basis of Outerplanar Graphs. Electron. J. Combin. 5 (1998): R16 (14 pages)

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[3] PM Gleiss, PF Stadler, A Wagner, DA Fell: Relevant Cycles in Chemical Reaction Network. Adv. Complex. Syst. 4 (2001) in press.

Tiled partial cubes

Boštjan Brešar, Wilfried Imrich, Sandi Klavžar,
Henry Martyn Mulder, Riste Škrekovski

In the quest to better understand the connection between median graphs, triangle-free graphs and partial cubes, a hierarchy of subclasses of partial cubes has been introduced. In this paper we study the role of tiled partial cubes in this scheme. For instance, we prove that almost-median graphs are tiled and that tiled partial cubes are semi-median. We also describe median graphs as tiled partial cubes without convex Q_3^- and extend an inequality for median graphs to a larger subclass of partial cubes.

2-local 4/3-competitive algorithm for multicoloring hexagonal graphs

Petra Šparl, Janez Žerovnik

An important optimization problem in the design of cellular networks is to assign sets of frequencies to transmitters to avoid unacceptable interference. A cellular network is generally modeled as a subgraph of the infinite triangular lattice. The distributed frequency assignment problem can be abstracted as a multicoloring problem on a weighted hexagonal graph, where the weight vector represents the number of calls to be assigned at vertices. In this paper we present a distributed algorithm for multicoloring hexagonal graphs using only the local clique number $\omega(v)$ at each vertex v of the given hexagonal graph, which can be computed from local information available at the vertex. We prove the algorithm uses no more than $\left\lceil \frac{4\omega(G)}{3} \right\rceil$ colors for any hexagonal graph G, without explicitly computing the global clique number $\omega(G)$. Hence the competitive ratio of the algorithm is 4/3. We also prove that our algorithm is 2-local.

Vertex-symmetric graphs: associated van den Dries-Wilkie metric spaces and vertex stabilizers

V. I. Trofimov

In the first part of the talk the van den Dries-Wilkie metric spaces associated with connected locally finite vertex-symmetric graphs are considered. The kernels of the natural actions of groups of automorphisms of graphs by isometries of these spaces are investigated. Applications to finitely generated groups are given.

In the second part of the talk the vertex stabilizers of connected locally finite graphs in vertex-transitive groups of automorphisms are considered. The problem of reconstruction of the stabilizer of a vertex by the action on the set of adjacent vertices is discussed. Applications to finite groups are given.

Computer search of graph invariants in products with cycles

Aleksander Vesel

A computer based approach in searching graph invariants of products with cycles is considered. Three new results: the λ number on the Cartesian product of a cycle and a path, the independence number on the family of graphs $C_7 \boxtimes C_7 \boxtimes C_{2k+1}$ and the domination number on the Cartesian product of C_6 with a cycle are presented.

Growth-sensitivity of context-free languages

Wolfgang Woess

The growth coefficient of a language L over a finite alphabet Σ is defined as $\gamma(L) = \limsup |\{w \in L : |w| = n\}|^{1/n}$. Let F be a finite set of sub-words of words in L, and define L^F as the sub-language of L consisting of those elements of L that have no sub-word in L. A language L is called growth-sensitive, if $\gamma(L^F) < \gamma(L)$ strictly for every such set F of forbidden sub-words.

It is shown that every context-free language L that is generated by a non-linear, unambiguous, ergodic context-free grammar is growth-sensitive. Here, "ergodic" means that the dependency-digraph of the grammar is strongly connected.

Graph theoretical aspects of the rational version of the Beckman-Quarles statement

Joseph Zaks

Beckman-Quarles Theorem states that every mapping from the Euclidean d-space to itself that preserves distance one is an isometry, provided d is at least 2. The analogue statement for the rational cases (replacing the Euclidean d-space by the rational d-space) is much more interesting: it is false in dimensions 2, 3 and 4; it is true in all even dimensions d which are at least 6, and all odd dimensions d of the form $d = 2k^2 - 1$, for all k greater than or equal to 3.

In most of these cases one actually treats some finite subgraphs of the unit-distance graph in (rational) d-space.

Cores decomposition of graphs

Vladimir Batagelj, Matjaž Zaveršnik

The structure of large graphs can be revealed by partitioning them into smaller parts, which are easier to handle. One of such decompositions is based on k-cores, proposed in 1983 by Seidman. A k-core of graph G is a maximum subgraph H in which each vertex $v \in H$ has a degree at least k inside H. An efficient, $\mathcal{O}(m)$, where m is the number of lines, algorithm for determining the cores decomposition of a given graph will be presented. The notion of cores can be generalized to valued graphs. An efficient, $\mathcal{O}(m \log n)$, where n is the number of vertices, algorithm for this case will be also presented. Some applications of cores decomposition will be discussed. The described algorithms are implemented in the program for large networks analysis **Pajek** (`http://vlado.fmf.uni-lj.si/pub/networks/pajek`).

Behzad-Vizing conjecture and Cartesian product graphs

Blaž Zmazek, Janez Žerovnik

We prove the following theorem: If the Behzad-Vizing conjecture is true for graphs G and H, then it is true for the Cartesian product $G \Box H$.

Weak reconstruction of strong product graphs

Janez Žerovnik

Wilfried Imrich and I have shown that a graph which is a Cartesian product of at least two factors can be reconstructed in an up to isomorphism unique way from any of its one vertex deleted subgraphs [W. Imrich, —: On the weak reconstruction of Cartesian-product graphs, *Discrete Math.* **150** (1996) 167–178]. An algorithm is given in [J. Hagauer, —: An algorithm for the weak reconstruction of Cartesian-product graphs, preprint, 1995 (to appear in *J. Combin. Inform. System Sci.*)]. Equivalent result saying that Cartesian product graphs are semistable was independently proved by Sims and Holton [J. Sims, D. A. Holton: Stability of Cartesian products, *JCTB* **25** (1978) 258–282]. Together with Blaž Zmazek we have recently extended the result to uniqueness of the reconstruction of Cartesian graph product graphs with at least $k+1$ factors with at least $k+1$ vertices each, from any k vertices deleted subgraph H.

I will discuss the analogous theorem(s) for the strong product of graphs. A strong product of at least two factors can be reconstructed from any of its one vertex deleted subgraphs. An interesting though not surprising difference is that uniqueness here is even more strict, i.e., the neighborhood of the missing vertex is always unique.

Resonance graph of catacondensed benzenoid graphs are median

S. Klavžar, P. Žigert, G. Brinkmann

Let G be a planar embedded 2-connected graph. Then the vertices of its resonance graph $R(G)$ are the 1-factors of G, two 1-factors being adjacent whenever their symmetric difference is a bounded face of G. For chemically important catacondensed benzenoid graphs we show that the resonance graphs are median. In particular, if G belongs to this class, $R(G)$ has an isometric embedding into Q_f, where f is the number of bounded faces of G.