How Many Types Are There?

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Abstract

We consider an revealed preference based method which will partition consumer microdata into an approximate *minimal* number of preference types such that the data are perfectly rationalisable by standard utility theory. This provides a simple, nonparametric and theory-driven way of investigating unobserved preference heterogeneity in empirical data, and easily extends to any choice model which has a revealed preference characterisation. We illustrate the approach using survey data and find that the number of types is remarkably few relative to the sample size - only 4 or 5 types are necessary to fully characterise all observed choices in a dataset with 500 observations of choice vectors.

Key Words: applied econometrics, unobserved heterogeneity, revealed preference, salamanders.

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1 Unobserved Heterogeneity in Microdata

One of the most striking features of consumer microdata is the great heterogeneity in choice behaviour which is evident, even amongst economic agents which are otherwise similar in observable respects. This presents researchers with a difficult problem - how to model behaviour in a way which accommodates this heterogeneity and yet preserves theoretical consistency and tractability.

One rather robust response is to demand that everything should be explainable by the theory in terms of observables alone. This view is typified by Becker and Stigler (1977):

> "Tastes neither change capriciously nor differ importantly between people." G. Becker and G. Stigler
> *De Gustibus Non Est Disputandum,* AER, 1977

The research agenda which follows from this view is one which tries to explain differences in observed behaviour without recourse to unobserved heterogeneity in tastes, but instead purely in terms of the theory and observable differences in constraints, characteristics of market goods and characteristics of agents. From this point of view, resorting to unobserved preference heterogeneity in order to rationalise behaviour is a cop-out; it is an admission of failure on the part of the theory.

From this perspective it is therefore a matter of some regret that measures of fit in applied work on microdata are typically very low - that is, the theory performs poorly (see, e.g., Banks, Blundell, Lewbell 1997 and Lewbel and Pendakur 2009 who report R^2 as low as 20% in consumer demand microdata). As a result, the belief that unobserved heterogeneity is an inescapable and essential part of the modeling problem has become the dominant view in the profession. This approach was summarised by the joint 2000 Nobel laureates as follows.

> "In the 1960's, rapidly increasing availability of survey data on individual behavior ... focussed attention on the variations in demand across individuals. It became important to explain these variations as part of consumer theory, rather than as *ad hoc* disturbances". D. McFadden, *Nobel Lecture*

> "Research in microeconometrics demonstrated that it was necessary to be careful in accounting for the sources of manifest differences among apparently similar individuals. ... This heterogeneity has profound consequences for economic theory and for econometric practice." J. Heckman, *Nobel Lecture*

In applied microeconometrics, the standard approach has been to pool data across agents and to model the behaviour of individuals as a combination of a common component and an idiosyncratic component which reflects unobserved heterogeneity. In its least sophisticated form, this amounts to interpreting additive error terms as unobserved preference heterogeneity parameters. Recently, it has become clear that such an approach typically requires a combination of assumptions on the functional form of the statistical model and the distribution of unobserved heterogeneity. Contributions here include McElroy (1987), Brown and Walker (1989), Lewbel (2001) and Lewbel and Pendakur (2009). Broadly, the current consensus on unobserved heterogeneity is that: it is a fundamental feature of consumer microdata; if neglected it makes econometric estimation and identification difficult; and it is rather hard to deal with convincingly, especially in non-linear models and where heterogeneity is not additively separable.

Whilst the dominant empirical methods, by and large, proceed by *pooling* agents, the approach which we develop here is based on *partitioning*. The spirit of pooling agents is to account for heterogeneity with a small number of extra parameters (e.g., one) per type or characteristic, as in fixed-effects models with lots of covariates. Here, most parameters (e.g., those relating to covariates) are shared across the agents in the pooled model, but each agent has one or more agent-specific parameters. In contrast, the spirit of partitioning is to allow each type to be arbitrarily different from every other, for example, by giving each type a completely different set of parameters governing the effects of covariates.

We work from the basis of revealed preference (RP) restrictions (Afriat 1967, Diewert 1973 and Varian 1982). At heart, RP restrictions are inequality restrictions on observables (prices, budgets and demands), which provide necessary and sufficient conditions for the existence of an unobservable (a well behaved utility function representing the consumer's preferences which rationalises the data). RP restrictions are usually applied to longitudinal data on individual consumers and are used to check for the existence and *stability* of well-behaved preferences. In this paper we apply this kind of test to cross-section data on many different consumers (though, as we describe below, our idea applies to many contexts with optimising agents). In this context, RP restrictions are interpretable as a check for the *commonality* of well-behaved preferences.[1]

Of course, this is a rather simplistic idea. The very notion that such a check might pass and that the choices of all of the consumers in a large microeconomic dataset could be explained perfectly by a single common utility

[1] We are not the first to make this observation. Gross (1995) also applies RP tests to cross sectional consumer data in order to look at the evidence for and against the assumption of commonality.

function is, as Lewbel (2001) points out, "implausibly restrictive". The real problem is what to do if (or more likely when) the data do not satisfy the RP restrictions. It is important to recognise that there are many reasons that a model which assumes homogeneous preferences might fit the data poorly including mistakes by the data collector (measurement error), mistakes by the individuals themselves (optimisation error) and mistakes by the theorist (specification error, which is to say applying the wrong model). The truth is doubtless a mixture of all three but this paper focuses primarily on the last of these and in particular on the issue of preference heterogeneity and asks how far we can get by assuming that this is the sole cause of poor fit[2]. Dean and Martin (2010) provide one type of solution along these lines: they show how to find the largest subset of the data that *do* satisfy (some of) the RP restrictions. However, their approach leaves some of the data as unexplained by the optimising model.

The contribution of this paper is to provide a different (and complementary) set of strategies for the case where the pooled data violate the RP restrictions. Here, some amount of preference heterogeneity is necessary in order to model those data—we need more than just one utility function. The question is *how many do we need*? Is it very many (perhaps as many as there are observations), or just a few? This paper shows how to find out the minimum number of types (utility functions) necessary to fully explain all observed choices in a data set. In seeking the *minimum* number of utility functions necessary to rationalise behaviour, we keep with Friedman's (1957) assertion that we don't want the true model, which may be unfathomably complex; rather, we want the simplest model that is not rejected by the data. Occam's Razor applies here: we know that we can fully explain behaviour with a model in which every agent is arbitrarily different from every other, but that model is not useful for modeling or predicting behaviour. Instead, our aim is to group agents into types to the maximum possible degree that is consistent with common preferences. If the minimum number of types (utility functions) is very large relative to the number of observations, then modeling strategies with a continuum of types, or with one type for each agent (such as fixed effects models), might be appropriate. In contrast, if the minimum number of types is small relative to the number of observations, then modeling strategies with a small number of discrete types, such as those found in macro-labour, education choice, and empirical marketing models, might be better.

We argue that our approach offers two main benefits which may complement the standard approaches to unobserved heterogeneity in empirical work. Firstly, it provides a framework for dealing with heterogeneity which is driven by an economic model of interest and it thereby provides a practical method of partitioning data so that the observations in each group are fully theory-consistent. This contrasts with approaches wherein only part of the model (the part which excludes the unobserved heterogeneity) satisfies the theory.[3] Secondly, it is elementary: our approach does not require statements about the empirical distributions of objects we can't observe or functional structures about which economic theory is silent. This contrasts with the standard approach of specifying *a priori* both the distribution of unobserved preference heterogeneity parameters and its functional relationship with observed variables.

We implement our strategy with cross-sectional dataset consumer microdata. These data happen to record milk purchases but, importantly, they have individual-level price, quantity and product characteristics information, and so are ideal for the application of RP methods. We find that at the number of types needed to completely explain all of the observed variation in consumption behaviour is quite small relative to the number of observations in our data. For our main application, with a cross-sectional dataset of 500 observations of

[2] In fact the approach considered here can be augmented to allow for measurement and optimisation errors as well. The methods involved are not original to this paper but we give a brief account of them in the Appendix.

[3] We note that by having a model in which the data are theory-consistent by construction, one cannot test the theory. Indeed, in our context, testability amounts to precluding unobserved heterogeneity.

quantity vectors, we find that 4 or 5 types is enough. Furthermore it seems that two-thirds of the data are consistent with a single type and two types are sufficient to model 85% of the observations.

The paper is organised as follows. We begin with a description of the cross-sectional data on household expenditures and demographics which we use in this study. We then investigate whether these data might be rationalised by partitioning on observed variables which form the standard controls in microeconometric models of spending patterns. We then set out a simple method for partitioning on revealed preferences, and consider whether the results from these partitioning exercises can be a useful input to econometric modelling of the data. We then consider the problem of inferring the number of types in the population from which our sample is drawn. The final section draws some conclusions.

2 The Data

In this paper we focus on the issue of rationalising cross-sectional household-level data on spending patterns with the standard static utility maximisation model of rational consumer choice. This approach can readily be extended to other more exotic economic models which have a nonparametric/revealed preference characterisation (examples are given in the discussion and in the Appendix). The data we use are on Danish households and their purchases of milk. These households comprise all types ranging from young singles to couples with children to elderly couples. The sample is from a survey of households which is representative of the Danish population. Each household keeps a strict record of the price paid and the quantity purchased as well as the characteristics of the product. We aggregate the milk records to a monthly level, partly to ease the computational burden and partly to allow us to treat milk as a non-durable, non-storable good, so that the intertemporally separable model which we are invoking is appropriate. Six different types of milk are recorded in the data: skimmed, semi-skimmed or full-fat versions of either organic or conventionally produced milk. Quantity indices are computed by simply adding the volume of each variety purchased and a corresponding unit price (total expenditure on a given variety divided by the total volume of that variety purchased) is used as the price index. That we can differentiate varieties is a particularly attractive feature of the these data because it means that variation in these prices in the cross section is principally due to supply and demand variation across markets (defined by time and location) and not due to unobserved differences in product qualities and characteristics[4]. Our full dataset has information on 1,917 households. Since some of the following calculations are computationally quite expensive we begin by drawing a smaller random sample of 500 households from our data. In section 6 we return, gradually, to the full sample size.

Descriptive statistics are given in Table 1. In what follows let $I = \{i : i = 1, ..., 500\}$ denote the index set for these observations and let $\{\mathbf{p}_i, \mathbf{q}_i\}_{i \in I}$ denote the price-quantity data. We will also make use of a list of observed characteristics (these are standard demographic controls used in demand analysis) of each household and these are represented by the vectors $\{\mathbf{z}_i\}_{i \in I}$.

Given these data the question of interest is whether it is possible to rationalise them with the canonical utility maximisation model. The classic result on this issue is provided by *Afriat's Theorem* (see especially Afriat (1967), Diewert (1973) and Varian (1982, 1983)). Afriat's Theorem shows that the generalised axiom of revealed preference (GARP) is a necessary and sufficient condition for the existence of a well-behaved utility function $u(\mathbf{q})$ which exactly rationalises the data. Such rationalisability requires that for every observed choice, $\mathbf{q}_i$, the choice made weakly utility-dominates all other affordable choices: $u(\mathbf{q}_i) \geq u(\mathbf{q})$ for all $\mathbf{q}$ such that $\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}$. Let

[4] See Deaton (1988) for a discussion of the problems which arise when unit prices which combine multiple varieties of goods are used.

$\mathbf{p}_i'\mathbf{q}_i \geq \mathbf{p}_i'\mathbf{q}_j \Leftrightarrow \mathbf{q}_i R^0 \mathbf{q}_j$ denote a direct revealed preference relation and let R be the transitive closure of R^0. The generalised axiom of revealed preference (GARP) is defined by the restriction that $\mathbf{q}_i R \mathbf{q}_m \Rightarrow \mathbf{p}_i'\mathbf{q}_i \leq \mathbf{p}_i'\mathbf{q}_m$. If observed demands $\{\mathbf{p}_i, \mathbf{q}_i\}_{i \in I}$ satisfy these GARP inequalities, then there is a single utility function (preference map) that can rationalise all observed demands. If not, then there is not.

Table 1: Descriptive Statistics

	Mean	Min	Max	Std. Dev
$\{\mathbf{w}_i\}_{i=1,..,500}$	**Budget Shares**			
Conventional Full Fat	0.1688	0	1	0.3158
Conventional Semi-skimmed	0.4255	0	1	0.4102
Conventional Skimmed	0.1521	0	1	0.2932
Organic Full Fat	0.0374	0	1	0.1394
Organic Semi-skimmed	0.0977	0	1	0.2237
Organic Skimmed	0.1185	0	0.9951	0.2669
	Total Expenditure (*DK*)			
Total Expenditure	66.1986	4.8222	345.1279	58.5765
$\{\mathbf{p}_i\}_{i=1,..,500}$	**Prices** (*DK litre*)			
Conventional Full Fat	6.1507	3.3068	11.3289	0.4652
Conventional Semi-skimmed	5.4104	4.0919	7.9567	0.4304
Conventional Skimmed	5.1524	4.1619	6.2075	0.1814
Organic Full Fat	7.3335	6.1188	8.6597	0.1860
Organic Semi-skimmed	6.4968	5.0565	8.5374	0.2187
Organic Skimmed	6.2679	5.5312	7.9684	0.1501
$\{\mathbf{z}_i\}_{i=1,..,500}$	**Demographics**			
Singles $\{0,1\}$	0.3260	0	1	0.4692
Singles Parents $\{0,1\}$	0.0420	0	1	0.2008
Couples $\{0,1\}$	0.3500	0	1	0.4774
Couples with children $\{0,1\}$	0.2300	0	1	0.4213
Multi-adult $\{0,1\}$	0.0520	0	1	0.2222
Age ($Years$))	47.8600	18	87	15.5240
Male HoH $\{0,1\}$	0.92	0	1	0.27156

We checked the data for consistency with GARP and it failed[5]. No single utility function exists which can explain the choices of all of these households - Lewbel's (2001) warning seems to be justified. So we now turn to the question: *how many well-behaved utility functions are required to rationalise these price-quantity microdata?* Obviously 500 utility functions, each one rationalising each observation, will be over-sufficient. The next two section explore the idea of conditioning on observed demographic variables and revealed preferences in order to find a *minimal* necessary partition of these data.

[5] We use the method described in Varian (1982) which uses an algorithm due to Warshall (1962) to check for cycles whch violate GARP. The time required is proportional to the number of observations cubed. See the Appendix in Varian (1982) for details.

3 Partitioning on Observed Variables

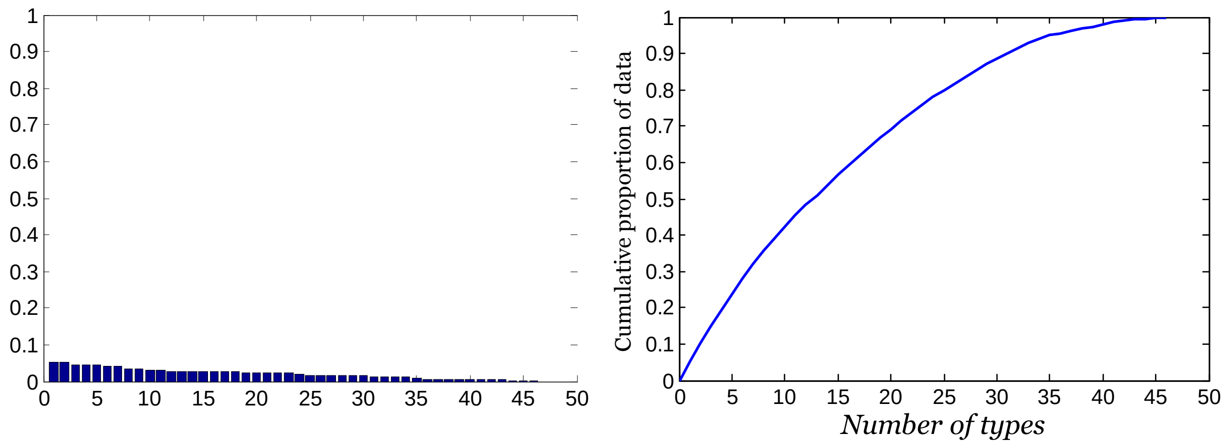
We begin by investigating whether it is possible to achieve a parsimonious partition of the data using "standard observables" - i.e. the sorts of variables which are often used as conditioning variables in microeconometric demand systems. To do this we used information on the structure of the household (defined according to 5 groups: single person households, single parents, couples, couples with children and multi-adult households), the age of the head of household (3 roughly equally-sized groups: less than 40 years old, 40 to 60 years old and over 60), region (there are 9 regional indicators observed in the data), the gender of the head of household and the size of the household's budget (deciles). Using these variables to partition observations there were 341 non-empty cells - the distribution of groups sizes were 235, 68, 29, 5, 2 and 2 for singletons, pairs, triples and groups of 4, 5 and 6 households respectively. Despite the fact that this partitioning of the data was clearly very fine, and that by creating groups composed of very small numbers of households we improve the prospect that within-group tests of GARP are satisfied (indeed singletons cannot fail) we found this did not produce a partition which was consistent with within-group commonality of preferences. It seems that even very small groups of households with similar observable characteristics exhibit preference heterogeneity. An interesting implication of this exercise[6] is that one can immediately conclude that no combination of two or three of these conditioning variables can produce consistent partitions. This is because combined data from multiple cells will always violate GARP if any of the data in the contributing cells violate. Thus, if a fine partition cannot rationalise the data, then neither can any coarser partition constructed from it.

Instead of using pre-defined cells to partition the data it is also possible to take a more data-driven, adaptive approach. This is essentially a question of designing a search algorithm which uses the results of a sequence of GARP tests to tell the investigator where to place the partitions. The simplest example of such an approach would be to order the data by some observable like age then to start with the youngest household and add successively older households until the current group violates GARP. The data are partitioned at this point and the last household to be added is then used to start the next group and so on. If the investigator wishes to consider other conditioning variables then the resulting partition is naturally path-dependent (the order in which one selects the variables with which to order the data affects the final result). As the number of conditioning variables grows the number of potential paths grows very quickly as does the computational complexity of finding the best solution. Nonetheless, while it may not be computationally feasible to find a fully efficient solution by checking all paths, such an approach does hold out the possibility of finding a more parsimonious partition than might be available through the use of pre-defined groups. To investigate we first stratified the data according to the household structure variable described above, and next ordered the household of each structure by the age of the head of household and, beginning with the youngest, we sequentially tested the RP condition in order to see whether we could rationalise behaviour by a further partition on age into contiguous bands. This proved impossible because there were instances of households with the same structure whose heads of household were the same age whose behaviour was not mutually rationalisable. Having first split by household structure, and then split by age and not yet found a rationalisation for the data we further split by region. This too failed to rationalise the data as there were instances of households with identical structure and age living in the same region who were irreconcilable with a common utility function. We then looked at the gender of the household head. This, finally, produced a rationalisation of the data. In contrast to the exercise which used 341 pre-defined cells and still could not rationalise the data, this adaptive procedure produced a consistent partition with 46 types defined by household structure/age/region/gender.

[6] We are grateful to an anonymous referee for suggesting this.

The left hand panel of Figure 1 shows the distribution of group sizes with the groups ordered largest to smallest. This shows that the largest groups consist of approximately 5% of the data (there are two such groups) whilst the smallest (the 44th, 45th and 46th on the left of the histogram) consist of singletons. The right hand panel of Figure 1 shows the cumulative proportion of the data explained by the rising numbers of types. The first ordinate shows that approximately 5% of the data are rationalisable by one type (the most numerous) and approximation 10% by two most numerous types. Ten types are needed to rationalise half the data.

FIGURE 1. Partitioning on Observed Demographics



It appears, therefore, that efforts to find a partition of the data in to types which admit common within-type preferences on the basis of the sorts of variables typically observed in microdata on consumer choices does not seem to produce a parsimonious result. Whilst a search algorithm does a great deal better than the simpler fixed-cell type of approach the results are still not impressive - each type only accounts for around 2% of the data on average.

4 Partitioning on Revealed Preferences

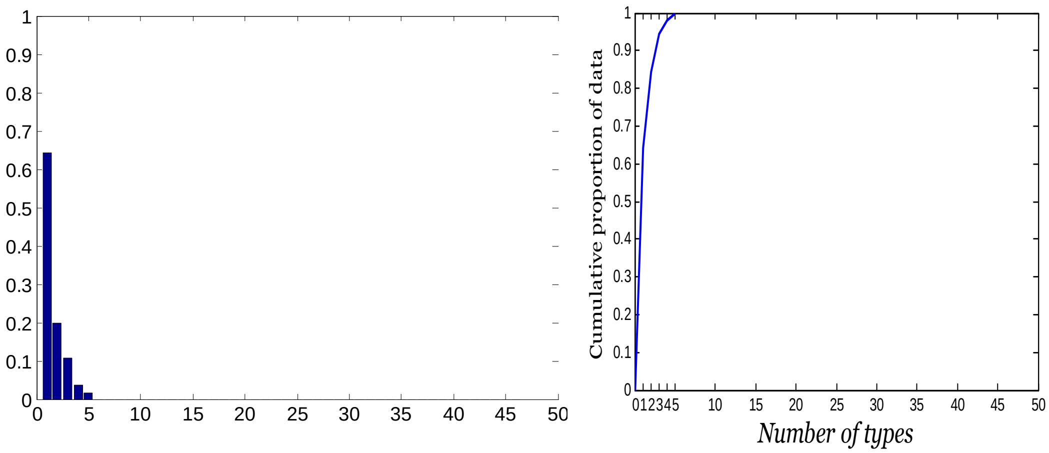
In this section we consider partitioning on revealed preferences. As before we are interested in trying to split the data into as few (and as large) groups as we can such that all of the households within each group can be modelled as having a common well-behaved utility function. However this time we will not use observables like those used above to guide/constrain us. The simplest "brute force" approach would be to check the RP restrictions within all of the possible subsets of the data and retain those which form the minimal exclusive exhaustive partitions of the data. This is computationally infeasible as there are 2^{500} such subsets. Instead we have designed two simple algorithms which will provide two-sided bounds on the minimal number of types in the data. The details of the algorithms need not detain us here (they are described in the Appendix).

We ran the algorithms on our data and found that the minimal number of types was between 4 and 5. That is one needs at least 4, and not more than 5, utility functions to completely rationalise all the observed variation in choice behaviour observed in these data in terms of income and substitution effects.[7] For our upper

[7] Recalling that our data are a random sample of 500 observations from a larger dataset of 1,917 observations, we also investigated the variability of these bounds induced by (re)sampling. We took 25 samples of 500 observations with replacement and calculated the bounds on the number of types in each sample. In all cases the bounds remained [4,5]. We are very grateful to an anonymous referee for suggesting this exercise and conclude from it that the bounds on the number of types, for a given sample size, is reasonably robust to sampling variation. We investigate the effects of varying the size of the sample below.

bound of 5 types our algorithm also delivers a partition of the data into the groups, such that within-groups a single utility function is sufficient to rationalise all the observed behaviour[8]. Table 2 gives the average budget shares for each group delivered by our upper bound algorithm and Figure 2 shows the distribution of types and gives the same information as Figure 1 on the same scale for ease of comparison and in order to emphasize how parsimonious this partition is in comparison. In contrast to Figure 1 we can see that a single utility function can rationalise the observed choices of around two-thirds of the sample. And two utility functions is all that is needed to rationalise nearly 85% of the data.

FIGURE 2. Partitioning on Unobserved Variables



Our expectation was that, even though conditioning on observables did not seem to be able to produce a perfect and parsimonious partition of the data, nonetheless observable characteristics of households would be important correlates of type-membership. However, a multinomial logit model of group membership conditional on demographic characteristics (age and sex of household head, number of members, number of children and geographic location) has a (McFadden unadjusted) pseudo-R^2 of only 5.4%.[9] That is, observed characteristics of households are essentially uninformative regarding which of the five types to which a household is assigned. The implication here is that, in a framework where we want to find the minimum number of types, preference heterogeneity is vastly more important than demographic heterogeneity.

TABLE 2: Average Budget Shares Across Types

		Conventional Milk			Organic Milk		
Sample Means	Group N	Full-fat	Semi	Skim	Full-fat	Semi	Skim
pooled	500	0.168	0.425	0.152	0.037	0.097	0.118
Type 1	321	0.160	0.496	0.143	0.024	0.075	0.100
Type 2	100	0.155	0.285	0.205	0.070	0.121	0.162
Type 3	53	0.239	0.351	0.074	0.044	0.144	0.147
Type 4	18	0.134	0.256	0.148	0.032	0.258	0.170
Type 5	8	0.292	0.195	0.357	0.128	0.017	0.009

[8] Note that the allocation of households to groups is not necessarily unique - it might be feasible to allocate any given housheold to more than one group. We return to this point below.

[9] We note that the low value of the pseudo-R^2 is not driven by the large number of classifications (5). If we drop the 5th type (the smallest group), the pseudo-R^2 drops to 4.5%, and if we drop the 4th and 5th types (the two smallest groups), it drops to 4.1%. We also not that the mean value of each regressor is not significantly different across groups.

5 Estimation of Preferences

The incorporation of unobserved preference heterogeneity into demand estimation is a theoretically and econometrically tricky affair. Matzkin (2003, 2007) proposes a variety of models and estimators for this application, all of which involve nonlinearly restricted quantile estimators, and most of which allow for unobserved heterogeneity which has arbitrary (but monotonic) effects on demand. These models are difficult to implement, and, as yet, only Matzkin (2003, 2007) has implemented them. Lewbel and Pendakur (2009) offer an empirical framework that incorporates unobserved preference heterogeneity into demand estimation that is easy to implement, but which requires that unobserved preference parameters act like fixed effects, pushing the entire compensated budget share function up or down by a fixed factor.

Given the difficulty of incorporating unobserved preference heterogeneity beyond a fixed effect, it is instructive to evaluate how our 5 utility functions differ from each other. Since group 5 has only 8 observations assigned to it, we leave it out of this part of the analysis. For the remaining groups we estimate group-specific demand systems. Since we know that, within each type, there exists a single preference map which rationalises all of the data we need not worry about unobserved heterogeneity in our estimation. We know that there is a single integrable demand system which exactly fits the data for each group. The problem we face is that we do not know the specification of that demand system so our main econometric problem is finding the right specification. We take the simplest possible route here and estimate a demand system with a flexible functional form - the quadratic almost ideal (QAI) demand system (Banks, Blundell and Lewbel 1997)). The idea is that such a model should be flexible enough to fit the mean well and that the interpretation of the errors is solely specification error[10].

The QAI demand system has budget shares, w_i^j, for each good $j = 1, ..., K$ and each household $i = 1, ..., N$ given by

$$w_i^j = a^j + \sum_{j=1}^{K} A^{jk} \ln p_i^k + b^j \ln \widetilde{x}_i + q^j \left(\ln \widetilde{x}_i\right)^2 / \widetilde{b}_i + e_i^j,$$

where

$$\begin{aligned} \ln \widetilde{x}_i &= \ln x_i - \sum_{k=1}^{K} a^k \ln p_i^k - \frac{1}{2} \sum_{k=1}^{K} \sum_{l=1}^{K} A^{kl} \ln p_i^k \ln p_i^l, \text{ and} \\ \widetilde{b}_i &= \prod_{k=1}^{K} (p_i^k)^{b^k}, \end{aligned}$$

and p^k are prices, x is total expenditure on all (milk) goods and e^j are error terms. The rationality restrictions of homogeneity and symmetry require that $\sum_k a^k = 1$, $\sum_k b^k = \sum_k q^k = 0$, $\sum_k A^{kl} = 0$ for all l, and $A^{kl} = A^{lk}$ for all k, l. We impose these restrictions and report the coefficients a^k and b^k in Table 3 below. Here, Engel Curves (defined as budget-share functions over expenditure holding price constant) are roughly quadratic in the log of total expenditure. Blundell and Robin (1999) show that this budget-share system may be estimated by iterated seemingly unrelated regression (SUR), and we use that method. For estimation, we normalise each price to its median value and normalise expenditure to its median value, so that at median prices and expenditure, $\ln p^k = \ln x = 0$. In practise, the estimates from this iterated model are 'close' to estimated coefficients from OLS regression of budget shares w^j on a constant (a^j), log-prices (A^{jk}), log-expenditure (b^j) and its square (q^j).

[10] Measurement error is much more cumbersome to consider in a revealed-preference context, so we do not consider it here.

By estimating budget-share equations for each of our four largest groups we characterise what their Engel curves look like and test whether or not including group dummies in budget share equations (as in Lewbel and Pendakur (2009)) is sufficient to absorb the differences across these utility functions.

TABLE 3: Predicted Budget Shares and Semi-Elasticities, QAI Estimation

		Conventional Milk			Organic Milk		
Group	Group N	Full-fat	Semi	Skim	Full-fat	Semi	Skim
Levels, a^j							
group 1	321	0.155	0.434	0.173	0.020	0.085	0.133
		(0.022)	*(0.030)*	*(0.021)*	*(0.007*	*(0.014)*	*(0.017)*
group 2	100	0.153	0.287	0.194	0.089	0.092	0.184
		(0.032)	*(0.044)*	*(0.041)*	*(0.021)*	*(0.028)*	*(0.032)*
group 3	53	0.195	0.330	0.091	0.070	0.130	0.184
		(0.055)	*(0.064)*	*(0.033)*	*(0.027)*	*(0.036)*	*(0.041)*
group 4	18	0.084	0.171	0.295	0.052	0.209	0.190
		(0.061)	*(0.075)*	*(0.079)*	*(0.028)*	*(0.061)*	*(0.057)*
Semi-Elasticities wrt Expenditure, b^j							
group 1	321	-0.040	0.004	0.021	-0.003	0.002	0.016
		(0.018)	*(0.025)*	*(0.017)*	*(0.006)*	*(0.011)*	*(0.014)*
group 2	100	-0.044	0.066	-0.030	-0.033	-0.002	0.043
		(0.032)	*(0.043)*	*(0.041)*	*(0.021)*	*(0.028)*	*(0.032)*
group 3	53	-0.013	0.010	-0.044	0.016	-0.019	0.049
		(0.056)	*(0.065)*	*(0.034)*	*(0.027)*	*(0.037)*	*(0.042)*
group 4	18	-0.099	0.034	-0.175	0.028	0.268	-0.056
		(0.069)	*(0.087)*	*(0.092)*	*(0.032)*	*(0.070)*	*(0.065)*

The top panel of Table 3 gives predicted budget-shares for each group, evaluated at a common constraint defined by the vector of median prices and the median milk expenditure level. (These are the level coefficients in the QAI regressions for each group, where prices and expenditures are normalised to 1 at the median constraint.) The point estimates differ quite substantially across groups, and a glance at the estimated standard errors shown in parentheses (and *italics*) shows that the hypothesis that these point estimates are the same value is heartily rejected.

The bottom panel of Table 3 gives estimated slopes of budget-shares with respect to the log of expenditure (expenditure semi-elasticities) at a common constraint defined by the vector of median prices and the median milk expenditure level. These are the slope coefficients in the QAI regressions for each group, and they differ somewhat across groups. We can weakly reject the hypothesis that the slopes are the same across all 4 groups: the sample value of the Wald test statistic for the hypothesis is 26, and under the Null it is distributed as a χ^2_{15} with a p-value of 3.7%. In fact, the restriction that we can bring in heterogeneity via group dummies implies that all these groups have the same slope and curvature terms. This hypothesis is also weakly rejected—the sample value of the test statistic is 45.4, and under the Null it is distributed as a χ^2_{30} with a p-value of 3.5%. Individually, only groups 2 and 4 show evidence that they differ from group 1 in terms of the total expenditure responses of budget shares (they test out with p-values of 8% and 1%, respectively).

Whereas expenditure effects differ only modestly across groups, the estimated price responses of budget shares differ greatly across groups. We do not present coefficient estimates here because there are 15 of them for each group, but we can assess their difference across groups via testing. The test that all 4 groups share the same price responses has a sample value of 382, and is distributed under the Null as a χ^2_{45} with a p-value of less than 0.1%. Further, any pairwise test of the hypothesis that two groups share the same price responses rejects at conventional levels of significance.

One can also test the hypothesis the heterogeneity across the types can be absorbed into level effects. Not surprisingly, given that we reject both the hypotheses that total expenditure effects are identical and that price effects are identical, this test is massively rejected. The test statistic has a sample value of 405, and is distributed under the Null as a χ^2_{75} with a p-value of less than 0.1%.

One problem with using the QAI demand system to evaluate the differences across groups is that there is no reason to think that the functional structure imposed by the QAI demand system is true. An alternative approach is to use nonparametric methods. These methods have the advantage of not imposing a particular functional form on the shape of demand. They have the disadvantage of suffering from a severe curse of dimensionality, because in essence one needs to estimate the level of the function at every point in the support of possible budget constraints. The dimensionality problem is that this support grows fast with the number of goods in the demand system. A nonparametric approach that does not suffer from the curse of dimensionality is to try to estimate averages across the support of budget constraints.

Table 4: Predicted Budget Shares and Semi-Elasticities, Nonparametric Estimation

		Conventional Milk			Organic Milk		
Group	Group N	Full-fat	Semi-fat	Skimmed	Full-fat	Semi-fat	Skimmed
Average Levels							
group 1	321	0.157	0.465	0.158	0.027	0.078	0.115
		(0.015)	*(0.020)*	*(0.014)*	*(0.006)*	*(0.014)*	*(0.013)*
group 2	100	0.168	0.292	0.191	0.074	0.092	0.184
		(0.022)	*(0.035)*	*(0.029)*	*(0.013)*	*(0.015)*	*(0.028)*
group 3	53	0.201	0.362	0.073	0.034	0.128	0.201
		(0.056)	*(0.056)*	*(0.019)*	*(0.017)*	*(0.032)*	*(0.056)*
Average Semi-Elasticity wrt Expenditure							
group 1	321	-0.020	0.013	0.003	-0.001	-0.016	0.022
		(0.022)	*(0.028)*	*(0.016)*	*(0.007)*	*(0.009)*	*(0.014)*
group 2	100	-0.048	0.068	-0.084	-0.010	0.000	0.074
		(0.028)	*(0.061)*	*(0.040)*	*(0.013)*	*(0.025)*	*(0.050)*
group 3	53	0.028	-0.074	-0.029	-0.002	-0.054	0.132
		(0.082)	*(0.094)*	*(0.027)*	*(0.017)*	*(0.035)*	*(0.099)*

In the top panel of Table 4, we present the average over all observed budget constraints of the nonparametric estimate of budget shares for each group. For the nonparametric analysis, we study only the 3 largest groups, totaling 474 observations. For each group, we nonparametrically estimate the budget share function evaluated at each of the 474 budget constraints, and report its average over the 474 values. Nonparametric estimates of budget-shares given prices and expenditures are computed following Haag, Hoderlein and Pendakur (2009), and the averages of these estimates are presented in the Table. The nonparametric estimate of the budget-share vector at a particular expenditure level and price vector is the locally-weighted average of budget-shares, with weights declining for observations with 'distant' prices or expenditures. Haag, Hoderlein and Pendakur (2009) show how to estimate such a locally weighted model while maintaining the restrictions of Slutsky symmetry and homogeneity. Simulated standard errors are in parentheses.[11]

The top panel of Table 4 shows average levels that are broadly similar to the sample averages reported in Table 2. However, those reported in Table 4 differ in one important respect: whereas those shown in Table 2 are averages across the budget constraints in each group, those reported in Table 4 are averages across the budget constraints of all groups. That is, whereas the sample averages in Table 2 mix the effects of preferences and constraints, the nonparametric estimates in Table 4 hold the budget constraints constant. These numbers suggest that there is a quite a lot of preference heterogeneity. For example, Group 1 and Group 2 have statistically significantly different average budget shares for most types of milk.

Given that unobserved heterogeneity which can be absorbed through level effects can fit into recently proposed models of demand (Lewbel and Pendakur 2009), it is more important to figure out whether or not the slopes of demand functions differ across groups. The bottom panel of Table 4 presents average derivatives with respect to the log of expenditure (that is, the expenditure semi-elasticities of budget share functions), again averaged over the 474 observed budget constraints, with simulated standard errors shown in parentheses (and *italics*).

Clearly, the estimated average derivatives are much more hazily estimated than the average levels. But, one can still distinguish groups 1 and 2: the skimmed conventional milk budget share function of group 2 has a statistically significantly lower (and negative) expenditure response than that of group 1. No other pairwise comparison is statistically significant. However, the restriction that the average derivatives are the same across groups combines 10 z-tests like this, two restrictions for each of the 5 independent equations. One can construct a nonparametric analogue to the joint Wald test of whether or not the three groups share the same expenditure responses in each of the 6 equations. This test statistic has a sample value of 24.3 and has a simulated p-value of 0.7%.[12]

The picture we have of the heterogeneity in the consumer microdata is as follows. First, we can *completely* explain all the variation of observed behaviour with variation in budget constraints and 4 or 5 preference maps (i.e. ordinal utility functions). Second, the groupings are not strongly related to observed characteristics of households. That is, the primary heterogeneity here is *un*observed. Third, the groups found by our upper bound algorithm are very different from each other, mainly in terms of how budget shares respond to prices, but also in expenditure responses. That the budget-share equations of the groups differ by more than just level effects suggests that unobserved preference heterogeneity may not act like 'error terms' (or fixed effects) in regression equations, and thus do not fit into models recently proposed to accommodate preference heterogeneity in consumer demand modeling.

6 How Many Types in the Population?

Up to now, we have concerned ourselves with the question of how many types are needed to characterise preferences in a *sample* of micro-economic choice data. This begs the question of how many types are needed

[11] It is well-known that average derivative estimators suffer from boundary bias. Although the estimates in Table 4 do not trim near the boundaries, estimates which do trim near the boundaries yield the same conclusions. Standard errors are simulated via the wild bootstrap using Radamacher bootstrap errors. Nonparametric estimators only suffer from specification error in the small sample. Such error disappears as the sample size gets large. Further, unobserved heterogeneity need not cause a deviation from the regression line, because such heterogeneity is not necessary after our grouping exercise. Thus, the wild bootstrap, which bases simulations on resamples from an error distribution, is actually an odd fit to the application at hand. An alternative is to resample from budget constraints (rather than from budget shares) to simulate standard errors. These simulated standard errors are much smaller, and make the groups look sharply different from each other in terms of both average levels and average slopes.

[12] If we use the alternative resampling strategy which provides tighter standard errors (outlined in the previous footnote), then the test that the average derivatives are the same for all 3 groups is rejected in each of the 5 independent equations, and, not surprisingly, rejected for all 5 together.

to characterise preferences in the *population* from which the sample is drawn. This is similar in some ways to the famous coupon collector's problem (see, e.g., Erdős and Rényi (1961)) and other classical problems in probability theory like the problem of estimating how many words Shakespeare knew, based on the Complete Works (see, e.g., Efron and Thisted (1975)). It is also a difficult problem to answer credibly - especially when the unseen types in the population are not abundant and there is consequently a high probability that you will miss them in any given sample.

Biologists have long concerned themselves with a question which is closely analogous to ours, that of the number of species which exist in the population of animals. Biostatisticians have developed a variety of estimators for this object. Most are based on the 'frequency of frequencies' of species in a sample (see, e.g., surveys by Bunge and Fitzpatrick (1993) and Colwell and Coddington (1994)). The frequency of frequencies records the number of singletons, defined as species observed only once in a sample, the number of doubletons, defined as the number of species observed twice, and so on.

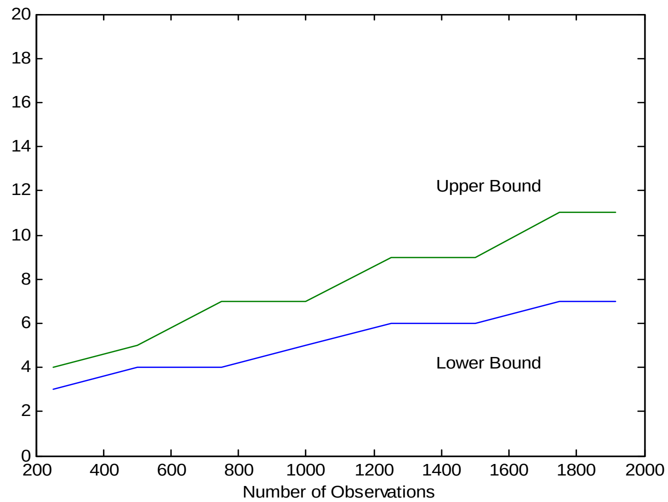
Perhaps the simplest of these estimators is that of Chao (1984) who proposes a lower bound estimator of the number of species equal to $s_{obs} + s_1^2/2s_2$, where s_{obs} is the number of species observed in the sample, s_1 is the number of singletons, and s_2 is the number of doubletons. This estimator has the property that it equals s_{obs} when there are no singletons ($s_1 = 0$). A variety of other (nonparametric) estimators have been proposed since Chao (1984), but all those we found have this same property regarding the dependence on the number of singletons.

Another approach is to characterise the number of species via extrapolation of the number of species observed in increasingly large samples from a finite population (Colwell and Coddington (1994) survey this literature). The idea is intuitively appealing: if the graph of $s_{obs}(N)$, the number of observed species as a function of sampling effort measured by the sample size N, asymptotes to a fixed number, then this may be taken as an estimate of the number of species in the population.

The analogy between animal species and preference types is worth considering for a moment. Whether or not two individuals could have the same utility function, and thus could be of the same type, is verifiable (via RP tests). However, when revealed preference restrictions are used to identify types, it is often possible to fit individuals into more than one type. That is to say that the definition of a type is not "crisp" and the allocation of individuals to types is not unique. It may be that persons A and C violate an RP test when pooled together, and so have different preferences, but that B passes an RP test when combined with either - where should we put B? In assessments of biodiversity which apply the statistical methods described above, the literature proceeds as if there is no such uncertainty as to which species an observation should be assigned. It is worth pointing out that biologists know that this is not entirely true. There exist "ring species" (the Ensatina salamanders which live in the Central Valley in California are the famous example) where (sub)species A and C cannot breed successfully, but species B can breed with either A's or C's - where should B lie in the taxonomy? The biostatistics literature treats this as an ignorable problem. It may or may not be an ignorable problem for economists. Nonetheless we too will ignore it.

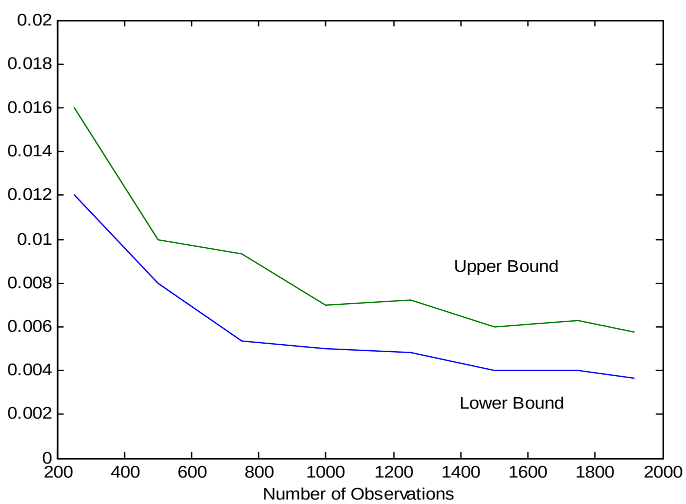
As shown in the previous section we did not find any singletons in our dataset of 500 observations. Therefore the frequencies of frequencies approach cannot be applied fruitfully in our data - it will simply give an estimate of the number of types in the population equal to the number of types in our sample - we adopt the idea of plotting $s_{obs}(N)$ and extrapolating. From the full data set of 1917 observations, we took random sub-samples of sizes 250,500,...,1750, and the full sample of 1917 observations, and ran our upper and lower bound algorithms to determine bounds on the minimum number of types necessary to rationalise all the observed choices in each sample. Figure 3 shows results for $s_{obs}(N)$: the upper line traces out the upper bound, and the lower line traces out the lower bound. Figure 4 shows the ratio of types to sample size: $s_{obs}(N)/N$.

FIGURE 3. The bounds on the number of types against sample size



Examination of Figure 3 does not immediately suggest an asymptote for $s_{obs}(N)$. However, it is clear from Figure 4 which shows the ratio of types to sample size that the number of types rises slower than linearly with the number of observations in the sample. Because the number of observations in the sample does not get anywhere near the size of the population (about 2.5 million, the number of households in Denmark), we cannot pick out this asymptote in a nonparametric way. Raaijmakers (1987) suggests the use of the parametric Eadie-Hoffstee equation ($s_{obs}(N) = s_{pop} - Bs_{obs}(N)/N$, where s_{pop} and B are unknown parameters) to estimate the asymptote, and provides a maximum likelihood estimator for the asymptote s_{pop}, which may be taken as the estimated number of species in the population. Implementation of this estimator using the upper bound on the number of types results in an estimate of 10.75 with a standard error of 0.6. This suggests that the number of types in the population is at most 12.

FIGURE 4. The bounds on the ratio of types to sample size



7 Discussion

We consider an elementary method of partitioning data so that it can be explained perfectly by the theory, and in a way which admits the minimal necessary heterogeneity. We argue that our approach offers two benefits which may complement the more established microeconometrics treatment of unobserved heterogeneity. Firstly it provides a framework in which to study heterogeneity which is driven by the economic model of interest. In doing so it provides a practical method of partitioning data so that the observations in each group are precisely theory-consistent rather than just approximately so. This allows researchers to estimate group-specific demand models without fear of the complications which arise in the presence of unobserved heterogeneity. Secondly it does not require statements about the distributions of objects we can't observe or functional structures about which economic theory is silent.

Throughout this paper we have focused on consumer data and on the canonical utility maximisation model. This is mainly for expositional reasons and it is important to point out what we are proposing can easily be applied to the analysis of heterogeneity in any microeconomic model of optimising behaviour which admits a RP-type characterisation. This is an increasingly wide class which includes profit maximisation and cost-minimisation models of competitive and monopolistic firms, models of intertemporal choice, habits, choice under uncertainty, collective household behaviour, characteristics models, firm investment as well as special cases of all of these models which embody useful structural restrictions on preferences or technology (e.g. weak separability, homotheticity and latent separability)[13]. To adapt our approach to any of those models, one simply replaces the GARP check in all the algorithms with the appropriate RP check (see the Appendix). The point is that our strategy for assessing heterogeneity in the consumer demand framework is in principle applicable to any environment where agents are assumed to be optimising something.

In the empirical illustration we characterise the amount of heterogeneity necessary to completely rationalise the observed variation in our consumer microdata. We find that very few types are sufficient to rationalise observed behaviour completely. Our results suggest that Becker and Stigler had it wrong in *De Gustibus*: preferences do indeed differ both capriciously and importantly between people. The capriciousness is that although in the three decades since Becker and Stigler's assessment, we have learned much about how to deal with preference heterogeneity that is correlated with observed variables, it seems that the more important kind of heterogeneity is driven by unobserved variables. Our results also suggest that models which use a small number of heterogeneous types—such as those found in macro-labour models, education choice models, and a vast number of empirical marketing models—may in fact be dealing with unobserved heterogeneity in a sufficient fashion. In contrast, models like Lewbel and Pendakur (2009), in which unobserved preference heterogeneity is captured by a multidimensional continuum of unobserved parameters could well be overkill.

[13] Afriat (1967), Diewert (1973), Varian (1982, 1983a, 1983b, 1984), Hanoch and Rothschild (1972), Browning (1989), Bar-Shiva (1992), Cherchye, De Rock and Vermuelen (2007),Blow *et al* (2008),

Appendix

Partitioning Algorithms

Notation: For an arbitrary set A, $\mathcal{P}(A)$ denotes the power set (set of all subsets) of A The number of elements of A is denoted by $|A|$. For arbitrary sets A and B, $A\backslash B$ denotes A minus B: all elements from A that are not in B. In what follows I is th e index set $\{1, ..., N\}$.

Brute Force Algorithm

Inputs: $I, \{\mathbf{p}_i, \mathbf{q}_i\}_{i\in I}$. Outputs: N and G.

1. If $\{\mathbf{p}_i, \mathbf{q}_i\}_{i\in I}$ satisfies RP set $N = 1, G = I$ and goto (6).
2. Set $H = \left\{h : h \in \mathcal{P}(I)\,,\ \{\mathbf{p}_i, \mathbf{q}_i\}_{i\in h} \text{ satisfies } RP\right\}$
3. Set $J = \left\{j : j \in \mathcal{P}(H)\,,\ \cup_{s_k\in j} s_k = I \text{ where } s_k \in j\right\}$
4. $N = \min\{|j| : j \in J\}$.
5. Set $G = \{j : j \in J,\ |j| = N\}$.
6. Stop.

The outputs of the "Brute Force" algorithm are N = the number of types and G = a set containing all of the exclusive and exhaustive partitions of the data into N subsets such that the data in each type satisfy RP. This algorithm works by simply enumerating all of the subsets of the data, checking RP conditions within those subsets and then finding the minimal partition based on those subsets which satisfy RP.

Upper Bound Algorithm.

Inputs: $I, \{\mathbf{p}_i, \mathbf{q}_i\}_{i\in I}$. Outputs: $\overline{N}$ and $\overline{G}$

1. If $\{\mathbf{p}_i, \mathbf{q}_i\}_{i\in I}$ satisfies RP set $\overline{N} = 1, \overline{G} = I$ and goto (8).
2. Select $i \in I$ with uniform probability, set $I = I\backslash i$.
3. Set $\overline{G}_1 = \{i\}$, set $\overline{G} = \left\{\overline{G}_1\right\}$
4. If $I \neq \varnothing$, select $i \in I$ with uniform probability, set $I = I\backslash i$,
   set $E = \overline{G}$. Else if $I = \varnothing$ goto (8)
5. If $E \neq \varnothing$, select $g = \arg\max\left\{|g| : g \in \overline{E}\right\}$,
   if g contains more than one element set $g = \min_i \left\{\overline{G}_i \in g\right\}$,
   set $E = \overline{G}\backslash g$.
   Else goto (7)
6. If $\{\mathbf{p}_j, \mathbf{q}_j\}_{j\in\{g,i\}}$ satisfies RP set $\overline{G} = \overline{G}\backslash g$, set $g = g \cup i$,
   set $\overline{G} = \overline{G} \cup g$ and goto (4); else goto (5).
7. Set $\overline{G}_{|\overline{G}|+1} = \{i\}$, set $\overline{G} = \overline{G} \cup \overline{G}_{|\overline{G}|+1}$, goto (4).
8. $\overline{N} = |\overline{G}|$
9. Stop.

The outputs of the Upper Bound algorithm are $\overline{N}$ = the upper bound on the number of types and $\overline{G}$ = a set containing an exclusive and exhaustive partition of the data into $\overline{N}$ subsets such that the data in each type satisfy the RP conditions. The algorithm works on the principle of randomly ordering the data and trying to construct groups which satisfy RP conditional on that ordering. As new observations are drawn it tries to add them to the existing partition and starts by placing them in the largest group available. If an observation cannot be added to an existing group it is used to initialise a new group. The upper bound algorithm begins by picking a single observation at random without replacement. This forms the basis for the first group. It then chooses the next observation at random also without replacement and tests whether the two satisfy RP. If they do they are placed together in the first group. If they don't the new observation is used to begin a new group. The next observation is then drawn and, starting with the largest existing group an RP test is used to determine whether it can by placed in that group. It is placed into the first group where it satisfies RP. If no such group can be found amongst the exists groups the observation is used to start a new group. The algorithm continues in this way until the dataset is empty and all observations have been assigned to groups. Since this algorithm relies on a random ordering of the data we run it a number of times and retain the minimum partition over these independent runs. In all of the empirical work in this paper we used 50 runs of the algorithm.

Lower Bound Algorithm.
Inputs: $I, \{\mathbf{p}_i, \mathbf{q}_i\}_{i \in I}$.. Output: $\underline{N}$

1. If $\{\mathbf{p}_i, \mathbf{q}_i\}_{i \in I}$ satisfies RP set $\underline{N}= 1, g = 1$ and goto (6).
2. Select $i \in I$ randomly with uniform probability, set $I = I \backslash i$ and $g = i$
3. If $I \neq \varnothing$, select $j \in I$ with uniform probability, set $I = I \backslash j$,
   else if $I = \varnothing$ goto (6)
4. If the dataset $\{\mathbf{p}_j, \mathbf{q}_j, \mathbf{p}_i, \mathbf{q}_i\}_{\forall j,i \in \{j,g\}}$ violates RP set $g = g \cup j$. Goto (3).
5. $\underline{N}= |g|$
6. Stop.

The output of the Lower Bound algorithm is $\underline{N}$ = the lower bound on the number of types. This algorithm works on the principle that if we can find $\underline{N}$ observations which violate RP in all pairwise tests conducted between themselves, then there must be at least $\underline{N}$ groups (since none of these observations could ever be placed in the same group). It begins by selecting an initial observation at random without replacement. It then picks another observation without replacement and tests RP. If the pair satisfy RP the second observation is dropped and a new observation selected. However if the pair violate RP the new observation is retained. We now have two observations which violate RP. A third observation is now selected from the data without replacement. This is tested against each of the observations currently held. If it violates in pairwise tests against all of them then it is retained. Otherwise it is dropped. The algorithm continues in this way until the dataset is exhausted. At the end of the process the algorithm has collected together a set of observations which all violate pairwise RP test conducted between each of them. The number of these observations gives the lower bounds $\underline{N}$. As before the algorithm is reliant on a random ordering of the data. We therefore run the process a number of times and retain the maximum value of $\underline{N}$ we find.

Allowing for Optimisation Errors

In the body of the paper we have treated the data as error free. We now consider optimising error by consumers and measurement errors by the data collector. Our treatment of these issues is not original to this paper[14]. The point of this section is merely to show, briefly, that these treatments can be applied in our context.

[14] The treatment of optimisation errors in RP tests is due to Afriat (1967) and that of measurement error is due to Varian (1985).

Afriat (1967) interpreted RP checks as a conflation of two sub-hypotheses: theoretical consistency *and* the idea that economic agents are efficient programmers. If the data violate the conditions then it may be that some consumers have made optimisation errors. His suggestion was that, instead of requiring exact efficiency, a form of partial efficiency is allowed. This is achieved by introducing a parameter $e \in [0,1]$ (the Afriat efficiency parameter) such that

$$e\mathbf{p}_t'\mathbf{q}_t \geq \mathbf{p}_t'\mathbf{q}_s \;\Leftrightarrow \mathbf{q}_t R_e^0 \mathbf{q}_s$$

The weaker form of GARP is then

$$\mathbf{q}_j R_e \mathbf{q}_i \Rightarrow e\mathbf{p}_i'\mathbf{q}_i \leq \mathbf{p}_i'\mathbf{q}_j$$

where R_e is the transitive closure of R_e^0. The interpretation of e is as the proportion of the consumer's budget which they are allowed to waste through optimisation errors. This parameter is used to modify the restrictions of interest to allow for a weaker form of consistency (see Afriat (1967)). To admit optimisation error into the partitioning approach all we need to do is to specify a level for e in advance and insert the modified RP restriction into the algorithms at the appropriate steps (step (2) in the exact algorithm, step (6) in the upper bound and step (4) in the lower bound algorithm). It is then straightforward to examine how the results vary with the required efficiency level. The effects of allowing for optimisation errors is to reduce the amount of heterogeneity which is needed to rationalise the data. Admit enough error and it is possible to rationalise almost anything. As a result, running the algorithms without these adaptations will give a "worst-case" assessment, delivering the "maximum minimum" number of groups.

We implemented this methodology with our sample of 500 observations of household milk demands. Clearly if there is enough optimisation error, then one can explain any behaviour with just one utility function. This is what we observe for $e \leq 0.781$. For e greater than this, more than one utility function may be required to explain the variation in behaviour that we observe. For $e \geq 0.90$, more than one utility function is definitely required to explain the variation in observed behaviour.

Table A.1: Bounds and Afriat Efficiency

e	Bounds
0.78	1
0.80	2
0.85	2
0.90	[2,3]
0.95	[3,4]
1.00	[4,5]

A method for dealing with classical measurement error is described in Varian (1985) and it is as follows. Suppose that the demands are measured with error which is assumed normal with mean zero and variance σ^2. Suppose further that we knew the true demands $\mathbf{q}_t^*$. Then $\mathbf{e}_t = \mathbf{q}_t^* - \mathbf{q}_t$ and the statistic $\sum_t \mathbf{e}_t'\mathbf{e}_t/\sigma^2$ is chi-squared. Thus if we knew the true values we could, given this probablistic model for the measurement error, proceed on this basis to compute the probability with which the true data satisfy the RP criteria. The trouble is we do not know the truth so statistics based on the distance between the observations and their true values are not immediately useful. However Varian (1985) suggests replacing $\mathbf{e}_t = \mathbf{q}_t^* - \mathbf{q}_t$ with $\mathbf{u}_t = \widehat{\mathbf{q}}_t - \mathbf{q}_t$ where $\widehat{\mathbf{q}}_t$ solves the quadratic programming problem

$$\min_{\{\widehat{\mathbf{q}}_t\}_{t=1,..,,T}} \sum_t \mathbf{u}_t'\mathbf{u}_t/\sigma^2$$

subject to the constraint that $\{\mathbf{p}_t, \widehat{\mathbf{q}}_t\}_{t=1,...,T}$ satisfies the RP restrictions. Thus the $\{\widehat{\mathbf{q}}_t\}_{t=1,.,,,T}$ represent the closest (in the least-squares sense) set of demands which satisfy the RP restrictions. Under the null hypothesis that the true data satisfy the theory the distance between the observed demands and the true demands cannot be less than the distance between the observed demands and these closest, theory-consistent points. Thus, it is argued, basing a statistical test on whether $\sum_t \mathbf{u}_t'\mathbf{u}_t/\sigma^2$ exceeds an α critical value gives a conservative approach to inference in the sense that the probability of rejecting the hypothesis that the true data satisfy the RP restrictions will be less than α. The essence of Varian (1985) is to use the quadratic programming problem to bound the unobserved random variable. In the present context this is, in principle, a straightforward bolt-on to the methods described in the paper: whenever an RP test is used to detect whether observations are of a common type a statistical test along these lines can be implemented *given* a suitable assumption about the form of the measurement error. The computational cost is likely to considerable though because ever test would involve solving a quadratic programming problem with, depending on the number of agents involved, very many parameters.

Panel Methods

So far we have considered cross-section data. Clearly in cross section data, where each consumer is observed only once, some degree of commonality in preferences is necessary in order to make progress in applied work. However panel data generally holds out the hope of identifying more about individuals than is possible with cross section data. Indeed panel data has two important features in terms of identifying types in our framework. Firstly, repeated observation on individuals allows them to distinguish their type more clearly through their behaviour. Secondly, repeated observations mean that stability of preferences becomes an important factor.

Recalling our main question: how many sets of preferences are needed to rationalise the data? a natural way to proceed is to first check GARP for each individual consumer and then to seek to allocate consumers into type groups. Given a set of individually GARP-consistent consumers, the algorithms described above can be applied almost without modification.

Of course some consumers will individually fail GARP and the question arises what to do with them. One answer is to simply set them aside as their behaviour is not rationalisable with the model of interest. However, this would not be in the spirit of taking rationality as a maintained assumption. A second possibility is to allow for enough optimisation errors in the way we describe above. However, this strategy would also affect the grouping of people (because admitting optimisation error would tend to decrease the amount of RP violations). Thirdly, we could consider alternative models for their intertemporal behaviour.

People change. Although economists like to invoke immutable preferences, we all know that our preferences can change, sometimes in a dramatic fashion. So, a final alternative is to allow for "multiple personalities". By that we mean that we can take the data on an individual and search within it for contiguous sub-periods during which their behaviour is rationalisable. We can then treat each of these sub-periods as a separate individual (which they are in the sense that each one potentially requires a different utility function to model it) and run the partitioning algorithms as before. Because this approach sits wholly inside our basic framework, without the need for discarding data, including optimisation errors or writing down a dynamic structure for utility functions, it is in some sense the simplest option, and therefore our preferred one.

We use the same 500 households as in our cross-sectional analysis, but use a sequence of up to 24 months of milk consumption data for each household. Using the "multiple personality" mentioned above, which preserves all of the data from our 500 households, we implement our model. The number of groups needed to completely rationalise these data is at least 12 and not more than 31. This is quite surprising. After all the standard approach to panel data in applied econometric analysis is to use "fixed effects", which in this context would imply 434 groups. Our results show that this is at least 15 times as many groups as are really necessary, and therefore is radically over-specified.

Fixed effects are a bad match to these data for 2 more important reasons. First, Blundell, Duncan and Pendakur (1998) show that fixed effects in budget shares are consistent with rationality restrictions only if budget shares are linear in the natural logarithm of total expenditure. Second, in our exploration of cross-sectional data above, we showed that *both* levels *and* derivatives of budget-share equations vary across groups. Thus, fixed level effects do not adequately capture the differences across groups.

Other Contexts

The methods outlined in this paper can be easily adapted to other optimising models. Corresponding restrictions are available for models of intertemporal choice (Browning 1989), habits (Crawford 2010), choice under uncertainty (Bar Shira 1992), profit maximisation by firms (Hanoch and Rothschild 1972), cost minimisation by firms (Hanoch and Rothschild 1972), collective household behaviour (Cherchye, De Rock and Vermuelen 2007), characteristics models (Blow *et al* 2008), as well as special cases of all of these models which embody useful structural restrictions on preferences or technology (e.g. weak separability, homotheticity and latent separability). In order to apply our methods to another optimising model, one simply replaces the GARP restrictions used in the body of this paper with the corresponding restrictions on optimising behaviour driven by the model of interest. Below, we briefly demonstrate how this works by applying our methods to a model of firm cost minimisation. Instead of seeking the minimum number of distinct preference maps necessary to rationalise observed consumption choices, we seek the minimum number of distinct technologies necessary to rationalise the observed input demand choices.

Here we provide an illustration of the application of partitioning to firm data. The data relate to 281 Danish Farms observed in 1990. These are Danish Farm Association Service data gathered through a voluntary consultancy scheme and for each farm the data includes detailed annual accounts of variable costs and earnings for each production line with corresponding accounts measures of most inputs and outputs. We measure five outputs {milk, two types of beef, and two types of crops} and we observed 46 inputs - like fodder, cattle, fertiliser, pesticides, and the services from labour, land, building and machine capital. The farms recorded the transactions prices for each of their inputs and outputs. In this application we are interested in unobserved technological heterogeneity and the economic model of interest is the canonical cost minimisation model:

$$\min_{\mathbf{x}} \ \mathbf{w}_i'\mathbf{x} \text{ subject to } \mathbf{x} \text{ is in } V(\mathbf{q}_i)$$

where $\mathbf{q}_i$ denotes a vector recording the quantities of the outputs of firm i and $\mathbf{x}_i$ is a vector recording the quantities of the firm's inputs. Technology is denoted by the input requirement set $V(\mathbf{q}_i)$ which is a closed, non-empty, monotonic, nested and convex set which consists of all input vectors $\mathbf{x}$ that can produce at least the output vector $\mathbf{q}_i$.The observable consequences of this model are summarised in the following theorem.

Theorem: (Hanoch and Rothschild (1972), Diewert and Parkan (1983), Varian (1984)). *The following conditions are equivalent:*
(1) there exists a family of nontrivial, closed, convex, positive monotonic input requirement sets $\{V(\mathbf{q})\}$ *such that the data* $\{\mathbf{q}_t, \mathbf{w}_t, \mathbf{x}_t\}$ *solves the problem* $\min_{\mathbf{x}} \ \mathbf{w}_i'\mathbf{x}$ subject to $\mathbf{x}$ is in $V(\mathbf{q}_i)$ for each $i = 0, 1, ..., N$.
(2) if $\mathbf{q}_j \geq \mathbf{q}_i$ *then* $\mathbf{w}_i'\mathbf{x}_j \geq \mathbf{w}_i'\mathbf{x}_i$ *for all* i *and* j.

The condition in (2) is the Weak Axiom of Cost Minimisation (Varian, 1983) and it provides the partitioning criteria: if the data for two firms are such that (2) does not hold then the two firms concerned cannot have the same technology. We simply replace GARP with WACM in the algorithms described in the paper and find the following bounds on the number of technological types is between 3 and 4. It appears that very few production technologies are required to model (precisely) these very disaggregated data in which firms are able to choose from many inputs and produce several outputs. Furthermore as with the consumer cross section data it turns out that a single production technology will fit the majority of the data:

Table A.2: The cumulative distribution of technological types

Number of Types	1	2	3	4
Percent of Sample Explained	80%	94%	98%	100%

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