University of Ljubljana
Institute of Mathematics, Physics and Mechanics
Ljubljana, Slovenia

Abstracts for The 4th Slovenian International Conference in Graph Theory

Bled, June 28 – July 2, 1999
Slovenia

Ljubljana, 1999

Foreword

It is our pleasure to welcome you at Bled, the site of the Fourth Slovenian Conference on Graph Theory.

This conference has made a long way from its first meeting in Dubrovnik (now in Croatia) in 1985, organized by Tomaž Pisanski, the "father" of Graph Theory in Slovenia whose 50th anniversary along with the 70th anniversary of Gert Sabidussi we will celebrate during this week. The second meeting was held at Bled in 1991 and coincided with the declaration of Slovenian independence. This caused a slight inconvenience to the 30 participants but the meeting will be remembered as a successful albeit adventurous event. In 1995 the number of participants more than tripled and the number of registered participants for the Fourth Slovenian Conference is more than hundred.

In the tradition of the former meetings, the conference is strongest in the areas of algebraic and topological graph theory, but we are glad that also other branches of graph theory are well represented. The received abstracts promise an interesting and fruitful contribution to mathematics. We express our thanks to all of you for attending this conference and wish you a mathematically productive week, but most of all a pleasant and relaxed stay in Slovenia.

This collection contains only abstracts of the talks. The proceedings of the conference will be published as a special volume of Discrete Mathematics after a thorough refereeing procedure following the standards of the journal.

The organizers are grateful to all those who helped make this meeting possible. Special thanks go to the Slovenian Ministry of Science and Technology, to the Institute of Mathematics Physics and Mechanics at the University of Ljubljana, to the Faculty of Mathematics and Physics at the University of Ljubljana, and to the companies Telekom, Ljubljana, ECS Tel, Trzin, Telemont, Koper, Iskra-Avtoelektrika, Šempeter, and software companies Hermes SoftLab and Zamisel for their financial support.

Sandi Klavžar,
Dragan Marušič,
Bojan Mohar

Ljubljana, June 18, 1999

Automorphism Groups of Walecki Tournaments

Janez Aleš

Lucas tournaments, today known as Walecki tournaments, were defined by Alspach in 1966. A mapping between cycles of the complementing circular shift register and isomorphism classes of Walecki tournaments was found. A upper bound on the number of isomorphism classes of Walecki tournaments was determined. Alspach conjectured that the bound is tight. The problem of enumerating Walecki tournaments has not been solved to date. However, it was published as an open problem in a paper by Alspach, Bermond, and Sotteau in 1987, and as a research problem by Alspach in 1989. In an attempt to prove this 33 years old conjecture we determine the automorphism groups of Walecki tournaments for all but those whose defining binary sequences are aperiodic. Techniques used in the proof originate from a diverse range of topics: permutation groups, tournaments, and bijective enumeration, to mention a few.

Cycle decomposition problems

Brian Alspach

I shall survey some of the outstanding unsolved problems dealing with cycle decompositions.

The representativity of planar graphs

Dan Archdeacon

The representativity, or face-width, is a parameter that measures how well an embedded graph represents a non-spherical surface. Alternatively, it measures the local planarity of an embedded graph. The parameter is considered undefined for planar graphs. In this talk I'll discuss efforts to define the representativity of planar graphs.

Analyses of Erdős Graph

Vladimir Batagelj, Andrej Mrvar

Paul Erdős was one of the most prolific mathematicians in history, with more than 1500 papers to his name, many in collaboration with others. Erdős number is defined as follows: Erdős himself has Erdős number 0, people who have written a joint paper with Paul Erdős have Erdős number 1, their co-authors, with yet undefined Erdős number, have Erdős number 2, etc. Patrick Ion (Mathematical Reviews) and Jerry Grossman (Oakland University) maintain a collection of related data available at

`http://www.oakland.edu/~grossman/erdoshp.html`

The data determine the ***Erdős collaboration graph*** – a graph containing all people with Erdős number up to 2, without Erdős himself and without connections to him. Edges in the graph connect two authors, if they wrote a joint paper. Connections between co-authors with Erdős number 2 are not available. In the last edition of the data (February 1999) there are 492 co-authors of Erdős (Erdős number 1) and 5608 their co-authors (Erdős number 2) – the graph contains 6100 vertices (authors) and 9939 edges (connections among co-authors).

In the paper some analyses of the Erdős graph (analysis of cores, different centrality measures) and visualizations of its parts, using program `Pajek`, are presented.

`Pajek` is a program for analysis and visualization of large networks. It is freely available for noncommercial use at

`http://vlado.fmf.uni-lj.si/pub/networks/pajek/`

Labeled $K_{2,t}$ minors in plane graphs

Thomas Böhme, Bojan Mohar

Let G be a 3-connected planar graph and let $U \subseteq V(G)$. It is shown that G contains a $K_{2,t}$ minor such that t is large and each vertex of degree 2 in $K_{2,t}$ corresponds to some vertex of U if and only if there is no small face cover of U. This result cannot be extended to 2-connected planar graphs.

Vertex Accumulation Points in Infinite Planar Graphs

C. Paul Bonnington, R. Bruce Richter, Carsten Thomassen

A vertex accumulation point of a planar embedding of an infinite graph is a point in plane such that every local neighbourhood contains an infinite number of vertices of the graph. Halin (1966) characterised those infinite planar graphs that have planar embeddings without accumulation points. We present a result in the direction of a generalisation of Halin's work and give classes of graphs that do not have a planar embedding with at most k accumulation points. Furthermore, we present an extension to Maclane's (1937) characterisation of planar graphs to infinite graphs which characterises embeddings with k accumulation points. Finally, we present interesting combinatorial and topological properties of the set of vertex accumulation points when the number of such points is infinite.

Chiral Hypermaps

Antonio Breda D'Azevedo, Roman Nedela

By a topological *map* we mean a cellular decomposition of an orientable closed surface. A common way of constructing maps is by imbedding a graph into a surface. A map is *3-coloured* if there is a colouring of its faces, say black (0), grey (1) and white (2), such that every vertex of the map is incident with three faces coloured by mutually different colours. A topological *hypermap* $\mathcal{H}$ is a 3-coloured map where every face has even size, or equivalently, a cellular imbedding of a hypergraph into a surface. The coloured faces of the map are called the *hypercells* of $\mathcal{H}$ while the black, grey and white coloured faces (the 0-,1-, and 2-faces of the coloured map) are called *hypervertices*, *hyperedges* and *hyperfaces* of $\mathcal{H}$ respectively. The *size* of a hypercell is half the size of the corresponding face of the 3-coloured map. The vertices of the underlying 3-coloured map are called *flags* of the hypermap. Every map $\mathcal{M}$ gives rise to a hypermap $\mathcal{H}(\mathcal{M})$ where the hyperedges are of size 2.

An *automorphism* of $\mathcal{H}$ is a permutation of flags which extends to a self-homeomorphism of the surface mapping hypervertices onto hypervertices, hyperfaces onto hyperfaces and hyperedges onto hyperedges and preserving the incidence relation between them. The orientation preserving subgroup of automorphisms of $\mathcal{H}$ is denoted by Aut^+H. It is well-known that $|\mathrm{Aut}^+\mathcal{H}| \leq |F|/2$ and $|\mathrm{Aut}\,\mathcal{H}| \leq |F|$, where F is the set of flags. $\mathcal{H}$ is said *orientably regular* if $|\mathrm{Aut}^+| = |F/2|$ and *regular* if $|\mathrm{Aut}\,\mathcal{H}| = |F|$.

A hypermap $\mathcal{H}$ is *chiral* if it is not isomorphic to its mirror image. Although chirality of maps and hypermaps seems to be an interesting invariant of maps and hypermaps on orientable surfaces, generally not too much is known about this phenomena. Most of the known orientable regular maps and hypermaps are reflexible, i. e. they are not chiral. For instance, it follows from the classification of orientably regular maps that surfaces of genus g where $2 \leq g \leq 6$ admit no chiral orientably regular maps. We present some fundamental results on chiral hypermaps and construct several infinite families of such hypermaps. In particular we classify all chiral hypermaps up to genus 4.

The relational expansion on graphs

Boštjan Brešar

The concept of expansion of a graph has proved to be useful in the study of median, quasi-median and partial Hamming graphs. Recently, several new classes of graphs (e.g. semi-median) have been introduced via this concept. The basic idea of expansion is that given a graph G we obtain a graph G' by enlarging a subgraph of G according to a certain rule in such a way that G' inhertis certian properties of G. In this talk we consider binary, relational expansion where as the rule of expansion a direct (relational) product is used. Especially we discuss a preservation of connectivity by this expansion.

Homomorphisms to powers of digraphs

Richard C. Brewster, Pavol Hell

Given a digraph G and a sufficiently long directed path P, a classical result is that G is homomorphic to P if and only if all cycles in G are *balanced.* The purpose of this paper is to study digraphs that contain unbalanced cycles. In this case, we may be able to find a homomorphism from G to a *power* of P. Our main result is that the minimum power of P to which G admits a homomorphism equals the maximum imbalance of any cycle in G. We conclude by relating our result to the chromatic number of the underlying graph of G and a result of Minty.

Game chromatic index of k-degenerate graphs

Leizhen Cai, Xuding Zhu

We consider the following edge coloring game on a graph G: Two players Alice and Bob, with Alice moving first, alternately select an uncolored edge e of G and assign it a color from the color set $\{1, 2, \ldots, c\}$ that has not been assigned to any edges adjacent to e. Bob wins if, at any stage of the game, there is an uncolored edge adjacent to edges in all c colors; otherwise Alice wins. Note that when Alice wins, all edges of G are properly colored. The *game chromatic index* of a graph G is the smallest c for which Alice has a winning strategy.

In this talk, we consider the edge coloring game on k-degenerate graphs, which contain planar graphs and partial k-trees. For this purpose, we introduce an edge ordering game on graphs, and use it to show that the game chromatic index of a k-degenerate graph is at most $\Delta + 3k - 1$, where Δ is the maximum vertex degree of the graph. We also show that the game chromatic index of a forest of maximum degree 3 is at most 4 when the forest contains an odd number of edges.

On the Ramsey number $R(3, 6)$

David Cariolaro

The Ramsey number $R(3, 6)$ is defined as the minimum positive integer n with the property that every graph with n vertices contains either a triangle or an independent set of size 6. It is known that $R(3, 6) = 18$ and it is easy to find an example of a triangle-free graph with 17 vertices and no independent set of size 6 (thus proving $R(3, 6) > 17$), but the author is unaware of any not computer-aided proof of the fact that $R(3, 6) \leq 18$. In this talk we give a proof of this inequality.

Minimal base sizes and proper k-covers

Nicholas Cavenagh

In this talk a link is given between permutation groups and combinatorial enumeration. Given a group G acting on a finite set X, a *base* is a list of elements of X whose pointwise stabilizer is the identity. Bases are used in computational group theory to store information about a group action; so the smaller the base size, the better. A *proper k-cover* of a finite set S is a set of distinct non-empty subsets of S such that each element is in exactly k of these subsets. Using results on the enumeration of proper k-covers we can calculate the minimal base size for a class of group actions. For these actions G is the automorphism group of a complete multipartite graph and X consists of subgraphs that intersect each partite set.

Combinatorial group-theoretic methods in graph theory

Marston Conder

This lecture will be a brief survey of some useful aspects of combinatorial group theory and associated computational methods, with particular reference to finitely-presented groups and their applications in the theory of graphs and maps. Specific topics include low index subgroup procedures, Schreier coset graphs, semi-direct product constructions, and relevant theorems of Schur and Zassenhaus, with applications to regular maps and arc-transitive graphs. A few recent results and open questions will be described along the way.

Isometric embedding of mosaics into cubic lattices

Michel Deza, Mikhail Shtogrin

We review mosaics T (edge-to-edge tilings of Euclidean plane by regular polygons) with respect of possible embedding, isometric up to a scale, of their skeletons or skeletons of their duals T^*, into some cubic lattice Z_n. Main result of this paper is the classification, given in Table 1, of all 58 such mosaics among all 165 mosaics of the catalog (D. Chavey, *Tilings by regular polygons - 2; a catalog of tilings*, Computers Math. Applic. **17** (1989) 147–165), including all main classifications of mosaics.

Some applications of monotonic functions

Tamás Fleiner

In the talk we explain how does a proposition, well-known from lattice theory imply a general lemma on so called comonotonic set-functions. Using this lemma one can unify and extend several earlier results, like the stable marriage theorem of Gale and Shapley, the linear description of the stable matching polytope due to Vande Vate and Rothblum, a theorem on monochromatic paths by Sands et.al., or the linking theorem of Pym. We can also deduce new results as well, like on minimum cost spanning sets of matroids or on common antichains of partial orders. The algorithmic consequence of these observations is that the proposal algorithm of Gale and Shapley can be extended to efficient algorithms for the above problems.

Bipartizing matchings and Sabidussi's compatibility conjecture

Herbert Fleischner

The proof of the Cycle-Plus-Triangles-Theorem (every 4-regular graph decomposable into a hamiltonian cycle and triangles, is 3-colourable) as given by M. Stiebitz and myself, required the proof that for such graphs the number of eulerian orientations = 2 (mod 4). This implies the existence of an eulerian orientation for any such graph such that the triangles in the decomposition are transitively oriented. This, in turn, implies that if G is a cubic graph with dominating cycle C, then there exists a matching M in G such that

a) M contains no edge of C ;
b) M covers all vertices not in C ;
c) $G - M$ is homeomorphic to a cubic (hamiltonian) bipartite graph.

Consequently we call such M a bipartizing matching.

Utilizing the concept of bipartizing matchings, it has been shown before that if G is a cubic graph with dominating cycle C and bipartizing matching M, then G has a NZ6F such that C is cyclically oriented, where the flow is assumed to consist of positive integers only. It has also been shown that if G contains two disjoint bipartizing matchings, then the analogous conclusion holds w.r.t. NZ5F. This result led to the following conjecture.

Conjecture. If G is a snark with dominating cycle C , then it has two disjoint bipartizing matchings.

In this talk it will be shown that if a cubic graph G satisfies this conjecture, then it has a cycle double cover containing C , and that a closely related eulerian graph with eulerian trail T (derived from C) has a cycle decomposition compatible with C. Consequently, if the above conjecture is true, then both the CDCC and the NZ5FC are being reduced to the dominating cycle conjecture which states that every snark has a dominating cycle.

Scheduling Czech National Basketball League

Dalibor Fronček

The Czech National Basketball League used to be traditionally scheduled as a round-robin tournament of "twins" of teams. It means that $4n$ teams were divided into $2n$ pairs according to their geographical location. Then a schedule for $2n$ teams was used to determine the weekends when the pairs of teams would meet. After Saturday games, the teams of the same pair mutually switched their opponents for Sunday games. An extra round in which the twin teams met was added. Mathematically speaking, the one-factorization GK_{2n} of the graph K_{2n} was used for a construction of a special one-factorization of K_{4n}.

The traditional model was later (partially) abandoned. Now only some rounds are played as twin-rounds while the other games are played once a week. The twin-rounds were preserved because there are eccentrically located pairs of geographically close teams.

The factorization GK_{2n} leads necessarily to sequences of up to five consecutive games played by a given team either in the home field or in the opponent's fields. However, as the schedule is not fully twin-teams schedule, it is possible to use other one-factorizations to make the home-away patterns of teams more regular. We discuss in this paper some aspects of such weak twin-teams schedules, especially their home-away patterns.

On Quadratic Modulo 2^n Cayley Graphs

Reinaldo E. Giudici, Aurora A. Olivieri

A family of undirected Cayley graph $Cay\,(Z_{2^n}, QR^*(2^n))$ is studied, where Z_{2^n} denotes the aditive group of integers modulo 2^n and the set $S^* = S \cup \{-S\}$, where $S = QR^*(2^n)$ denotes the set of quadratic residues of Z_{2^n}, zero excluded. In this paper we show that the eccentricity of the graphs $Cay\,(Z_{2^n}, QR^*(2^n))$ is 2 and we give recursive formulas for the number of triangles in the graph. In addition we discuss the number of k-residues modulo p^n, p prime and $n \geq 1$.

Hamilton Cycles in Cayley Graphs

Henry Glover

We study the question whether every Cayley graph of a finite group presentation has a Hamilton cycle. We give an affirmative answer for presentations $G = \langle a, b | a^2 = b^s = (ab)^3 = 1 \rangle$ where $s = 2k+1$, $4k$, and $4k+2$ when $|G| = 4l+2$. We will also mention our inabilty to find further negative answers to the question: Does every connected, vertex transitive graph have a Hamilton cycle?

Pentomino Exclusion Problem

Sylvain Gravier, Charles Payan

We are interested in the *Pentomino Exclusion Problem* due to Golomb: *Find the minimum number of unit-square to be placed on a $k \times n$ chessboard so as to exclude all pentominos.*

Using an appropriate definition of density of a tiling, we obtain the asymptotical value of this number, and we establish this number for the $4 \times n$ chessboard when $n \equiv 0$ [4].

Configurations between geometry and combinatorics

Harald Gropp

Definition *A configuration* (v_r, b_k) *is a finite incidence structure of* v *points and* b *lines such that*

(i) there are k *points on each line and* r *lines through each point, and*

(ii) there is at most one line through two given points.

For further information see [1]. The word configuration in the above sense was used in Theodor Reye's book *Geometrie der Lage* (1876) for the first time.

There are two main questions for given parameters v, r, b, k. Does a configuration (v_r, b_k) exist? How many nonisomorphic of these configurations are there?

At least the following obvious necessary conditions must hold:

$$vr = bk, \qquad v \geq r(k-1)+1, \qquad b \geq k(r-1)+1.$$

However, they are not sufficient. For example, there is no configuration 22_5.

Concerning the problem of determining all nonisomorphic configurations with parameters v, r, b, k already in the last century all configurations n_3 with $n \leq 11$ were constructed. During the last ten years further results have been obtained. For example, the number of configurations $(12_4, 16_3)$ is 574.

From the point of view of pure combinatorics the problems of realizing and drawing configurations may be artificial. However, because of the origin and the early history of configurations they are quite natural. Realizations and drawings have already been discussed in [3] and [2].

In order to find a drawing of a configuration in the plane it is necessary to assign coordinates to the points such that points are collinear in the plane if and only if they are on a common line of the configuration.

A realization of a configuration with point set P and line set L over a field K is a mapping from P to K^3 such that for all distinct $i, j, k \in P$ we have $det(x_i, x_j, x_k) = 0$ if and only if i, j, k are collinear.

Since 1997 there has been a cooperation with Tomaž Pisanski and his group in Ljubljana in order to investigate configurations and there properties, in particular to obtain nice drawings of configurations.

Hence my talk is dedicated to the special anniversary of *Tomaž Pisanski*.

References

[1] H. Gropp, Configurations, in The CRC Handbook of Combinatorial Designs, (eds.) C.J. Colbourn, J.H. Dinitz, Boca Raton (1996), 253–255.

[2] H. Gropp, The drawing of configurations, in Lecture Notes in Computer Science 1027, (ed.) F.J. Brandenburg, Graph Drawing GD'95 Proceedings, Springer (1995), 267–276.

[3] H. Gropp, Configurations and their realization, Discrete Math. 174 (1997), 137–151.

An Optimal Fault-Tolerant Message Routing Algorithm for Double-Loop Networks

D. J. Guan, C. Y. Chou, K. L. Wang

Messages routing is an important function of a computer network. A message routing algorithm is *optimal* if every message is sent along a shortest path from its source node to its destination node. In this paper, we consider routing algorithms for double-loop networks. A weighted double-loop network can be modeled by a weighted directed graph $G(n; h_1, h_2; w_1, w_2)$ with vertex set $Z_n = \{0, 1, \dots, n-1\}$, and edge set the union of $E_1 = \{(u, u+h_1) | u \in Z_n\}$ and $E_2 = \{(u, u+h_2) | u \in Z_n\}$. Assume that the weight of each edge in E_1 is w_1, and the weight of each edge in E_2 is w_2. Guan has presented an optimal message routing algorithm for weighted double-loop networks without using routing tables. In this paper, we present an optimal message routing algorithm when a vertex fault or an edge fault is detected. At each vertex, for any destination, the algorithm needs only constant time and constant spaces to determine the next vertex on the shortest path to which the message must be sent.

On the Independence Number of a Graph

Jochen Harant

For a finite undirected graph G on n vertices some continuous optimization problems taken over the n-dimensional unit cube are presented and it is proved that their optimum values equal the independence number of G.

Special numbers of crossings for complete graphs

Heiko Harborth

Different drawings of the complete graph with the edges inside or outside of a cycle without crossings are compared with general drawings. Do numbers of crossings exist which cannot be realized by these special drawings?

On supersimple graph designs

Sven Hartmann

A supersimple (n, G, λ) graph design is a family $\mathcal{G}$ of subgraphs of the complete graph K_n satisfying the following conditions:

(i) All elements of $\mathcal{G}$ are isomorphic to a given graph G.

(ii) Every edge of K_n occurs in exactly λ elements of $\mathcal{G}$.

(iii) Any two distinct elements of $\mathcal{G}$ have at most one edge in common.

Supersimple (n, K_k, λ) graph designs were introduced and first studied by Gronau and Mullin [1].

Our objective is to generalize their concept to arbitrary graph designs, to give examples of supersimple graph designs and to prove the asymptotic existence of supersimple graph designs whenever certain necessary conditions hold.

Moreover, we shall discuss consequences of our results to orthogonal double covers of complete graphs, i.e. $(n, G, 2)$ graph designs where any two distinct elements of $\mathcal{G}$ share precisely one edge.

References

[1] H.-D.O.F. Gronau and R.C. Mullin. On supersimple 2-(v, k, λ) designs. *J. Combin. Math. Combin. Comput.*, 11:113–121, 1992.

Channel assignement and multicolouring subgraphs of the triangular lattice

F. Havet, J. Žerovnik

A basic problem in the design of mobile telephone networks is to assign sets of radio frequency bands (colours) to transmitters (vertices) to avoid interference. Often the transmitters are laid out like vertices of a triangular lattice in the plane. We investigate the corresponding colouring problem of assigning sets of colours of given size k to vertices of the triangular lattice so that the sets of colours assigned to adjacent vertices are disjoints. We prove that every triangular-free induced subgraph of the triangular lattice is 7-[3]colourable.

We also give an alternative proof of the fact that every triangular-free induced subgraph of the triangular lattice is 5-[2]colourable that yields a distributed algorithm a constant time algorithm for finding a 5-[2]colouring of such a graph.

List partitions and finding skew cutsets

T. Feder, P. Hell, S. Klein, R. Motwani

List partitions generalize list colourings and list homomorphisms, and can model problems such as finding clique cutsets, recognizing split graphs and their generalizations, finding homogeneous sets, and finding skew partitions. I will describe polynomial algorithms for certain list partition problems, and subexponential algorithms for certain others.

On Possible Counterexamples to Negami's Planar Cover Conjecture

Petr Hliněný, Robin Thomas

A graph H is a cover of a graph G if there exists a mapping φ from $V(H)$ onto $V(G)$ such that for every vertex v of G, φ maps the neighbours of v in H bijectively to the neighbours of $\varphi(v)$ in G. Negami conjectured in 1987 that a connected graph has a finite planar cover if and only if it embeds in the projective plane.

It follows from the results of Archdeacon, Fellows, Negami, and the first author that the conjecture holds as long as $K_{1,2,2,2}$ has no finite planar cover, but those results seem to imply little about possible counterexamples. We show that that there are, up to obvious constructions, at most 16 possible counterexamples to Negami's conjecture.

Some new aspects of the Buneman graph

K. T. Huber, V. Moulton

One of the main goals in phylogenetic analysis is to represent the phylogenetic relationships between the elements of a finite set X of taxa by a bifurcating tree whose leaves are labelled by X. However, in most cases such a tree is not unambiguously supported by the data. If this is the case and the phylogenetic data induces *splits* (i. e. bipartitions) of X, the so called *Buneman graph* can provide an alternative way to represent the data. This median graph has many attractive properties, although it does suffer from a drawback, in that, even for fairly small data sets, the Buneman graph can be very complex.

In this talk, we look at two ways to deal with this problem: First, we describe a way to construct the Buneman graph locally. Second, we describe a way to thin out the Buneman graph, based on techniques coming from *T-theory*. We finish with a look at a generalization of the Buneman graph to arbitrary partitions of X, which often arise in real data sets, called the *relation graph*.

Four-coloring Eulerian triangulations of surfaces

Joan P. Hutchinson, Bruce Richter, Paul Seymour

C. Thomassen has shown that every graph embedded on a surface $S(g)$ of genus $g > 0$ with sufficiently large edge-width can be 5-colored. It is well known that a planar, Eulerian (i.e., all vertices of even degree) triangulation can be 3-colored. We show that every Eulerian triangulation of an orientable surface $S(g)$, $g > 0$, with sufficiently large edge-width (or representativeness) can be 4-colored and that such a result does not hold for the projective plane.

Median graphs and triangle free graphs

Wilfried Imrich

This is primarily a talk about median graphs, isometric subgraphs of hypercubes and connections with triangle-free graphs. But it is also a talk about structure and recognition of graph classes and graph decompositions.

It turns out that median graphs, a very modestly defined class of bipartite graphs which can also be characterized as retracts of hypercubes, are rich in structure and pose many subtle properties which make it possible that median graphs, once recognized as such, can be quickly embedded into hypercubes, but not easily recognized. Their recognition is connected with the decomposition of graphs with respect to a metrically defined relation on the edge-set and properties of convex subgraphs. Moreover, there exists a natural bijection between a subclass of all median graphs and the class of triangle-free graphs which can be exploited for the construction of recognition algorithms.

These concepts and results carry over from hypercubes, i.e. Cartesian products of complete graphs on two vertices, to Hamming graphs, i.e. Cartesian products of complete graphs on arbitrarily many vertices. In other words, in this talk hypercubes, median graphs, semimedian graphs, isometric subgraphs of hypercubes and the generalization of these concepts to Hamming graphs, quasimedian graphs and isometric subgraphs of Hamming graphs are treated. The structure of these graphs is described as well as decomposition methods for their recognition. Moreover, the complexity of recognition and embedding algorithms for these graph classes is discussed and a close relationship with the recognition of triangle-free graphs established.

Skew-morphisms of regular Cayley maps

Robert Jajcay, Jozef Širáň

A Cayley map is a 2-cell embedding of a Cayley graph in an orientable surface with the same local orientation of outgoing arcs at each vertex. Due to their underlying group structure, Cayley maps are inherently highly symmetrical, and are the natural first choice for the study of regular maps.

Regular Cayley maps possess the richest automorphism group possible, and have received a considerable amount of attention. First results concerning regular Cayley maps are due to Biggs, who introduced the concept of a balanced Cayley map and noticed the sufficiency of the existence of a certain group automorphism of the underlying group for the regularity of the resulting Cayley map. His results were extended by Širáň and Škoviera who proved the existence of the Biggs's group automorphism to be both sufficient and necessary, introduced antibalanced Cayley maps, and characterized regular antibalanced Cayley maps via the existence of anti-automorphisms. These concepts were further generalized by Jajcay using the concepts of rotary mappings and rotary extensions.

In our talk we will unify all the above ideas by introducing the concept of a skew-morphism of a group. We reprove and extend several of the known results and address one of the two central problems of the theory of regular maps, namely the problem of the classification of finite groups admitting a regular Cayley map.

Directed strongly regular graphs: Cayley graph constructions

Leif K. Jørgensen

A directed strongly regular graph with parameters (n, k, μ, λ, t) is a regular directed graph on n vertices with degree k, every vertex is incident with t undirected edges, and the number paths of length 2 directed from a vertex x to a vertex y is λ if there an $x \to y$ edge and μ otherwise. These graphs were first investigated by Duval [1].

In this talk we consider constructions of directed strongly regular graphs as (generalized) Cayley graphs. More than half of the parametersets with $n \leq 30$ can be realized as Cayley graphs of non-abelian groups and some infinite series of Cayley graph constructions are known (see [2]).

Cayley graphs of abelian groups cannot be directed strongly regular. But we prove that in the case $\mu = \lambda = t - 1$ a directed strongly regular graph can be constructed as a generalized Cayley graph (see [3]) of a cyclic group if μ divides $k - 1$.

[1] A. Duval, A directed version of strongly regular graphs *J. Comb. Th. A* **47**, (1988), 71–100.

[2] M. Klin, A. Munemusa, M. Muzychuk, P.-H. Zieschang, Directed strongly regular graphs via coherent (cellular) algebras, preprint.

[3] D. Marušič, R. Scapellato, N. Zagaglia Salvi, Generalized Cayley graphs, *Discr. Math.* **102** (1992) 279–285.

Krein parameters of antipodal distance-regular graphs of diameter 4

Aleksandar Jurišić, Jack Koolen, Paul Terwilliger

We determine which Krein parameters of nonbipartite antipodal distance-regular graphs with diameter three and four can vanish. We prove that in an antipodal distance-regular graph Γ with diameter four and vanishing Krein parameters q^4_{11} and q^4_{44} every second subconstituent graph is again an antipodal distance-regular graph of diameter four. Finally, if Γ is also a double-cover, i.e., Q-polynomial, then it is 2-homogeneous.

We also study tight distance-regular graphs with small diameter. It turns out that in the case of diameter three these are precisely the Taylor graphs and in the case of antipodal distance-regular graphs with diameter four these are precisely the graphs for which the Krein parameter q^4_{11} vanishes.

On locally claw-free graphs

Vladislav Kabanov

*No abstract received

Novel Highly Symmetrical Trivalent Graphs which Lead to Negative Curvature Carbon and Boron Nitride Chemical Structures

R. B. King

Consider a trivalent graph constructed from non-hexagons. A leapfrog transformation, which consists of omnicapping (stellation) followed by dualization, of this graph triples the number of vertices while preserving the automorphism group of the original graph and provides the minimum number of new hexagons to dilute the non-hexagons so that no pair of the non-hexagons has a common edge. Such a process can be used to construct the truncated icosahedron graph of the C60 fullerene from the regular dodecahedron. The most symmetrical trivalent graphs containing heptagons or octagons do not lead to analogous finite polyhedral structures but instead can be embedded into infinite periodic minimal surfaces based on unit cells with a genus 3 surface. A graph described by Klein in the 19th century consisting of 24 heptagons can be used to generate possible but not yet experimentally realized carbon structures through such a leapfrog transformation. The automorphism group of the Klein graph is the simple PSL(2,7) group of order 168, which can be generated from 2 x 2 matrices in a seven-element finite field F7 analogous to the generation of the icosahedral group of order 60 by a similar procedure using F5. Similarly a graph described by Walther Dyck, also in the 19th century, consisting of 12 octagons on a genus 3 surface can generate possible carbon or boron nitride structures consisting of hexagons and octagons through a leapfrog transformation. The automorphism group of the Dyck graph is a solvable group of order 96 but does not contain the octahedral group as a normal subgroup and is not a normal subgroup of the automorphism group of the four-dimensional analogue of the octahedron. The spectra of the Klein and Dyck graphs and their duals exhibit many features similar to the spectra of the dodecahedron/icosahedron and cube/octahedron dual pairs, respectively.

Metric and other properties of benzenoid graphs

Sandi Klavžar

A benzenoid graph is formed by the vertices and edges of the hexagonal lattice which lie on, and in the interior, a given circuit of the lattice. Benzenoid graphs are quite important in chemical graph theory. In this talk we will present several recent results related to these graphs. In particular, we will show a linear and a sublinear recognition algorithm which are based on isometric embeddings of benzenoid graphs. Some other properties will also be presented, for instance, benzenoid graphs have a convenient dismantling scheme. Different parts of the talk are join work with I. Gutman, B. Mohar and V. Chepoi.

Embedding graphs into Hypercubes

Mohamed Kobeissi, Michel Mollard

The aim of this talk is to prove that certain trees are spanning trees of the hypercube Q_n. We introduce a new family of graphs which span hypercubes and we characterize the doubke starlike trees with maximum degree six that span a hypercube. We conclude about some open problems about spanning graphs and partitioning the hypercube into vertex-disjoint cycles of even length.

k-Cross-Free Set Systems

A. Dress, M. Klucznik, J. Koolen, V. Moulton

Two subsets Y and Z of a set X are crossing if all of $Y \cap Z$, $\overline{Y} \cap Z$, $Y \cap \overline{Z}$, and $\overline{Y} \cap \overline{Z}$ are not empty (where $\overline{Y} = X - Y$.) A set system $\mathbf{S} \subset 2^X$ is called k-cross-free if for all $S_1, \ldots, S_k \in \mathbf{S}$ we have $i \neq j$ such that S_i does not cross S_j.
3-cross-free set systems were introduced by Karzanov and Lomonosov in their so-called Locking Theorem. Karzanov conjectured that 3-cross-free set systems $\subseteq 2^X$ have $O(n)$ elements, where $n = \#X$. Let $karz(n)$ be defined as the maximal number of elements of a 3-cross-free set on X with $\#X = n$. Pevzner (1987) showed that $karz(n) \leq 12n$ and Fleiner (1998) showed $karz(n) \leq 10n$. In this talk, we show that $karz(n) = 8n - 12$. We will give an outline of the proof in our talk.

Matchings and Hadwiger's Conjecture

Andrei Kotlov

Assuming that G on n vertices is a minimal counterexample to Hadwiger's Conjecture $\chi(G) \leq \eta(G)$, we apply Gallai-Edmonds Structure Theorem to its complement, H, to find that H has a matching of size $\lfloor n/2 \rfloor$. Hence $\chi(G) \leq \lceil n/2 \rceil$. Further, H is two-connected and of tree width at least three. Ditto for Colin de Verdière's Conjecture $\mu(G) + 1 \geq \chi(G)$.

Local spanning trees in graphs and hypergraph decomposition with respect to edge connectivity

Matthias Kriesell

We present a sufficient condition to a subset A of vertices of a graph G for the existence of a system of k edge-disjoint Steiner trees with respect to A, i. e. subtrees of G which cover A.

As a corollary, we prove that every $k \cdot (k+1)$-edge-connected hypergraph of rank at most $k+1$ is decomposable into k connected spanning subhypergraphs.

Orthogonal Double Covers of Complete Graphs by Paths and Cycles

Uwe Leck

An orthogonal double cover (ODC) of the complete graph K_n by some given graph G is a collection $\mathcal{G} = \{G_1, G_2, \ldots, G_n\}$ of spanning subgraphs of K_n such that the following conditions are satisfied:

1. Every edge of K_n is contained in exactly two members of $\mathcal{G}$.
2. Every two distinct members of $\mathcal{G}$ share exactly one edge.
3. G_i is isomorphic to G for $i = 1, 2, \ldots, n$.

We present a new construction for the case that $G = P_n$, the path on n vertices. In combination with a result of Horton and Nonay, our construction provides an ODC of K_n by P_n for all $n = 2^\alpha p_1^{2\alpha_1} \ldots p_m^{2\alpha_m}$ such that the p_i's are primes with $p_i \equiv 3 (\mathrm{mod}\ 4)$ and $\alpha \neq 1, 2$. Together with a result of Heinrich and Nonay this implies the existence of an ODC of K_{4n} by an almost-hamiltonian cycle for the same n's.

On finite s-arc transitive graphs

Cai Heng Li

A finite graph Γ is said to be (G, s)-arc transitive if all directed paths of size at most s are equivalent under the G-action, where s is a positive integer and G is a group of automorphisms of Γ.

This class of graphs has been investigated for more than half a century. There are two different methods for studying these graphs in the literature, that is, the local-action analysis and the global-action analysis. The former has led to some fundamental results, for example, proving $s \leq 7$ and classifying the point-stabilizer G_α for $s \geq 4$; while the latter has led to some fundamental results about the structure of G.

In this talk, I will report on some recent results of mine obtained by a combination of the two different methods, for example, it is shown that there exist no 4-arc transitive graphs of odd order or a prime-power order, and that there are only very 'few' 4-arc transitive graphs which are vertex-primitive or vertex-biprimitive. Also some new 4-arc transitive graphs are constructed.

Towards a characterisation of Pfaffian graphs

C.H.C. Little, F. Rendl

In 1967 Kasteleyn devised a method for enumerating the 1- factors of a planar graph. The idea was to assign plus or minus signs to the 1-factors by using an orientation of the graph. If the orientation can be chosen so that the resulting signs of the 1- factors are all equal, then Kasteleyn showed that the problem of enumerating the 1-factors reduces to that of evaluating a certain determinant. Graphs which can be given the required orientation are said to be Pfaffian. Kasteleyn showed that all planar graphs are Pfaffian, but some non-planar graphs are also Pfaffian. Pfaffian bipartite graphs have been characterised. We show how the problem of characterising Pfaffian graphs may be divided into four cases, and indicate what progress has been made.

Regular pakings of regular graphs

A. Gutierrez, A. S. Llado

A graph H is G-decomposable if it contains subgraphs $G_1, \cdots, G_m$, $m \geq 2$,isomorphic to G whose sets of edges partition $E(H)$. G is said to be an isopart of H. Wilson proved that every nonempty graph is an isopart of infinitely many regular, connected graphs.

Let $r_0(G)$ and $n_0(G)$, the minimum degree and the minimum order of a G-decomposable graph, and $m_0(G)$, the minimum number of graphs in a G-decomposition. Fink et al. studied the values of these parameters for K_n, $K_{n,m}$ and the $n-$cube. We obtain tight bounds for these parameters when G is a regular graph.

Cayley Graphs of order 10

Reinaldo Giudici, Gabor Loerincs

We generate all Cayley graphs of order 10, that is, when the Γ group is Z_{10} or D_{10}. A method based on $Aut(\Gamma)$ is used to decide when two Cayley graphs coming from the same Γ group are isomorphic. Some general results are obtained for graphs coming from the diedral group D_n. We list and classify all Cayley graphs obtained.

Regular graphs and their extensions

Alexandre A. Makhnev, Dmitrii V. Paduchikh

A geometry of rank 2 is a structure with two types of elements, which we normally call points and blocks and an incidence relation between points and blocks without repetition. Thus any block coincides with set of points incident to this block. We shall use informal language, such as *contains, lies on* etc.

If $\mathcal{S} = (\mathcal{P}, \mathcal{B})$ is a geometry of rank 2 then the point-graph $\Gamma = \Gamma(\mathcal{S})$ is the graph whose vertices are the points of $\mathcal{S}$, with two distinct vertices being adjacent if there is a block containing them both. We call two distinct points of $\mathcal{S}$ collinear if they are adjacent in Γ. The geometry $\mathcal{S}$ will be said to be connected, regular and so on, according as Γ has these properties.

Let $\mathcal{S}$ be a geometry of rank 2. The residue $\mathcal{S}_{\mathcal{P}}$ of $\mathcal{S}$ at a point P is defined to be $(\mathcal{P}_P, \mathcal{B}_P)$, where $\mathcal{P}_P$ consists of the points collinear with P and $\mathcal{B}_P$ is the set of all blocks incident with P (with the *natural* induced incidence). Let $\mathcal{G}$ be a family of geometries of rank 2 and every residue $\mathcal{S}_P$ is in $\mathcal{G}$, then we say that $\mathcal{S}$ is an extension of $\mathcal{G}$. A geometry $\mathcal{S}$ is said to be triangular if whenever X, Y, Z are three pairwise collinear points, then there is at least one block of $\mathcal{S}$ which contains all three of X, Y, Z.

A point-block pair (P, y) is a flag if $P \in y$, and an antiflag otherwise. If (P, y) is antiflag, then $\varphi(P, y)$ is the number of points on y which are collinear with P. We say that $\mathcal{S}$ is (strongly) φ-uniform if $\varphi(P, y) = 0$ or φ ($\varphi(P, y) = \varphi$) for any antiflag (P, y) and some constant φ. The blocks of $\mathcal{S}$ are called lines if distinct blocks intersects in at most one point.

If $\mathcal{S}$ is a geomerty such that every line has $s+1$ points, every point is on $t+1$ lines (with $s > 0,\ t > 0$) and $\mathcal{S}$ is strongly α-uniform ($\alpha > 0$), then $\mathcal{S}$ is a α-partial geometry of order (s, t), or $pG_\alpha(s, t)$ or a pG_α. A generalized quadrangle $GQ(s, t)$ of order (s, t) is a $pG_1(s, t)$. A geometry $pG_t(s, t)$ is a net and $pG_{s+1}(s, t)$ is 2-design with $\lambda = 1$.

A point graph of partial geometry $pG_\alpha(s, t)$ is strongly regular with parameters $v = (s{+}1)(1{+}st/\alpha)$, $k = s(t{+}1)$, $\lambda = (s{-}1){+}(\alpha{-}1)t$, $\mu = \alpha(t{+}1)$. A strongly regular graph Γ with this parameters is called pseudo-geometric graph for this geometry. This graph is called geometric if it contains such family $\mathcal{B}$ of $(s+1)$-cliques that $(\Gamma, \mathcal{B})$ is $pG_\alpha(s, t)$. If Γ is pseudo-geometric graph for partial geometry $pG_\alpha(s, t)$, then it has eigenvalues k, $r = s -$

$\alpha,\ -m=-(1+t)$ of multiplicities 1, $f=\dfrac{s(s+1)t(t+1)}{\alpha(s+t+1-\alpha)}$, $g=\dfrac{s(s-\alpha+1)(st+\alpha)}{\alpha(s+t+1-\alpha)}$ accordingly.

A connected extension of the family of pG_α's is an EpG_α or an EpG. It may be shown that if $\mathcal{S}$ be an EpG_α, then for any antiflag (P,y) with $\varphi(P,y)\neq 0$, we have $\varphi(P,y)\geq \alpha+1$, and that $\mathcal{S}$ is triangular iff it is $(\alpha+1)$-uniform.

Consider partial geometry $\mathcal{S}$ with a small number points on line. If $s=1$, then $\mathcal{S}$ is either $GQ(1,t)$ (the geometry of vertices and edges of the complete bipartite graph with parts of size $t+1$) or $pG_2(1,t)$ (the geometry of vertices and edges of the triangular graph $T(t+2)$).

If $s=2$, then $\mathcal{S}$ is either $GQ(2,t)$, $t=1,2,4$ or dual net $pG_2(2,t)$ or 2-design $pG_3(2,t)$ (Steiner triple system).

If $s=3$, then $\mathcal{S}$ is either $GQ(3,t)$, $t=1,3,5,6,9$ or $pG_2(3,t)$, $t=1,2,4$ or dual net $pG_3(3,t)$ or Steiner 2-design $pG_4(3,t)$. It is known (the author [1] and independently Haemers [2]) that pseudo-geometric graph for $GQ(3,6)$ does not exist. Apart from, partial geometry $pG_2(3,4)$ does not exist [3].

Finally, if $s=4$, then $\mathcal{S}$ is either $GQ(4,t)$, $t=1,2,4,6,8,11,12,16$ or $pG_2(4,t)$, $t=1,2,3,7,9,12,17,27$ or $pG_3(4,t)$, $t=3,6$ or dual net $pG_4(4,t)$ or Steiner 2-design $pG_5(4,t)$. At present, the existence $GQ(4,11)$, $GQ(4,12)$, $pG_2(4,7)$ and $pG_2(4,9)$ are unknown (moreover the existence pseudo-geometric graphs for this geometries are unknown). The family pseudo-geometric graphs for $pG_2(4,t)$ contains triangular graph $T(6)$, the quotient of Johnson graph $\bar{J}(8,4)$, Hermitian graph $\mathcal{H}(5)$, Haemers graph (geometric graph for $pG_2(4,17)$) and McLaughlin graph.

Locally $GQ(2,t)$ graphs were classified by F.Buekenhout and X.Hubaut [4]. Let Γ be a locally $GQ(3,t)$ graph. There are all cases were considered: $t=1$ by A.Brouwer and A.Blokhuis [5], $t=3$ by A.Makhnev [6] and independently by D.Pasechnik (with using computer) [7], $t=5$ by A.Makhnev [8], $t=9$ by D.Pasechnik [9] (with using computer) and independently by A.Makhnev and D.Paduchikh [10]. Locally $GQ(4,2)$ graphs consider in the next theorem.

Theorem 1 (A.Makhnev, D.Paduchikh). *Let Γ be a connected amply regular locally $GQ(4,2)$ graph. Then Γ is one of the following graphs:*

(1) *strongly regular graph with parameters* $(126,45,12,18)$ *with automorphism group $O_6^-(3)$;*

(2) *the unique distance regular graph on* 378 *vertices with intersection array* $(45,32,12,1;1,6,32,45)$ *(it is the* 3*-cover of graph from* (1)*).*

The step on the way of classification of a uniform $EpG_\alpha(4,t)$ is the following theorem (Makhnev A., Paduchikh D.).

Theorem 2. *Let $\mathcal{S}$ be a φ-uniform $EpG_2(4,t)$, $\varphi \leq 5$. Then one of the following statements holds:*

(1) $\varphi = 3$, *if also* Γ *be amply regular, then* Γ *is either Taylor graph or strongly regular graph with parameters* $(210, 95, 40, 45)$*;*

(2) $\varphi = 5$, Γ *is complete multipartite graph with* 6 *parties of size* $2t + 1$ *and* $t = 1,\ 2$ *or* 7.

The existence of a strongly regular graph with parameters $(210, 95, 40, 45)$ is open.

References

1. Makhnev A. On pseudo-geometric graphs of some partial geometries. In: Problems in Algebra 1997. V. 11. Homel-University Press, 60–67.

2. Haemers W. There exist no (76,21,2,7) strongly regular graph. In: Finite Geometries and Combinatorics, London Math. Soc. Lecture Notes. Eds F. De Clerck et.al. 1993. V. 191. Cambridge Univ. Press: Cambridge, 175–176.

3. Brouwer A.E., van Lint J.H. Strongly regular graphs and partial geometries. Enumeration and Design, Proc. Silwer Jubilee Conf. on Combinatorics. Acad.Press, Toronto 1984, 85–122.

4. Buekenhout F., Hubaut X. Locally polar spaces and related rank 3 graphs. J. Algebra 1977. V. 45, 391–434.

5. Blokhuis A., Brouwer A.E. Locally 4-by-4 grid graphs. J. Graph Theory 1989. V. 13, 229–244.

6. Makhnev A.A. Finite locally $GQ(3,3)$-graphs. Sibirean Math. J. 1994. V. 35, 1314–1324.

7. Pasechnik D. The triangular extensions of a generalized quadrangle of order (3,3). Simon Stevin 1995. V. 2, 509–518.

8. Makhnev A.A. Locally $GQ(3,5)$-graphs and geometries with short lines. Discr. Matem. 1998. V. 10, 72–86.

9. Pasechnik D. The extensions of the generalized quadrangle of order (3,9). Europ. J. Comb. 1996. V. 17, 751–755.

10. Makhnev A.A., Paduchikh D.V. On the structure of connected locally GQ(3,9) graphs. Discret. Anal. and Issled. Operacii 1998. V. 5, 61–77.

Action graphs and coverings

Aleksander Malnič

Epimorphisms of group actions induce coverings of action graphs. Problems related to reconstruction from quotients and lifting automorphisms with some applications to maps on surfaces will be discussed.

On Computation of the Local Density of a Graph

Jože Marinček, Bojan Mohar

*No abstract received.

On transitive permutation groups with a non-self-paired suborbit of length 2 and their graphs

Dragan Marušič

Finite transitive permutation groups having a non-self-paired suborbit of length 2 will be discussed (with the emphasis on their point-stabilizers). In graph-theoretic language, this corresponds to vertex- and edge- but not arc-transitive group actions on finite tetravalent graphs. In particular, a construction of a tetravalent half-arc-transitive graph with vertex stabilizer isomorphic to the dihedral group D_8 will be presented. The graph has 5376 vertices and it is (to my knowledge) the first example of a tetravalent vertex- and edge- but not arc-transitive graph with a nonabelian vertex stabilizer.

Embedding partial bipartite directed cycle systems

C. C. Lindner, L. Milazzo

The object of this talk is to prove that a partial directed 2k-bicycle system of order (s, t) can always be embedded in a directed 2k-bicycle system of order (ks, kt).

$K_{3,k}$ minors in 7-connected graphs

T. Böhme, J. Maharry, B. Mohar

It will be shown that for any positive integers k and w there exists a constant $N = N(k, w)$ such that every 7-connected graph of tree-width less than w and of order at least N contains $K_{3,k}$ as a minor. This is the first result of this type where fixed connectivity forces large (nontrivial) minors.

Our more recent result shows that the same conclusion can be obtained without the assumption on the bounded tree-width.

Domination Conditions for Tournaments

Patty McKenna, Margaret Morton, Jamie Sneddon

Let T be a tournament on n vertices. For $1 \leq k \leq n-3$, the k-domination graph of T is defined to have the same vertex set as T, and an edge between two distinct vertices if and only if that pair of vertices dominates in one step all of the other vertices of T with the possible exception of $k-1$ vertices. Properties of this family of domination graphs, including a full characterization for the rotational tournaments, are discussed.

On isomorphism criteria for circulant graphs

Mikhail Muzychuk, Reinhard Pöschel

We shall discuss the isomorphism problem for circulant graphs of arbitrary order n. Our appoach is based on the notion of a solving set. Using this notion we reformulate the old results about circulant graphs of prime-power order and also give an isomorphism criterion for some specific values of n.

Generating triangulations on closed surfaces with minimum degree at least 4

Atsuhiro Nakamoto, Seiya Negami

We show that every triangulation on a closed surface, except the sphere, with minimum degree at least 4 can be obtained from an irreducible one by two kinds of local deformations, called 4-splitting and addition of an octahedron and that every triangulation on the sphere with minimum degree at least 4 can be obtained from a double wheel by them.

An application of network flows to rearrangement of series

C. St. J. A. Nash-Williams, D. J. White

For each permutation f of the set of positive integers, we determine all triples s, t, u such that t and u are the lower and upper limits of the sequence of partial sums of the 'f-rearrangement' $\sum a_{f(n)}$ of some real series $\sum a_n$

with sum s. The proof uses the Max-Flow Min-Cut Theorem applied to flows in an infinite graph G_f associated with f.

More generally, we consider series $\sum \mathbf{a}_n$ where $\mathbf{a}_n \in \mathbb{R}^d$ (Euclidean d-dimensional space) for all n. The *generalised sum* of such a series is the set of cluster points in $\mathbb{R}^d$ of its sequence of partial sums. For any given permutation f of the set of positive integers, let $\mathcal{C}_f^d$ be the set of all generalised sums of f-rearrangements $\sum \mathbf{a}_{f(n)}$ of convergent series $\sum \mathbf{a}_n$ with terms in $\mathbb{R}^d$ and sum $\mathbf{0}$. We are able to determine all unbounded sets and all convex centrosymmetric sets which belong to $\mathcal{C}_f^d$ (a subset X of $\mathbb{R}^d$ being called *centrosymmetric* if there exists $\mathbf{c} \in \mathbb{R}^d$ such that $2\mathbf{c} - \mathbf{x} \in X$ for every $\mathbf{x} \in X$). In the case of convex centrosymmetric sets, the proof once again uses the Max-Flow Min-Cut Theorem.

Half-arc-transitive group actions on graphs of valency four

D. Marušič, R. Nedela

Let X be a graph. We say that a group $G \leq Aut\, X$ acts *half-arc-transitively* on X if its action is transitive on vertices and edges of X but not on arcs of X. It is known (Tutte) that if X admits such action then every vertex of X is of valency $2k$ for some integer k. The class of tetravalent graphs admitting a half-arc-transitive group action is studied. Equivalently, (in group theoretic terminology) transitive permutation groups having a non-self-paired suborbit of length two are investigated.

The aim of the talk is to present known results and to give an information on the recent progress in the field. In particular, we give a characterization of such actions, we show how they are related to regular maps and hypermaps, we present several structural results on vertex-stabilizers. Finally, we obtain a restricted list of 19 possible candidates when the vertex-stabilizers are of order ≤ 256 (and the respective tetravalent graph is connected).

Re-embedding structures of 5-connected projective planar graphs

Seiya Negami

In this talk, a projective-planar graph is a connected graph which is not planar and can be embedded on the projective plane, that is, one of nonorientable genus 1.

Kitakubo has already proved that every 5-connected projective-planar graph has exactly 1, 2, 3, 4, 6, 9 or 12 inequivalent embeddings on the projective plane.

His proof is included in his thesis, but was too long to be published as a paper.

In this talk, we shall show an alternative brief proof of his result and classify the structures of 5-connected projective-planar graphs which generate their re-embeddings on the projective plane.

Choice Number of Complete Multi-Partite Graphs

H. Enomoto, K. Ohba, K. Ota, J. Sakamoto

The *choice number* $\mathrm{ch}(G)$ of a graph G is the least integer k such that for any assignment $L : V(G) \to \binom{\mathbf{N}}{k}$, there exists a proper coloring $c : V(G) \to \mathbf{N}$ of G satisfying that $c(v) \in L(v)$ for every $v \in V(G)$. By the definition, $\mathrm{ch}(G) \geq \chi(G)$ holds for any graph G, where $\chi(G)$ denotes the chromatic number of G. However, it is known that bipartite graphs can have arbitrarily large choice number.

It is an interesting problem to find a large family of graphs which satisfy the equality $\mathrm{ch}(G) = \chi(G)$. *List coloring conjecture* asserts that the line graphs would be such a family.

Recently, Ohba proved the following theorem:

Theorem *For any graph G, there exists an integer n_0 such that for any $n \geq n_0$, $\mathrm{ch}(G + K_n) = \chi(G + K_n)$.*

This theorem suggests that if the order of a graph is not so large compared with the chromatic number, then the choice number may be equal to the chromatic number. In fact, he conjectures as follows:

Conjecture *If* $|V(G)| \leq 2\chi(G) + 1$, *then* $\mathrm{ch}(G) = \chi(G)$.

Note that in order to verify the assertion, we may assume that G is complete multi-partite.

In this talk, we give several observation to the above conjecture. We present some examples showing that the bound $|V(G)| \leq 2\chi(G) + 1$ cannot be weakened. Also, we prove the conjecture for some special cases. In particular, if G is a complete multi-partite graph with at most one partite set of size greater than two, then under a slightly weaker assumption than the conjecture, $\mathrm{ch}(G) = \chi(G)$ holds.

A new technique for the characterization of graphs with maximum number of spanning trees

LOUIS PETINGI, JOSE RODRIGUEZ

Let $\Gamma(n, e)$ denote the class of all simple graphs on n nodes and e edges. The number of spanning trees of a graph G is denoted by $t(G)$. A graph $G_0 \in \Gamma(n, e)$ is said to be t-optimal if $t(G_0) \geq t(G)$ for all $G \in \Gamma(n, e)$. The problem of characterizing t-optimal graphs for arbitrary n and e is still open, although characterizations of t-optimal graphs for specific pairs (n, e) are known.

We introduce a new technique for the characterization of t-optimal graphs, based on an upper bound for the number of spanning trees of a graph G in terms of the degree sequence and the number of vertex-induced paths of length two of the complement of G. The technique yields the following new results:

1. Complete, almost-regular multipartite graphs are t-optimal.
2. A complete characterization of t-optimal graphs in $\Gamma(n, e)$ for $n(n-1)/2 - 3n/2 \leq e \leq n(n-1)/2 - n$ is obtained for $n \geq n_0$, where n_0 can be explicitly determined.

We conclude with some open problems and conjectures regarding a general characterization of t-optimal graphs.

Letter graphs

Marko Petkovšek

Let Σ be a finite alphabet and P a set of words of length two over Σ. Given a word w over Σ, we construct a graph $G(w, P)$ by taking each symbol of w as a vertex and by connecting two vertices with an edge if the corresponding ordered pair of symbols belongs to P. For example, if $P = \{ac, cb, ba, bb\}$ then $G(abcabc, P) \cong C_6$. A graph is a *k-letter graph* if it can be obtained in this way using a k-letter alphabet. We give some results concerning the structure and enumeration of k-letter graphs.

Configurations and Graphs

Tomaž Pisanski

A *configuration* is an incidence structure $(P, \mathcal{B}, I)$ whose incidence graph (or Levi graph) is bipartite, semiregular, of girth ≥ 6. We present briefly some classical configurations in order to stress their importance in the development of modern combinatorics and finite geometries. We also single out some special classes of configurations and some problems on configurations that were recently studied by the author in several works co-authored by A. Betten, M. Boben, G. Brinkmann, M. Hladnik, H. Gropp, D. Marušič, and M. Randić. In particular we show the importance of cyclic configurations in connection with Haar graphs, and develop the connection between the notion of a weakly flag-transitive configurations and the theory of half-arc-transitive graphs.

Some classical configurations involve planes in adition to points and lines and can be conveniently studied by invoking the machinery of the theory of incidence geometry. Here Levi graphs become vertex-colored multipartite graphs. For instance, the so-called shadow spaces can be used to explain some operations on polyhedra such as the leapfrog operation. In particular, one obtains the buckminsterfullerene as a shadow space of the geometry of the dodecahedron. By reversing the argument one can generalize the notion of a leapfrog to more general incidence geometries.

On the domination number of a graph

Jochen Harant, Anja Pruchnewski

For a finite undirected graph G on n vertices some continuous optimization problems taken over the n-dimensional cube are presented. It is proved that their minimal values equal the domination number $\gamma(G)$ of G.

A Self-similarity Structure Generated by King's Walk

Marko Razpet

For all nonnegative integers i, j let $w(i, j)$ denote the number of all paths in the plane from $(0, 0)$ to (i, j) with steps $(1, 0), (0, 1)$, and $(1, 1)$. The array $w(i, j)$ is an eigenvector for the double difference operator with eigenvalue 1.

Let p be an odd prime and let $\bar{w}(i, j)$ denote the remainders of dividing $w(i, j)$ by p where $0 \leq \bar{w}(i, j) < p$. The Lucas' property of $w(i, j)$ explains the self-similar pattern of $\bar{w}(i, j)$. The principal clusters of higher orders are generated by the tensor powers of the principal cell containing $\bar{w}(i, j)$, where $0 \leq i < p$ and $0 \leq j < p$ in the natural way.

The principal cell has a symmetric pattern with special features for $p = 3, 5, 7, 11, 19$. This could be related to regular polyhedra with $p + 1$ faces. Some other symmetry properties of the principal cell will be explained, too.

Colouring graphs with high chromatic number

Bruce Reed

We show that we can determine whether the chromatic number of a graph is c in polynomial time, provided c is sufficently close to Δ. The proof blends probabilistic and structural decomposition techniques.

On orthogonal A-trails in medial graphs

Bill Jackson, R. Bruce Richter

An A-trail in an embedded Eulerian graph is an Euler tour for which every pair of consecutive edges of the tour are also consecutive on some face of the embedded graph. Two A-trails are orthogonal if they never use the same consecutive pair of edges. A medial graph is an embedded 4-regular graph whose dual is bipartite. The problem is to determine whether a given medial graph has two orthogonal A-trails (it cannot have three pairwise orthogonal A-trails).

Attention will be focussed on low-genus cases: algorithmic solutions are known and the general necessary conditions become sufficient as well.

Minimizing symmetric set functions

Romeo Rizzi

Mader proved that every loopless undirected graph contains a pair (u, v) of nodes such that the star of v is a minimum cut separating u and v. Nagamochi and Ibaraki showed that the last two nodes of a "max-back order" form such a pair and used this fact to develop an elegant min-cut algorithm. M. Queyranne extended this approach to minimize symmetric submodular functions. With the help of a short and simple proof, here we show that the same algorithm works for an even more general class of set functions.

Clique covering and degree conditions in claw-free graphs

Zdeněk Ryjáček

By using the closure concept in claw-free graphs, we prove that for any sufficiently large claw-free graph G satisfying a degree condition of type $\sigma_k(G) > n + k^2 - 2k$ (where k is an arbitrary constant), the closure of G can be covered by at most $k-1$ cliques. The degree condition can be strengthned to $\sigma_k(G) > n+k^2-4k+7$ under an additional assumption that G is nonhamiltonian. Using structural properties of the closure concept, we show a method for characterizing all such nonhamiltonian exceptional graphs with limited clique covering number.

The method is explicitly carried out for $k \leq 8$ and illustrated by proving that every 2-connected claw-free graph G of order $n \geq 153$ with $\delta(G) \geq 20$ and $\sigma_8(G) > n+39$ (or, as a corollary, with $n \geq 153$ and $\delta(G) > (n+39)/8$), is either hamiltonian or belongs to a certain family of exceptions. These clases of exceptions were found for $k = 7, 8$ with help of a cluster of parallel workstations.

Joint work with O. Favaron, E. Flandrin and H. Li, Orsay, and O. Kovářík and M. Mulač, Plzeň.

Connectivity of vertex-transitive graphs

Gert Sabidussi

*No abstract received.

Forbidden subgraphs and MIN-algorithm for independence number

Jochen Harant, Zdeněk Ryjáček, Ingo Schiermeyer

The well-known greedy algorithm MIN for finding a maximal independent set in a graph G is based on recursively removing the closed neighborhood of a vertex which has (in the currently existing graph) minimum degree. We present forbidden induced subgraph conditions under which algorithm MIN always results in finding a maximum independent set of G, and hence yields the exact value of the independence number $\alpha(G)$ of G in polynomial time.

A Local-Global Principle for vertex isoperimetric problems

S. Bezrukov, X. Portas, O. Serra

We consider the vertex isoperimetric problem (VIP) for cartesian powers of a graph G. A total order $\preceq$ on the set of vertices of G is isoperimetric if the boundary of sets of a given size k is minimized for initial segments of $\preceq$ and the ball of an initial segment is again an initial segment. We prove a local-global principle with respect to simplicial orders in G^n. Namely, we show that a simplicial order $\preceq_n$ is isoperimetric for each $n \geq 3$ iff it is so for $n = 1, 2$. Some structural properties of graphs which admit simplicial isoperimetric orderings are also derived.

Dual Eulerian graphs

Brigitte Servatius, Herman Servatius

A dual Eulerian graph is a plane graph which has an ordering defined on its edge set which forms simultaneously an Euler circuit in the graph and an Euler circuit in the dual graph. Dual Eulerian graphs were defined and studied in the context of silicon optimization of cmos layouts. Here we examine the dual Eulerian property, Petrie walks and graph connectivity. We will also consider the dual Eulerian property for graphs embedding in surfaces of higher genus.

Relevant Cycles in Biopolymers and Random Graphs

Petra M. Gleiss, Peter F. Stadler

Minimal Cycle Bases (MCB) are an important characteristic of molecular graphs in organic chemistry as well as in structural biology. In general, the MCB is not unique. The set of relevant cycles of a graph is the union of all its MCBs. While there is a unique MCB for outerplanar graphs such as RNA secondary structures, the set of relevant cycles my become very large in general, even in biomolecular graphs. For comparison we survey the number and length distributions of relevant cycles in random graphs as a function of their connectivity.

Cycle decompositions of K_n and $K_n - I$

Mateja Šajna, Brian Alspach

We establish necessary and sufficient conditions for decomposing the complete graph of odd order into cycles of even length and the complete graph of even order minus a 1-factor into cycles of odd length.

Coverings of graphs and maps, orthogonality, and eigenvectors

Jozef Širáň

Graph and map coverings are a powerful tool for constructing new graphs and maps from small quotients. The coverings are usually described in terms of voltage assignments of various types; a voltage assignment allows to "lift" the quotient to a "large" graph or map that covers the quotient. Along with this, one is often interested in lifting automorphisms of graphs and maps so as to obtain lifts that are as symmetric as possible. Voltage assignments that allow to lift an automorphism of the quotient will be called *compatible* with the automorphism; several necessary and sufficient conditions for compatibility of voltage assignments are known in the literature.

A number of important covering constructions in topological graph theory can be presented in the language of (ordinary) voltage assignments in *Abelian* groups. Such assignments are the primary object of investigation in our talk. We show that compatibility of a voltage assignment in an Abelian group with a graph automorphism is closely related to orthogonality in certain $\mathcal{Z}$-modules. In particular, compatible voltage assignments in *cyclic* groups directly correspond to eigenvectors (and eigenvalues) of certain matrices that are naturally associated with graph automorphisms. This correspondence is convenient in various applications, for instance, in constructions of highly symmetric maps.

Relations between median graphs, semi-median graphs and partial cubes

W. Imrich, S. Klavžar, H. M. Mulder, Riste Škrekovski

A graph (with distance function d) is called a *median graph*, if for any three vertices u, v, w of G there exists a unique vertex x such that $d(u, v) = d(u, x) + d(x, v)$, $d(u, w) = d(u, x) + d(x, w)$, and $d(v, w) = d(v, x) + d(x, w)$. A graph is a *partial cube*, if it has an isometric embedding in some hypercube. Denote by $\mathcal{M}$ and $\mathcal{PC}$ the classes of median graphs and partial cubes, respectively. A class of semi-median $\mathcal{SM}$ is between these two classes, i.e. $\mathcal{M} \subset \mathcal{SM} \subset \mathcal{PC}$. In the talk will be presented some relations between these classes of graphs.

A Hedetniemi-type problem for multicolourings

Benoit Larose, Claude Tardif, Xuding Zhu

We present an adaptation of Hedetniemi's conjecture to a variation of graph colourings devised by Stahl, where the vertices are assigned sets of colours instead of single colours. In this context, the analogue of Hedetniemi's conjecture is false, but many results related to the conjecture can be adapted to the variation.

Excluding Minors in Nonplanar Graphs of Girth at Least Five

Robin Thomas, Jan McDonald Thomson

We show that any nonplanar graph with minimum degree at least three and no cycle of length less than five has a minor isomorphic to the Petersen graph with one edge deleted ($P_{10} \backslash e$.) We deduce the following weakening of Tutte's Four Flow Conjecture: every 2-edge connected graph with no minor isomorphic to $P_{10} \backslash e$ has a nowhere zero four flow. This extends a result of Kilakos and Shepherd who proved the same for 3-regular graphs.

Asymptotic Growth in Graphs of Bounded Valence

Thomas Tucker, Tomaž Pisanski

Given an infinite graph G and vertex v, the sphere growth function $s_v(n)$ at vertex v (respectively, ball growth function $b_v(n)$ at vertex v) is the number of vertices whose shortest path to v has length n (respectively, length at most n). If G is a Cayley graph for an infinite group, these functions describe the classical growth of the group. Trofimov, Seifter, Imrich and others have studied growth in vertex-transitive graphs, especially for the case of polynomial growth. This paper begins a study of the asymptotics of growth functions for general graphs of bounded valence. Two natural kinds of asymptotic equivalence are introduced and related. Since sphere growth is in effect a derivative of ball growth, its asymptotics for general graphs can be rough. For example, ball growth functions at different vertices of the same graph are asymptotically equivalent, but sphere growth functions are not. In fact, given any increasing function g such that $g(n+1)/g(n)$ is bounded, there is a planar graph of bounded valence whose ball growth is aymptotically equivalent to g, but whose sphere growth function takes on the

same value infinitely often. On the other hand, if at any vertex the condition $s(n) < ds(n+1)$ holds for some d, which we call bounded decay, then all sphere growth functions are asympotitcally equivalent, making a general theory possible. The connection between growth functions and transient random walks on graphs is also considered via the isoperimetric function $g(n)/s(n)$.

Recognizing pseudo-median graphs

Aleksander Vesel

Pseudo-median graphs form a nonbipartite generalization of median graphs. We derive a characterization of pseudo-median graphs based on a sequence of gated expansions which allows us to recognize these graphs in $O(mn)$ time.

Polyhedral graphs with restricted number of faces of the same type

Margit Voigt, Hansjoachim Walther

Let $G = G(V, E, F)$ be a polyhedral graph with vertex set $V(G)$, edge set $E(G)$ and face set $F(G)$. A face $\alpha \in F(G)$ is an $\langle a_1, \ldots, a_l \rangle$-face if α is an l-gon and the degrees $deg_G(x_i)$ of the vertices x_i, $i = 1, \ldots, l$ incident with α in the cyclic order are $a_1, a_2, \ldots, a_l$. The lexicographical minimum $\langle b_1, \ldots, b_l \rangle$ such that α is an $\langle a_1, \ldots, a_l \rangle$-face is called the *type* of α.

We deal with polyhedral graphs where for each type the number of faces of that type occuring in the graph is restricted by a fixed integer z. By a discharging method we are able to prove that the number of such graphs is finite.

Light subgraphs of multigraphs embedded in compact 2-manifolds

Stanislav Jendrol', Heinz-Juergen Voss

Fabrici and Jendrol' proved that each 3-connected plane graph with a k-path (a path of k vertices), contains a k-path such that each vertex has a degree at most $5k$. Further they showed that each 3-connected plane graph with at least k vertices contains a connected subgraph on k vertices such that each vertex has a degree at most $4k+3$ for each $k \geq 3$. Both bounds are sharp. We generalized these results to compact 2-manifolds M of Euler characteristic $\chi(M) < 0$. Three of our results are: each polyhedral map of M with a k-path contains a k-path such that each vertex has a degree at most $k\lfloor(5+\sqrt{49-24\chi(M)}\,)/2\rfloor$. Equality for even $k \geq 2$. If G is a 3-connected multigraph on M without loops and 2-faces then the degree bound is $(6k-2\epsilon)(1+|\chi(M)|/3)$, where $\epsilon = 0$, if k is even, and $\epsilon = 1$, if k is odd. If G is a 3-connected graph on M, then the degree bound is $2+\lfloor(6k-6-2\epsilon)(1+|\chi(M)|/3)\rfloor$ for $k \geq 4, \chi(M) < 0$ and for $k \geq 2, \chi(M) < -3$. All the bounds for G are the best possible.

Modelling Finite Geometries on Surfaces

Arthur T. White

The usual model of the Fano plane has several deficiencies. These are remedied by interpreting $K(7)$ on the torus properly. This idea is generalized, in two directions. (1) Let q be a prime power. Efficient topological models are described for each projective plane $PG(2,q)$, and a study is begun for the affine planes $AG(2,q)$. (2) A 3-configuration is a geometry satisfying: (i) each line is on exactly 3 points; (ii) each point is on exactly r lines, where r is a positive integer; (iii) two points are on at most one common line. Surface models are found for 3-configurations of low order, including those of Pappus and Desargues. Special attention is paid to $AG(2,3)$. A study is begun of 3-configurations which are also partial geometries, and four general constructions are given.

Packing and cyclic permutations

Mariusz Wozniak

We improve some results on the packing of two copies of a graph by proving that actually there exists a cyclic packing permutation.

Highly arc-transitive digraphs with no homomorphism onto $\mathbb{Z}$

Aleksander Malnič, Dragan Marušič,
Norbert Seifter, Boris Zgrablić

In an infinite digraph D, an edge e' is *reachable* from an edge e if there exists an alternating walk in D whose initial and terminal edges are e and e'. Reachability is an equivalence relation and if D is 1-arc-transitive, then this relation is either universal or all of its equivalence classes induce isomorphic bipartite digraphs. In [1, Question 1.3], Cameron, Praeger and Wormald asked if there exist highly arc-transitive digraphs (apart from directed cycles) for which the reachability relation is not universal and which do not have a homomorphism onto the two-way infinite directed path (a Cayley digraph of $\mathbb{Z}$ with respect to one generator).

For each odd integer $n \geq 3$, a construction of such a digraph satisfying the additional properties that its indegree and outdegree are equal to 2 and that the reachability equivalence classes induce alternating cycles of length $2n$, is given. Furthermore, using the line digraph operator, digraphs having the above properties but with alternating cycles of length 4 are obtained.

References

[1] P. J. Cameron, C. E. Praeger and N. C. Wormald, Infinite highly arc transitive digraphs and universal covering digraphs, *Combinatorica* **13** (1993), 377–396.

Symmetric Graphs, 2-Arc Transitive Graphs and Near-polygonal Graphs

Sanming Zhou

Let Γ be a finite G-symmetric graph whose vertex set admits a nontrivial G-invariant partition $\mathcal{B}$ with block size v. A geometrical approach, introduced by Gardiner and Praeger, for studying such graphs Γ involves an analysis of the quotient graph $\Gamma_{\mathcal{B}}$ relative to $\mathcal{B}$, the bipartite subgraph $\Gamma[B, C]$ of Γ induced by adjacent blocks B, C of $\Gamma_{\mathcal{B}}$ and a certain 1-design $\mathcal{D}(B)$ induced by a block $B \in \mathcal{B}$, and the reconstruction of Γ from the triple $(\Gamma_{\mathcal{B}}, \Gamma[B, C], \mathcal{D}(B))$. I will review briefly the work with Li and Praeger in the case where the block size k of $\mathcal{D}(B)$ satisfies $k = v - 1 \geq 2$. We prove that $\mathcal{D}(B)$ contains no repeated blocks if and only if $\Gamma_{\mathcal{B}}$ is $(G, 2)$-arc transitive, and give a method for reconstructing such a graph Γ from a 2-arc transitive graph with a self-paired orbit on 3-arcs. We show further that each such graph Γ can be constructed by this method.

We find that, under the assumptions above (i.e., $k = v - 1 \geq 2$ and $\mathcal{D}(B)$ contains no repeated blocks), the possibilities for $\Gamma[B, C]$ (and hence for Γ) depend on $(\Gamma_{\mathcal{B}}, G)$, and vice versa. We prove that $\Gamma[B, C] \cong K_{v-1,v-1}$ occurs if and only if $\Gamma_{\mathcal{B}}$ is $(G, 3)$-arc transitive, and in this case Γ is uniquely determined by $\Gamma_{\mathcal{B}}$. In the case where $\Gamma[B, C] \cong (v-1) \cdot K_2$, we prove that either (i) $\Gamma_{\mathcal{B}} \cong K_{v+1}$ and (Γ, G) can be classified completely, or (ii) for some even integer $n \geq 4$, $\Gamma_{\mathcal{B}}$ is a $(G, 2)$-arc transitive near n-gonal graph with respect to a certain G-orbit on n-cycles of $\Gamma_{\mathcal{B}}$. Moreover, we show that every $(G, 2)$-arc transitive near n-gonal graph with respect to a G-orbit on n-cycles arises as such a quotient $\Gamma_{\mathcal{B}}$. (For $n \geq 4$, a connected graph Σ of girth at least four is said to be a near n-gonal graph with respect to $\mathcal{E}$ if $\mathcal{E}$ is a set of n-cycles of Σ such that each 2-arc of Σ is contained in a unique member of $\mathcal{E}$.)

Regular colouring of distance graphs and circulant graphs

Xuding Zhu

Let D be a finite subset of R^+. The distance graph $G(R, D)$ on the real line with generating set D has vertex R and edge set $\{xy : |x-y| \in D\}$. We are interested in finding the chromatic number and the circular chromatic number of $G(R, D)$. There is a simple colouring method that works quite well for many distance graphs: the regular colouring method. The method is easy to describe. Suppose we want to find a k-colouring of $G(R, D)$. Choose a positive real number r, and partition the real line into half open intervals of length r, say let $I_i = [ir, (i+1)r)$. Then colour I_i with colour $i \pmod{k}$. We call such a colouring a regular k-colouring of R. The problem is when such a colouring is a proper colouring for $G(R, D)$? How to choose the real number r ? These turn out to be related many other interesting problems.

For a real number x, let $||x||$ denote the distance from x to the nearest integer. For set D, let $||D|| = \min\{||x|| : x \in D\}$ and for any real number t, let $tD = \{tx : x \in D\}$. Define $\kappa(D) = \sup_{t \in R} ||tD||$. I shall show that there is regular k-colouring of R which is a proper colouring of $G(R, D)$ if and only if $\kappa(D) \geq 1/k$. Using estimations of the function $\kappa(D)$, the chromatic number of $G(R, D)$ with $|D| = 3$ is completely determined. The estimation of the function $\kappa(D)$ is a typical diophantine approximation problem and is also related geometrical problems and graph flows.

The regular colouring method can also be applied to circulant graphs. I shall also report a recent result of H. Yeh and myslef which determines the chromatic number of some circulant graphs by using the regular colouring method. The results confirms two conjectures proposed by Collin, Fisher and Hutchinson. The regular colouring method can also be used to determine the circular chromatic number of distance graphs and circulant graphs.

Factoring Cartesian Graph Bundles

Blaž Zmazek

Graph bundles generalize the notion of covering graphs and graph products. In [Imrich, T. Pisanski, J. Žerovnik, Recognizing Cartesian graph bundles, *Discrete Mathematics* **167/168** (1998) 393–403] authors constructed an algorithm that finds a presentation as a nontrivial Cartesian graph bundle for all graphs that are Cartesian graph bundles over triangle-free simple base using the relation δ^* having the square property. An equivalence relation R on edge set of a graph has the (unique) square property iff any pair of incident edges which belong to distinct R-equivalence classes span exactly one induced 4-cycle (with opposite edges in the same R-equivalence class). In this paper we show that any maximal equivalence relation possessing the unique square property, determines the fundamental factorization of a graph as a nontrivial Cartesian graph bundle over arbitrary base graph.

An algorithm for k-convex closure

Tomaž Pisanski, Janez Žerovnik

An algorithm for computing the k-convex closure of a subgraph relative to a given equivalence relation R among edges of a graph is given. For general graph and arbitrary relation R the time complexity is $\mathcal{O}(qn^2 + mn)$, where n is the number of vertices, m is the number of edges and q is the number of equivalence classes of R. A special case is an $\mathcal{O}(mn)$ algorithm for the usual k-convexity. We also show that Cartesian graph bundles over triangle free bases can be recognized in $\mathcal{O}(m^2)$ time and that all representations of such graphs as Cartesian graph bundles can be found in $\mathcal{O}(m^2 n)$ time.

Plane graphs with Eulerian Petrie walks

Arjana Žitnik

A *Petrie walk* in a plane graph G is such a walk that when we travel along it, we alternatively turn to the left edge and to the right edge of the current edge in the cyclic rotation about the common vertex. Petrie walks are also called *left-right walks.*

We consider some properties of plane graphs, having a Petrie walk which is Eulerian. First, we show that there are no simple plane graphs with minimal degree four, having an Eulerian Petrie walk. We also show, that it is an NP-complete problem to decide, whether a graph can be subdivided or parallel edges added to obtain a graph with Eulerian Petrie walk. We give a necessary and sufficient condition for a plane graph to have an Eulerian Petrie walk, which gives rise to a simple algorithm for constructing plane graphs with Eulerian Petrie walk. This algorithm can also be used for the construction of 2-connected plane graphs with prescribed face lengths and vertex degrees, having an Eulerian Petrie walk, if such a graph exists. In some special cases, for example if all the vertices are of degree 4, the algorithm is polynomial-time.