SECURITY ANALYSIS AND PORTFOLIO MANAGEMENT

More Solutions to Problem Set #1

Section II:

2. Let C = $6 T = 5 and y = 8%

$$PB^{A} = \$3 \sum_{t=1}^{10} \frac{1}{(1.04)^{t}} + \frac{100}{(1.04)^{10}} = \$91.89$$

$$PB^{Q} = \$1.50 \sum_{t=1}^{20} \frac{1}{(1.02)^{t}} + \frac{100}{(1.02)^{20}} = \$91.82$$

3. y = 12% (y/2 = 6%)

How to solve for the yield to maturity given the PB? As bonds are traded in terms of price, this is the typical calculation which has to be done in practice.

Solution Methods:

a) Get either a financial calculator or a calculator with a Present Value function and learn how to use it.

b) Use the trial and error method-- guess a yield and solve for the bond price. If the calculated price is above the observed price, then try a higher yield and recalculate the price. If the calculated price is below the observed price, then try a lower yield and recalculate the price. Continue this process until a yield is determined which has a desirable level of precision.

4. A perpetuity is a security which offers to pay a fixed or variable coupon, at regular intervals, forever (in perpetuity). If the coupon is variable, then it is referred to as a floating rate perpetuity. Almost all perpetuities issued in recent years have been floating rate perpetuities. The most well known perpetuity is a **consol**, originally issued by the British government in the 18th and early 19th century, which pays a fixed coupon.

For the Consol:

$$P^{perp} = \sum_{t=1}^{\infty} \frac{C}{(1+y)^t} = \frac{C}{(1+y)} \{1 + \frac{1}{1+y} + \frac{1}{(1+y)^2} + \frac{1}{(1+y)^3} ....\}$$

$$\textit{Recall:} \quad \frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + ...$$

$$\therefore \quad P^{perp} = \frac{C}{(1+y)} \{\frac{1}{1 - \frac{1}{(1+y)}}\} = \frac{C}{y}$$

If coupons are paid quarterly, the formula is the same, except that y will be different (lower) than in the annual coupon case.

For floating rate perpetuals which are riskless, set C = y M.

5. Macaulay duration

$$D = \frac{\sum_{t=1}^{T} \frac{t\,C}{(1+y)^t} + \frac{T\,M}{(1+y)^T}}{PB} = \frac{1+y}{y} - \frac{1 + y + T\,(\frac{C}{M} - y)}{\frac{C}{M}\{(1+y)^T - 1\} + y}$$

$$= \frac{1+y}{PB} \{\frac{C}{y} [\frac{1}{y} - \frac{1}{(1+y)^T\,y} - \frac{T}{(1+y)^{T+1}}] + \frac{T\,M}{(1+y)^{T+1}}\}$$

y = 8%	C = \$4	C = \$8	C=\$10
Duration	2.87918	2.78326	2.74236
PB	89.6916	100	105.154

y = .12

Duration	2.87211
PB	80.7853

Δ P = (Duration)* (Δ y/(1 + y_0) * P_0
= 9.56438

Actual ΔP = 89.6916 - 80.78653 = 8.90507

Conclusion: For large changes in yield, duration estimate deviates significantly from actual price change. (Recall interpretation of duration as the slope of the price/yield curve).

Duration of Portfolio:

$$Dur_p = \sum_{i=1}^{3} w_i \, Dur_i \qquad where \qquad w_i = \frac{\$A_i}{\sum_{i=1}^{3} A_i}$$

Duration of portfolio = (1/3) 2.87918 + (1/3) 2.78326 + (1/3) 2.74236 = 2.8016

6. Macaulay duration as an elasticity:

$$Dur = \frac{1 + y}{PB} \frac{dPB}{d(1 + y)}$$

$$\frac{dPB}{d(1 + y)} = \frac{dPB}{dy} = -\{\sum_{t=1}^{T} \frac{tC}{(1 + y)^{t+1}} + \frac{TM}{(1 + y)^{T+1}}\}$$

Multiplying through by (1+ y)/PB and cancelling produces the duration formula.

Section III: Term Structure Behaviour

1. $PZ = M/(1 + z_t)^t$ ----> $z_t = (M/PZ)^{1/t} - 1$

z_1 = .02987 z_2 = .03975 z_5 = .06007 z_{10} = .07007 z_{30} = .0701

Implied forward rates:

$$1 + f_{i,j} = \{\frac{(1 + z_j)^j}{(1 + z_i)^i}\}^{1/(j-i)} \qquad 1 + f_{1,2} = \frac{(1 + z_2)^2}{1 + z_1} \qquad 1 + f_{5,10} = \{\frac{(1 + z_{10})^{10}}{(1 + z_5)^5}\}^{1/5}$$

$f_{1,2}$ = .04972 $f_{1,5}$ = .06776 $f_{1,10}$ = .07463 $f_{1,30}$ = .07151 $f_{2,5}$ = .07384

$f_{2,10}$ = .07779 $f_{2,30}$ = .0723 $f_{5,10}$ = .08016 $f_{5,30}$ = .07212 $F_{10,30}$ = .07012

2. Under the unbiased expectations hypothesis, the implied forward rate is the market' s current unbiased prediction about what interest rates will be in the future. If rates are not expected to change in the future, this is consistent with the current yield curve being flat under the unbiased expectations hypothesis.

Under the liquidity premium hypothesis, the forward rate is an upwardly biased predictor, where the amount of bias is determined by a liquidity premium which causes current interest rates on long term bonds to be higher than on short term bonds. The longer is the maturity, the larger will be the liquidity premium (liquidity premia are montonically increasing). Under the liquidity premium hypothesis, if interest rates are not expected to change in the future, this is consistent with the current yield curve being upward sloping.

Under the market segmentation hypothesis, there are significant differences in the demands and supplies for bonds of different maturities. Under this hypothesis, it is not possible to make accurate inferences about market predictions of future interest rates from the shape of the observed yield curve.

3. a) No investment is required in the portfolio

Assets:		Liabilities:	
20 Year bond	$376,890	Short sale	$1,504,370
Cash	$1,127,480		
Total:	$1,504,370		

b) Duration: 20 (376,890/1,540,370) = 5 years, same as for liability
Because the value of the assets and liabilities are equal, this portfolio is immunized.

c) Yields rise from 5% to 6%

Assets:		Liabilities Plus Net Worth:	
20 Year bond	$311,800	Short sale	$1,434,740
Cash	$1,127,480	Net Worth	4,540
Total:	$1,439,280		

At the new interest rate, the value of the assets decreases less than the liability.

d) Yields fall from 5% to 4%

Assets:		Liabilities:	
20 Year bond	$456,390	Short sale	$1,573,100
Cash	$1,127,480	Net worth	5,770
Total:	$1,583,870		

At the new interest rate, the value of assets increases more than the liability

e) Yields stay the same, but one year passes:

Assets:		Liabilities:	
20 Year bond	$395,734	Short sale	$1,579,590
Cash	$1,127,480	Net Worth	($55,276)
Total:	$1,524,214		

If interest rates remain unchanged, the value of the assets is less.

This illustrates the role of convexity. Setting the durations of the portfolios equal, higher convexity portfolios outperform low convexity portfolios when interest rates go up or down. However, if rates do not change then higher convexity portfolios will underperform lower convexity portfolios.

ADDITIONAL QUESTIONS

PRACTICE QUESTIONS FROM CFA EXAMINATIONS

DURATION

Q1) Q88 1992 "An 8%, semi-annual coupon, 20 year corporate bond is priced to yield 9%. The Macaulay duration for this bond is 8.85 years. Given this information, the bond's modified duration is...."

Q2) Q48 1991 "An 8%, 15 year bond has a yield-to-maturity of 10% and a modified duration of 8.05 years. If the market yield changes by 25 basis points, how much change will there be in the bond's price?"

Q3) Q6 1994 "A 6% coupon bond paying interest semi-annually has a modified duration of 10 years and sells for $800, and is priced at a YTM of 8%. If the YTM increases to 9%, the predicted change in price, using the duration concept, decreases by:"

CONVEXITY

Q4) Q97 1994, "A 6% coupon bond with semi-annual coupons has a convexity (in years) of 120...and is priced at a YTM of 8%. If YTM increases to 9.5%, the predicted contribution to the percentage change in price, due to convexity, would be:"

A1) Macaulay duration / (1+ (y/2)) = 8.85/1.045 = 8.47

In words, **modified duration gives the percentage change in the bond price for a given change in yield** (not percentage change in yield).

A2) $\%\Delta P$ = modified duration x dy = 8.05 x .25 = 2.01%

NOTE: There are a number of possible complications which can be introduced. Two of these complications are: coupon payments (and compounding) occur more than once a year and dollar price change instead of % price change. Both these changes occurred in Q96 1994.

A3) Modified duration for semi-annual coupon bonds:

$$D^* = \textit{Modified duration} = (1 + \frac{y}{2})^{-1} \ \textit{Macaulay duration} \quad (\textit{in years})$$

$$dP = (D*)\ (dy)\ (P) \quad \rightarrow \quad PVBP = \frac{D^* \ x \ P}{10000}$$

where PVBP is the price value of a basis point. It follows: (10 x 800 x 100 basis points) / 10000 = $80. This formula for calculating price volatility is referred to a **dollar duration**, p.74 Fabozzi.

Why divide by 10000? This converts one bp = .0001 to 1 which is the unit of measurement for bp in the calculation.

A4) Evaluate the second term in the Taylor series:

1/2 (CON) $(\Delta y)^2$ = .5(120)(.015)(.015) = .0135 or 1.35%

Note: For non-option bonds convexity is positive. (Q30 1990: "Positive convexity implies that price increases at a faster rate as yields drop, than price decreases as yields rise.")