# TURTLES, TAILS, AND STEREOS: ARBITRAGE AND THE DESIGN OF FUTURES SPREAD TRADING STRATEGIES

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*Despite their practical importance, futures spread trading strategies have received only incomplete attention in the academic literature. This article provides an overview of a number of sophisticated spread techniques derived from cash-and-carry arbitrage restrictions. Trades examined include tandems, turtles, and stereos.*

*A general framework is provided for analyzing the profitability of trades. Detailed attention is given to deriving the profit function for two specific examples: the currency tandem and the golden turtle.*

Spread trading of futures contracts is an essential futures market activity, providing an important component of market liquidity, particularly in the deferred contract months. Market participants have used simple types of spread trades since the earliest trading in futures contracts. Modern futures traders employ a wide range of speculative spreading strategies, including some complicated and exotic trades that are derived from arbitrage fundamentals.

Despite the practical relevance of spread trading, treatment of this subject in the academic literature is incomplete and, typically, imprecise. Numerous interesting and widely used trades, such as the crack spread and stereo trades, have received little or no direct attention.

A primary objective of this article is to examine a class of spread trading strategies derived from the cash-and-carry arbitrage fundamentals underlying futures contracts for different commodities. Three important types of trades are considered: turtles, tandems, and stereos.

Developing the analysis requires introducing and exploiting a number of theoretical concepts: the arbitrage profit function; the spread tail; and the spread hedge ratio. The trades examined include intra- and intercommodity spreads in metals and currency markets. The profit functions for a number of relevant trades are derived and used to illustrate the design of futures spread trading strategies.

## I. THE BUILDING BLOCKS

The language of futures trading may be colorful, but it is not always revealing. The same concept may be referred to using different terminology, while the same terminology may refer to different concepts. To avoid semantic confusion, some attention is given to defining and explaining important basic concepts. Exhibit 1 summarizes the relevant trades.

Spread trades are of two general types. Intracommodity spreads, also referred to as *calendar spreads* or interdelivery spreads, involve taking a short position for one delivery date simultaneously with a long position for another delivery date.[1] While there are less stringent margin requirements and lower transaction costs associated with taking an equal number of short and long contracts, there are often analytical and practical advantages to having an unbalanced spread position.

The other general type of spread trade is the intercommodity spread, a category that includes a wide variety of possible trades including tandems, turtles, and stereos. In some cases, the profit function for an intercommodity spread can be developed from underlying production relationships. Examples are the soybean crush spread (see Johnson et al. [1991], Rechner and Poitras [1993]) and the crack spread (see Schap [1991, 1993]).[2] The profit functions for the types of intercommodity spreads considered in this article are derived using underlying cash-and-carry arbitrage conditions.

A basic building block for developing spread trading strategies is the profit function for the one-to-one intracommodity spread, a calendar spread involving equal position sizes on the two legs of the spread. With no loss of generality, assume that this trade is initiated at t = 0 and closed out at t = 1, and that the trader goes short the nearby (N) contract and long the deferred (T) contract for the same commodity.

For example, the trader could be short Dec 97 gold and long Dec 98 gold, both contracts being in the same commodity. Taking F(t, N) and F(t, T) to be the nearby and deferred futures prices observed at time t, the trading profile looks like:

| Date | Nearby Position | Deferred Position |
|---|---|---|
| t = 0 | Short 1 contract at F(0, N) | Long 1 contract at F(0, T) |
| t = 1 | Close out position by buying 1 contract at F(1, N) | Close out position by selling 1 contract at F(1, T) |

## EXHIBIT 1
### SPREAD TRADES

Spread trades are sometimes referred to as *straddle* trades, but this terminology is used also to describe a specific option trading strategy and can create semantic confusion. Schwager [1984, Part 5] provides a useful and practical introduction to spread trading.

A *calendar spread*, also referred to as an interdelivery spread, is a trade composed of a short and a long position in the same commodity involving different delivery dates. The number of contracts used for the short and long positions can be equal (a one-to-one spread) or unequal.

A *tailed spread* is a calendar spread using an unequal number of contracts for the short and long positions. The number of short and long contracts is chosen to achieve a specific type of trade payoff. It is possible to set the tail to have a spread trade payoff that depends on changes in the implied repo rate, an important feature for stereo and turtle trades.

A *tandem spread* is a trade combining calendar spreads in two different commodities. The component spreads can be either one-to-one or tailed. The trade involves a hedge ratio to be calculated, usually to equalize the starting values of the positions in the two commodities. There are a wide range of possible rationales for doing tandem trades. See, for example, Kilcollin [1982] and Poitras [1998].

A *stereo* trade is a specific type of tandem trade designed to speculate on changes in the implied repo rates for different commodities. Hence, a stereo is a specific type of tailed tandem where the tails are set so that the calendar spread payoffs depend only on changes in implied repo rates. The trade is usually triggered when the implied repo rates for different commodities are observed to deviate from typical historical relationships. See Yano [1989].

A *turtle* trade combines a tailed spread in one commodity with a short or long position in an interest rate future. The tail is set so that the calendar spread payoff depends on the change in the implied repo rate. The rationale for a turtle varies, depending on the specific commodity. For T-bonds and T-notes, the turtle is triggered when the implied repo rate is observed to deviate significantly from the cash repo rate. See, e.g., Jones [1981] and Rentzler [1986].

While this trading profile is specified using one contract for each leg of the spread, in practice spread trades almost always involve considerably larger position sizes.

The profit function $\Pi$ can now be specified by observing that the profit for each leg of the spread is equal to the contract selling (short) price minus the purchase (long) price:[3]

$$\Pi = [F(0, N) - F(1, N)] + [F(1, T) - F(0, T)]$$

$$= [F(1, T) - F(1, N)] - [F(0, T) - F(0, N)] \qquad (1)$$

In words, when the initial $t = 0$ term structure of futures prices is upward sloping (a contango market), as in the case of the precious metals, the one-to-one intracommodity spread that is short the nearby and long the deferred will be profitable if the difference between the deferred and nearby prices widens. The opposite would be true for the alternative spread; a long-the-nearby and short-the-deferred spread would be profitable when the deferred-nearby price difference narrows.

It is also possible for the $t = 0$ difference between the deferred and the nearby prices to be negative (a market in backwardation). This is the case with T-bond futures when the cash market yield curve is upward sloping, and it has also been the case with copper in recent years. In that case, profitability for the short nearby-long deferred spread requires that the difference between the prices become less negative. Similarly, a long-the-nearby and short-the-deferred spread would be profitable when the deferred-nearby price difference becomes more negative.

The profit function (1) is generic; it applies to any calendar spread. To develop the trades of interest in this article, we further develop the equation by introducing the general cash-and-carry arbitrage condition for futures contracts. See, for example, Duboffsky [1992], Poitras [1991], Siegel and Siegel [1990], Allen and Thurston [1988], Hegde and Branch [1985], and Kawaller and Koch [1984]:

$$F(t, T) \equiv F(t, N)[1 + C(t, N, T)] \qquad (2)$$

In (2), the implied carry, $C(t, N, T)$, is defined as the net cost of carrying the commodity from date N to date T implied in the futures prices $F(t, N)$ and $F(t, T)$ observed at time t.[4] Making appropriate substitutions of the arbitrage condition (2) into the profit function (1), and dropping the N, T notation for C gives the result:

$$\Pi = F(1, N)C(1) - F(0, N)C(0)$$

Defining $\Delta C = C(1) - C(0)$ and $\Delta F(N) = F(1, N) - F(0, N)$, basic algebra provides the fundamental result for the one-to-one spread profit function:

$$\Pi = C(0)\Delta F(N) + F(1, N)\Delta C \qquad (3)$$

This demonstrates that $\Pi$ for the one-to-one spread depends on the changes in two variables, $\Delta F$ and $\Delta C$.

In general, trading a one-to-one calendar spread requires predicting the behavior of two random variables in order to ascertain profitability. As an example of how futures price level changes can affect spread profitability, consider the case of gold for the period November 9, 1979, through February 15, 1980. Over this period, interest rates were relatively unchanged, with the benchmark three-month T-bill rising only 11 basis points from 12.25 to 12.36, while the Handy and Harmon spot gold price rose from \$389.75 to \$667.00.

According to the June 80-June 81 COMEX gold calendar spread for this period, the June 80 contract rose from \$420.80 to \$703.50, while the June 81 contract rose from \$471.20 to \$843.00. This resulted in a change in the futures spread from \$50.40 to \$139.50. Remembering that the C for gold is determined primarily by interest rates, the impact of interest rate changes on the gold spread is reflected over the period March 3, 1980–August 25, 1980, when the Handy and Harmon spot price was relatively unchanged, going only from \$633.75 to \$634.75, while interest rates, as reflected in the three month T-bill rate, fell from 13.38% to 9.41%.

An examination of the Oct 80-Oct 81 COMEX gold calendar spread over this period shows that the Oct 80 contract fell from \$709.50 to \$629.70, while Oct 81 fell from \$849.50 to \$719.40. This reflects a decline in the gold futures price spread from \$140.00 to \$89.70.

In many instances, and especially for the types of spread trades considered here, it is necessary to use an intracommodity spread profit function that depends on only the random change in the carrying cost, $\Delta C$. Significantly, the technique of *tailing* the spread can achieve this objective by altering the relative sizes of the nearby and deferred positions in the spread so that the $\Delta F$ term disappears from the profit function (see Jones [1981]). Unlike a one-to-one spread, the tailed spread involves a different number of nearby and deferred contracts.

In this fashion, tailed intracommodity spreads can be used to speculate on changes in the implied net cost of carry, $\Delta C$, without having to worry about trade profitability also being affected by changes in price levels. Tailed spreads thus become building blocks in more complicated trades that combine tailed spreads with other positions, such as tailed spreads in other commodities or naked money market futures positions, to create trading strategies such as the stereo and the turtle.

To understand the profit function for a tailed spread, consider the trading profile for an intracommodity spread with potentially unequal position sizes. Letting the contract amounts be $Q_N$ and $Q_T$, the short-the-nearby, long-the-deferred trade, can be depicted:

| Date | Nearby Position | Deferred Position |
|---|---|---|
| t = 0 | Short $Q_N$ units at F(0, N) | Long $Q_T$ units at F(0, T) |
| t = 1 | Close out position by buying $Q_N$ units at F(1, N) | Close out position by selling $Q_T$ at F(1, T) |

In this case, the profit function can be specified:

$$\Pi(1, T) = [F(0, N) - F(1, N)]Q_N + [F(1, T) - F(0, T)]Q_T \qquad (4)$$

The tail for an intracommodity spread can be set by holding either spread leg constant and varying the other leg. To see this, set $Q_T = 1$. It can now be verified that $Q_N = F(0, T)/F(0, N)$ will give a trade profit function that depends only on $\Delta C$. Observing that $F(0, T)/F(0, N) = [1 + C(0)]$ and substituting this result and $Q_T = 1$ into (4) gives:

$$\Pi(1) = F(1, N)\Delta C \qquad (5)$$

By correctly choosing the number of contracts in each leg of the spread, the effect of $\Delta F$ on the spread profit function is eliminated.[5]

This tailing method requires the legs of the spread to have equal dollar value in the underlying commodity, instead of an equal number of contracts as in the one-to-one spread. A similar technique is used in futures hedging, where the method is referred to as *dollar equivalency* hedging.

To see the practical calculation of a tail, consider the gold prices that were available in February 1989 for June delivery contracts: June 90, \$404.80, and June 89, \$379.00. For these contracts, the one-year spread gives $1 + C(0) = F(t, T)/F(t, N) = 1.068$. Using the tailing method involves taking 1.068 June 89 nearby contracts for every one June 90 deferred contract.

Because futures contracts are traded only in whole numbers, however, it is necessary to gross the number of contracts up until an acceptable ratio is found. The degree of precision achievable is usually restricted by limitation on the size of position a trader can initiate. In this case, 14(1.068) = 14.952. Hence, a ratio of fifteen nearby (June 89) for every fourteen deferred (June 90) contracts would appear to be acceptable, although as the size of the spread trade positions grows, the more accurate the tail can be.

This discussion raises an important practical question. Is it always necessary to tail a spread in order to have the profit function depend solely on $\Delta C$? The answer to this question is yes. Without a tail, spread profitability is *theoretically* dependent on price level adjustments.

In certain cases, however, the tail magnitude is small enough that it can be ignored, for all practical purposes. The need to tail a spread depends in practice on both the shape of the term structure of futures prices and the length of time between N and T. When prices across the delivery months used in the spread are at relatively the same level, so that the term structure of futures prices is flat and $F(t, T)/F(t, N)$ is approximately one, it is not necessary to implement a tail because dollar equivalency is already implicit in the futures prices. A flat term structure of futures prices is often observed for a number of commodities such as currencies.

For example, using the price quotes for October 31, 1994, and taking, say, a six-month (Jun 95/Dec 94) spread in German marks, gives a tail of (0.6676/0.6648) = 1.00421. Similarly, for the Canadian dollar the Jun 95/Dec 94 contracts give a tail of (0.7385/0.7392) = 0.999053. These numbers indicate that a tail is not required unless the trade involves hundreds of contracts.

It is not always the case that currency spreads can be untailed in practice. For example, the Sep 95/Dec 94 yen contracts on October 31, 1994, give a tail of (1.0637/1.0358) = 1.02694, which indicates a tailed spread of thirty-seven deferred and thirty-eight nearby contracts.

An interesting application of a tailed spread occurs in the case of T-bonds. In this case, the tailed spread can be used for speculating on changes in the shape of the yield curve:

| Date | Nearby (N) Position | Deferred (T) Position |
|---|---|---|
| t = 0 | Short [F(0, T)/F(0, N)]Q T-bonds at F(0 ,N) | Long Q T-bonds at F(0, T) |
| t = 1 | Buy [F(0, T)/F(0, N)]Q at F(1, N) | Sell Q at F(1, T) |

For example, because of the negatively sloped futures term structure, the August 8, 1994, price quotes give a Sept 95/Sept 94 tail of (100-17/32)/(103-9/32) = (0.973374). Making the tail an integer requires thirty-seven spreads with an additional Sep 95 contract, for a position that is long thirty-eight Sep 95 and short thirty-seven Sep 94. Applying Equation (5) to the specific case of a T-bond, the profit function for the short-the-nearby, long-the-deferred, tailed T-bond spread takes the form:

$$\Pi(1) = F(1, N)\Delta C$$

$$= F(1, N)[\Delta\rho(N, T) - \Delta R(N, T)] \qquad (6)$$

where $\rho$ is the implied repo rate (IRR), that is, the repurchase agreement financing rate implied in T-bond futures prices, and R is the percentage rate of coupon flow on the cash T-bond or, in other words, the bond coupon received (including amounts accrued but not paid out) during the period between the two delivery dates, N and T, as a percentage of F(0, N). Recognizing that the tailed T-bond spread IRR can be a proxy for a short-term interest rate while R can be taken as an approximation of the cash T-bond rate for > 15-year maturities, which is a long-term rate, the connection between the payoff on a tailed T-bond spread and shifts in the term structure of interest rates should be apparent.[6]

A final substantive comment about tailing needs to be made. More precisely, dollar equivalency is not the only possible tailing method. As will be seen when we discuss the specifics of intercommodity trades such as the turtle, the process of determining the tail also allows profit functions that are dependent on changes in specific components of C, and not just C itself. The tailed T-bond spread provides an important example.

If $[1 + \rho(0, N, T)]$ is used to determine the size of the tail instead of using $[1 + C(0, N, T)]$, as in the dollar equivalency case, the profit function of the tailed spread will be given by $F(1, N)\Delta\rho$. Instead of depending on the changes in difference between $\rho$ and R, spread profitability would depend on the change in only one interest rate, the implied repo rate. This result plays an essential role in specifying the turtle trade involving tailed T-bond spreads and T-bills. Similarly, with appropriate selection of the tailing strategy, it is possible to isolate other components of C, such as the convenience yield in a copper spread.

## II. TANDEMS, TURTLES, AND STEREOS

### Tandems and Butterflies

A *tandem* is a trade combining spreads in two different commodities. The component spreads can be tailed or untailed, depending on the rationale for the trade. Unlike the stereo and turtle trades, which are designed to speculate on differences in specific interest rates, there are a wide range of possible rationales for tandem trades. The trading rationale will depend on the commodities and contracts used to construct the component spreads.

In general, much like a calendar spread, the tandem involves taking a short position in one spread and a long position in the other spread. To derive the untailed tandem profit function, consider the profit function for the untailed short-the-nearby, long-the-deferred, spread in the first commodity:

$$\Pi_1 = Q_1\{[F(1, T) - F(1, N)] - [F(0, T) - F(0, N)]\}$$

And, for the second commodity, where the untailed spread is long the nearby, short the deferred:

$$\Pi_2 = Q_2\{[G(0, T) - G(0, N)] - [G(1, T) - G(1, N)]\}$$

Futures prices for the second commodity are denoted by G. Combining these two component spreads gives the general profit function for the tandem trade:

$$\Pi_{tan} = \{Q_1[F(1, T) - F(1, N)] - Q_2[G(1, T) - G(1, N)]\} - \{Q_1[F(0, T) - F(0, N)] - Q_2[G(0, T) - G(0, N)]\} \quad (7)$$

While similar in intuition to (1), the tandem profit function is considerably more complicated.

Because a tandem involves spreads in different commodities, it is necessary to determine the *spread hedge ratio*, the number of spreads in commodity 2 for each spread in commodity 1. The requirements for determining a dollar-equivalence hedge ratio can be achieved by dividing (7) through by $Q_1F(1, N)$. Doing this division and substituting in (3) where appropriate gives the cash-and-carry arbitrage form of the tandem profit function:

$$\frac{\Pi_{tan}}{Q_1F(1, N)} = \left[\Delta C_F + C_F(0)\frac{\Delta F(N)}{F(1, N)}\right] - \frac{Q_2G(1, N)}{Q_2F(1, N)}\left[\Delta C_G + C_G(0)\frac{\Delta G(N)}{G(1, N)}\right] \quad (8)$$

In this form, the spread hedge ratio is $[Q_2G(1, N)]/[Q_1F(1, N)]$, the relative values of the commodity contracts at $t = 1$. Choosing a dollar-equivalence hedge ratio involves setting $[Q_2G(1, N)] = [Q_1F(1, N)]$.

While this form of the tandem profit function gives the most revealing theoretical result, in practice F(1, N) and G(1, N) are not available when the spread is established, requiring G(0, N) and F(0, N) to be used instead.[7] It can be verified that using the $t = 0$ value to specify (8) leads to a more complicated formula, which is not substantively different from the result given.

To see an example of a tandem trade, consider establishing an untailed Dec 94/Dec 95 gold/copper tandem on October 31, 1994. For Dec 94 delivery, the 100 oz. gold contract is selling for \$384.90/oz., and the 25,000 lb. copper contract for \$1.2260/lb., providing for a dollar-equivalence spread hedge ratio of 0.79, or approximately four gold spreads for each five copper spreads. Hence, the trade would involve entering four gold spreads, which are short the Dec 94 and long the Dec 95 contracts, and simultaneously entering five copper spreads, which are long the Dec 94 and short the Dec 95.

Assuming that the possible impact of changes in gold and copper price levels can be ignored, the profitability of this trade would depend on differences in the C for gold and copper. Assuming also that the interest components of C for gold and copper cancel out, profitability would depend primarily on changes in the convenience yield for copper.

An important assumption in this gold/copper tandem example is that the impact of price level changes could be ignored. The conclusion that trade profitability depends on changes in the convenience yield for copper is facilitated by the assumption that C for gold depends primarily on interest charges, which could be canceled against the interest component in C for copper, leaving the convenience yield component.

For tandem trades involving commodities in the same complex, e.g., heating oil and unleaded gasoline, or soybean meal and soybean oil, ignoring the price level impact is a practical assumption.

Unfortunately, the presence of two commodities in the tandem trade means that interpretation of the profit function can be somewhat complicated. An example is the TED (Treasury bill/Eurodollar) tandem (Landau and Wolkowitz [1987], Kawaller and Koch [1992], Poitras [1998]).

To simplify the cash-and-carry profit function for the tandem, (8), it is convenient to require both sides of the trade to be tailed spreads. As in (4), this permits the price level impact to be eliminated for the spreads composing the tandem. In this case, (8) becomes:

$$\frac{\Pi_{tt}}{Q_1 F(1, N)} = \Delta C_F - \frac{Q_2 G(1, N)}{Q_1 F(1, N)} \Delta C_G \qquad (9)$$

Choosing a dollar-equivalence hedge ratio such that $[Q_2 G(1, N)] = [Q_1 F(1, N)]$ permits the profit function to depend solely on the difference in the C changes for the two commodities involved.

A natural extension of the intercommodity tandem trade occurs when spreads involved in the trade are calendar spreads in the same commodity. In this case, the trade is constructed by using nearby and deferred spreads featuring different delivery dates. One popular variation of this trade is the *butterfly*, where the spreads involve only three distinct delivery dates: short (long) one nearby contract; long (short) two contracts of an intermediate delivery date contract; and short (long) one distant delivery contract (see Schwager [1984]). The trade can be interpreted as a "spread of spreads," a combination of a short (long) nearby spread and a long (short) deferred spread.

For example, a Dec 96/Jun 97 spread that is short the nearby Dec 96 and long the deferred Jun 97 contract could be combined with a Jun 97/Dec 97 spread that is long the nearby Jun 97 and short the deferred Dec 97. The combined position would be short one Dec 96, long two Jun 97, and short one Dec 97. When there are two intermediate delivery dates, e.g., short one Dec 96, long one Jun 97, short one Dec 97, long one Jun 98, the trade is referred to as a *condor* (see Yano [1989]).

In general, the trading profile supporting the profit function for the butterfly can be described as:

| Date | Nearby (N) Position | Intermediate (T) | Distant Position ($T^*$) |
|---|---|---|---|
| t = 0 | Short 1 at F(0, N) | Long 1 at F(0, T), Long 1 at F(0, T) | Short 1 at $F(0, T^*)$ |
| t = 1 | Buy 1 at F(1, N) | Sell 1 at F(1, T), Sell 1 at F(1, T) | Buy 1 at $F(1, T^*)$ |

The profit function for the short-long-short butterfly is:

$$\begin{aligned}\pi_b/Q = {} & [\{F(1, T) - F(1, N)\} - \{F(0, T) - F(0, N)\}] + \\ & \{[F(1, T) - F(0, T^*)] - [F(0, T) - F(0, T^*)]\} \\ = {} & [F(0, N) - F(1, N)] + 2[F(1, T) - F(0, T)] + \\ & [F(0, T^*) - F(1, T^*)]\end{aligned}$$

For this trade to be profitable in a contango market, the nearby futures basis is expected to widen more than the deferred futures basis. Similarly, a long-short-long butterfly would be profitable in a contango market when the nearby futures basis widens less than the deferred futures basis.

Analysis of the butterfly proceeds expeditiously by assuming that each of the spreads in the trade has been "tailed" using the dollar-equivalency hedge ratios, $F(t, T)/F(t, N)$ and $F(t, T^*)/F(t, T)$, respectively. In this case the profit function can be expressed as:[8]

$$\pi_b/[QF(1, N)] =$$

$$\Delta C(N, T) - [F(1, T)/F(1, N)]\Delta C(T, T^*)$$

By further adjusting the number of deferred tailed spreads by the factor $[F(1, T)/F(1, N)]$, the profit function for a fully tailed butterfly can be determined as: $\Delta C(N, T) - \Delta C(T, T^*)$.

In general, profitability of the butterfly depends on the behavior of the *term structure of futures prices*. For non-exchange members subject to higher transaction costs, this type of trade would usually not provide interesting opportunities because the associated price movements are small compared to the costs of trading as a non-exchange member.[9] On the other hand, floor traders can use untailed butterflies, for example, to flatten out the futures term structure. This could occur if the price of an intermediate contract becomes mispriced due, say, to a large position that is being placed in a particular delivery month for cash market considerations.

### Stereos

A *stereo* trade (Yano [1989]) is a special case of a tandem where the profit function depends on the difference in changes for the cost-of-carry interest rates implied in arbitrages for selected futures contracts. A simple example of a stereo trade has already been encountered: a (dollar-equivalency) tailed tandem involving gold and silver contracts. For these commodities, there is no significant cash payout from holding the cash commodity, and non-interest carrying charges involved in the cash-and-carry arbitrage are negligible compared to interest charges, so C can be taken to depend only on interest carrying charges. As a result, the profit function for these tailed tandem trades depends on the difference in the changes for the carrying charge interest rates implied in gold and silver futures prices, and the dollar-equivalency tailed tandem is also a stereo trade.

Because C for other commodities can depend on more than just interest carrying charges, stereo trades for other commodities are more difficult to specify. For example, C for debt futures contracts has both an interest carrying cost, the implied repo rate, and an interest carrying return. Deriving the stereo trades for these commodities requires the dollar-equivalency tailing procedure to be adjusted so that the profit function for the spread in each commodity depends only on the implied repo rate changes.

When $C = \rho - R$, the general profit function for a stereo can be expressed:

$$\Pi_s^* \equiv \frac{\Pi_s}{F(1,N)Q_1} = \Delta\rho_1 - \frac{G(1,N)Q_2}{F(1,N)Q_1}\Delta\rho_2$$

where $\rho_j$ is the interest carrying cost implied by the cash-and-carry arbitrage for commodity j, i.e., the implied repo rate for a debt future, and R is the associated carry return, i.e., the percentage rate of coupon flow on the underlying bond for a debt future. To derive the appropriate position sizes for a stereo trade, the tailing method for the component spreads must be adjusted to convert the profit function to depend only on the change in interest carrying charges. This requires specification of the tail for each of the component intracommodity spreads so that the resulting profit function is of the form: $\Pi_{irr} = F(1, N)\Delta\rho$.

To identify the appropriate tail for this situation, observe that, when $C = \rho - R$, then $\rho = C + R$. More precisely:

$$\rho(0,N,T) = \frac{F(0,T) - F(0,N)}{F(0,N)} + \frac{A}{F(0,N)}\frac{T-N}{365} = C(0,T,N) + R(0) \qquad (10)$$

where A is the annual stated coupon on the underlying theoretical bond or note. Using this result, it is possible to specify a spread with a profit function depending on $\Delta\rho$ instead of $\Delta C$.

Taking $\Pi_{ts} = F(1, N)\Delta C = F(1, N)(\Delta\rho - \Delta R)$, to derive the appropriate tail observe that:

$$F(1,N)\Delta R = F(1,N)\left[\frac{A^*}{F(1,N)} - \frac{A^*}{F(0,N)}\right] = -\frac{A^*}{F(0,N)}\Delta F$$

where $A^* = A(T - N)/365$. Combining this with (4), where a dollar-equivalency tail has been used to specify the position sizes:

$$\Pi_{ts} = [1 + C(0)][F(0, N) - F(1, N)] + [F(1, T) - F(0, T)] = [1 + C(0)][-\Delta F(N)] + [F(1, T) - F(0, T)]$$

Substituting from the definition for $\Pi_{irr}$ gives:

$$\Pi_{irr} = \Pi_{ts} + F(1, N)\Delta R = \Pi_{ts} - R(0)\Delta F(N)$$

$$= [1 + C(0) + R(0)][F(0, N) - F(1, N)] + [F(1, T) - F(0, T)]$$

$$= [1 + \rho(0)][F(0, N) - F(1, N)] + [F(1, T) - F(0, T)]$$

Hence, for spreads that involve profit functions depending on changes in the implied repo rate, the appropriate tail is $[1 + \rho(0)]$ and not $[1 + C(0)]$ where IRR is calculated using (10). For example, in Section I the $[1 + C(0)]$ dollar-equivalency tail for the Sep 95/Sep 94 T-bond spread was $[F(0, T)/F(0, N)] = 0.973374$. Recognizing that the annualization factor can be ignored for a one-year spread, the $[1 + \rho(0)]$ tail would be $\{[F(0, T)/F(0, N)] + [A/F(0, N)]\} = 0.973374 + [8/100 - 17/32] = 1.053$.

The $[1 + \rho(0)]$ tailed tandem stereo trades are specific instances of "differential repo arbitrage" trades, a class of trades that also includes the turtle trades discussed below (Yano [1989]). The profit functions for these intercommodity trades depend either on the difference in the implied repo rates for two sets of financial futures contracts or on the difference in an implied repo rate and a surrogate for the cash market repo rate.

Trading opportunities are identified when the IRR for a given futures contract deviates significantly, either from the IRR for other futures contracts, which generates a stereo trade, or from the cash market, which generates a turtle trade. When an observed deviation of rates is "too large" or "too small" depends on various factors, conditioned on the history of previously observed deviations.

Specific examples of these trades are the stereo NOB, which trades the difference in the IRR between T-note and T-bond futures, and the stereo GUN, which trades the difference in IRRs between GNMA and T-note contracts. While the stereo trade is typically associated with debt futures, it is possible to design such trades for a wide range of commodities. Yano [1989] provides an elegant and slightly more precise method of arriving at the particular position sizes.

To understand the design of a stereo trade, consider a tailed tandem combining gold and silver contracts. Because C for the precious metals is, to a first approximation, determined by interest charges, a dollar-equivalency $[1 + C(0)]$ tail produces the same result as a $[1 + \rho(0)]$ tail, permitting this type of tailed tandem to also be considered as a stereo.

Again using the October 31, 1994, prices, the $F(0, T)/F(0, N)$ = Apr 96/Apr 95 gold spread has \$416.20/\$392.00 = 1.0617, implying a tailed spread of sixteen to seventeen. The Mar 96/Mar 95 silver spread has $F(0, T)/F(0, N)$ = \$5.737/\$5.346 = 1.0731, implying a tailed spread of approximately fourteen to fifteen. Once the tails have been determined, the dollar-equivalency spread hedge ratios are calculated so that $[G(0, N)Q_2] = [F(0, N)Q_1]$. Because $[5{,}000G(0, N)/100F(0, N)] = 0.682$, it follows that seventy-five nearby Mar 95 silver contracts have approximately the same dollar value as fifty-one nearby Apr 95 gold contracts.

Assuming that it is expected that $\Delta C_G - \Delta C_S > 0$, the appropriate contract positions for the tailed tandem would be: short fifty-one Apr 95 and long forty-eight Apr 96 gold contracts combined with long seventy-five Mar 95 and short seventy Mar 96 silver contracts.

**Turtles**

While similar in concept to stereo trades and some tailed tandem trades, turtle trades differ in construction. Instead of speculating on changes in the difference between IRRs of two different commodities, the turtle trade speculates on changes in the difference between an IRR and a surrogate for a cash market rate. To do this, the turtle trade substitutes a naked position, typically in a money market future, for one of the $[1 + \rho(0)]$ tailed spreads in the stereo.

The simplest version of this trade is a metal turtle, which is discussed in more detail below. This trade involves, for example, combining a $[1 + C(0)] = [1 + \rho(0)]$ tailed gold spread with a Eurodollar futures position (see Poitras [1987]). The objective is to speculate on changes in the difference between the implied interest rate in gold futures and the Eurodollar (Euro\$) rate.

One reason this trade is of interest is because there is a *one-sided* arbitrage relationship between gold futures prices and Eurodollar interest rates that can be used to fine-tune the spread trading decision.[10] Because absence-of-arbitrage associated with the long-the-cash arbitrage trade prevents the gold C from being greater than the relevant Eurodollar rate, comparison of the observed difference between the two rates with the past history of the difference can be used to identify trading opportunities.

In turtle trades involving debt futures, such as the turtle between T-bond spreads and T-bills, it is the IRR and not the C that is of interest (Rentzler [1986], Easterwood and Senchack [1986]). Much as in the stereo trades that speculate on changes in $(\Delta\rho_1 - \Delta\rho_2)$, the turtle is concerned with speculating on $\Delta\rho - \Delta i$, where i is the interest rate on the appropriate open (naked) interest rate futures contract

Compared to precious metal turtles, factors determining profitability of turtle trades involving debt futures are somewhat more complicated. As with the (1 + C) tailed T-bond spread, the turtle can be used to speculate on changes in yield curve shape, albeit only at the short end. More frequently, turtle trades involving debt futures are used to capture deviations of the implied repo rate from the actual or cash repo rate. These deviations emerge because the repurchase agreement used to finance cash transactions is primarily an overnight rate, with some term repo available in short maturities but effectively no terms to maturity that correspond to the deliveries of the relevant debt futures contract.

Because the cash market does not provide a direct financing vehicle for arbitrage involving, say, T-bonds, it is possible for the IRR associated with T-bonds to deviate substantially from the IRR observed in the cash market (Allen and Thurston [1988]). Turtles take the form of a cash-and-carry quasi-arbitrage trade designed to exploit the observed deviation. This intuition for the turtle trade relies on the T-bill position being a surrogate for the cash repo rate.

The motivation for effecting turtle trades using debt futures can be illustrated by considering the profit function for the turtle, which has been simplified by assuming the spread hedge ratio has been set appropriately:

$$\Pi_{turt} = [\Delta\rho(N, T)] - [\Delta b(N, T)]$$

where ρ is the implied repo rate for the [1 + ρ(0)] T-bond spread, and b is the interest rate reflected in the relevant T-bill futures. When the hedge ratio is set appropriately, this leaves the payoff on the turtle dependent on the difference in the T-bond implied repo rate and T-bill rate changes.[11] From this, the turtle trades can be generalized to trades involving (1 + IRR) tailed spreads and any other relevant money market futures contracts. Other possible configurations include (1 + IRR) tailed T-note spreads with Eurodollars.

Because the profit functions for the various possible turtle trades involve differencing two interest rates that are, invariably, determined by differing market forces, it is necessary to construct a behavioral foundation for explaining each specific trade's profitability. Yano [1989] recognizes this point:

> The turtle trade is not riskless arbitrage. There seems to be a widespread fallacy that the [difference in the implied repo rates is] zero on average, but there is no necessary reason for this to be true.

Referring to turtles derived from financial futures:

> Different configurations will have their idiosyncrasies due to, but not limited to, heterogeneous expectations along the yield curve [1989, p. 446].

As a specific example of calculating the turtle trade, consider the [1 + ρ(0)] tailed Sep 95/Sep 94 T-bond spread discussed above, where the tail is calculated as 1.053. This translates into twenty Sep 94 for every nineteen Sep 95 contracts. To determine the number of T-bill contracts, it is expeditious to use the technique of equating the dollar value of basis points (bp).

While the value of 1 bp for T-bill futures is given at $25, determining the bp value for T-bond futures is more complicated because this value depends on the level and direction of interest rates. Suppose, for example, that rates are expected to change by 100 bp from 8% to 7%. While the T-bond at 8% has a value of $100,000, the price of the T-bond at 7% is calculated as $110,590, producing a result of $105.90 per bp. Similarly, for an increase from 8% to 9%, the T-bond futures price falls by \$9,130 for a value of \$91.30 per basis point.

Observing that the August 8, 1994, F(0, N) = 100-17/32, a spread hedge ratio of four to one can be chosen. Assuming it is expected that $\Delta\rho - \Delta b > 0$, then the turtle trade that could be established on August 8, 1994, is: long twenty Sep 94 T-bonds, short nineteen Sep 95 T-bonds, and short five Sep 94 T-bills.

## III. THE CURRENCY TANDEM[12]

Currency tandems are one of the most interesting of all spread trades (see Adler [1983]). To develop the profit function for this trade requires the relevant cash-and-carry arbitrage condition for currency futures, *covered interest parity* (CIP) (see Poitras [1988]):

$$F(0, T) = \frac{1 + i(0, T - N)}{1 + i^*(0, T - N)}$$

where i(0, T − N) and $i^*(0, T - N)$ are the time 0 domestic (U.S.) and foreign forward interest rates adjusted by (T − N)/365 to account for the trading horizon.[13]

The U.S. dollar is taken to be the domestic currency because almost all currency futures are quoted as the amount of U.S.\$ per one unit of foreign currency. In spread form this becomes:

$$F(0,T) - F(0,N) = \frac{i(0,N,T) - i^*(0,N,T)}{[1 + i^*(0,N,T)]}F(0,N)$$

$$\equiv \theta(0)F(0,N)$$

From (2) it follows that:

$$[F(1, T) - F(0, T)] - [F(1, N) - F(0, N)] \equiv$$

$$\Delta[F(T) - F(N)] = \theta(0)\Delta F + F(1, N)\Delta\theta \qquad (11)$$

By working directly with the CIP condition, this exact result can be used to derive a precise expression for the profit function.

Evaluating the $\Delta\theta$ term gives:

$$\Delta\theta = \frac{\Delta(i - i^*)}{(1 + i^*)} + (i - i^*)\Delta\frac{1}{(1 + i^*)}$$

$$= \frac{(\Delta i - \Delta i^*)}{(1 + i^*)} - (i - i^*)\frac{\Delta i^*}{(1 + i^*)^2}$$

Observing that $(i - i^*)\Delta i^*$ is a product of differences in interest rates, it follows that this term on the right-hand side is second order to a first approximation, and can be set equal to zero. Using this result, substituting into (11) and collecting terms gives:

$$\Pi_{cs} = \Delta[F(T) - F(N)]$$

$$= F(1, N)\left[\frac{\Delta i - \Delta i^*}{(1 + i^*)}\right] + \frac{i - i^*}{1 + i^*}[\Delta F(N)]$$

If the spread is tailed, or tailing is unnecessary for practical purposes, the $\Delta F$ term can be removed, which leaves only the first term on the right-hand side to determine the short-the-nearby, long-the-deferred, spread profit function:

$$\Pi_{cs} \cong \frac{F(1, N)}{1 + i^*}[\Delta i - \Delta i^*] \qquad (12)$$

The upshot is that the *profitability of an intracommodity currency spread depends on the relative change in the appropriate interest rates for the U.S. and the foreign country*. This result extends naturally to a tandem, which can be used to speculate on changes in interest rates that are not U.S. The tandem permits speculation on relative foreign interest rate changes even when there are no liquid foreign currency futures contracts directly quoted in terms of the two foreign currencies (see Poitras [1988]).

One practical problem about the intracommodity currency spread needs to be considered. Under what conditions is it possible to simplify the profit function by ignoring the tail on the spread?

This question can be resolved by observing that the $\Delta F$ term in (11) is associated with the tail, with $\theta(0)$ representing the appropriate size of the tail. It follows that if foreign and domestic interest rates are approximately equal $[\theta(0) \cong 0]$, it is not necessary to tail the spread. When there is a significant difference, however, a tail may be required.

To see this, assume that $F(0, N) = 1$, $i = 0.1$, and $i^* = 0.04$. If the exchange rate falls by 20%, then $\Delta F = 0.2$ and $[\theta(0)\Delta F]$ is around 0.012. If $[\Delta i - \Delta i^*]$ changes by 0.02, then $F(1, N)\Delta\theta$ is around 0.016.

While there are definitely situations in which tailing a currency spread is advisable, it is also possible to construct examples for which a tail is not required for the currency spread. In general, because it is a product of two differences, the difference in foreign and domestic interest rate levels and the change in exchange rates, the $\theta(0)\Delta F$ term is of second order, unless the difference between foreign and domestic interest rates is large.

We assume for simplicity that it is not necessary to tail the two currency spreads constituting the tandem, and proceed to calculation of a spread hedge ratio for the tandem. Reexpressing the currency tandem profit function using (9):

$$\frac{1 + i_F}{Q_F F(1, N)} \Pi_{ct} = \Pi_{ct}^*$$

$$= (\Delta i - \Delta i_F) -$$

$$\frac{Q_G G(1, N)(1 + i_F)}{Q_F F(1, N)(1 + i_G)} [\Delta i - \Delta i_G]$$

If the hedge ratio is chosen to be dollar-equivalent $[Q_G G(1, N)(1 + i_F) = Q_F F(1, N)(1 + i_G)]$, then the U.S. interest rate terms $\Delta i$ will cancel, and the profit function will depend on the difference in the two foreign interest rates:

$$\pi_{ct}^* \cong \Delta i_F - \Delta i_G$$

Hence, when the hedge ratio is set appropriately, and tailing is effected when necessary, the profitability of a currency tandem depends on the difference in the two foreign interest rate changes, with the U.S. interest rate impact canceling out. By implication, the currency tandem can also be interpreted as a stereo trade.

In practice, calculation of the hedge ratio involves solving the approximation $[Q_2 G(0, N)] = [Q_1 F(0, N)]$. This requires equalizing dollar value on both legs of the tandem at $t = 0$. To see how this is accomplished for the currency tandem, consider a trade whose objective is to speculate on relative changes in Canadian and British interest rates using currency futures denominated in terms of the U.S. dollar. In this case, the U.S. dollar value of the Canadian dollar contract is: (U.S.\$/C\$) \$100,000 = $F(0, N)Q_1$. And the U.S. dollar value of the pound contract is: (U.S.\$/£)(£ 62,500) = $G(0, N)Q_2$. Hence: $(G(0, N)Q_2/F(0, N)Q_1)$ = [(U.S.\$/£) (£ 62,500)]/ (U.S.\$/C\$) \$100,000 = (C\$/£) (62,500/100,000).

The spread hedge ratio is the product of the current Canadian to British exchange rate times 0.625. For example, if it is expected that $\Delta i_C - \Delta i_£ > 0$, then using the October 31, 1994, price quotes for the Mar 95/Dec 94 contracts, the spread hedge ratio is [0.625 (1.6346/0.7392)] = 1.3863, which produces eighteen British pound spreads to twenty-five Canadian dollar spreads. Recognizing that $F(0, T)/F(0, N)$ is 0.999 for both the pound and the Canadian dollar, there is no need to tail, and the appropriate trade is short twenty-five Dec 94 and long twenty-five Mar 95 C\$ contracts, and long eighteen Dec 94 and short eighteen Mar 95 £ contracts.

## IV. GOLDEN TURTLES

The profit function for a golden turtle trade is more straightforward than the complicated profit function for the currency tandem. Ignoring the spread hedge ratio, $\Pi_{gt} = \Delta C - \Delta i = \Delta\rho - \Delta i$, where i is the interest rate on the Eurodollar futures contract, which is selected as the appropriate interest rate for the golden turtle trade.

One practical difficulty with a turtle occurs with specification of the time at which the trade is initiated. In the case of the precious metals, gold and silver, turtle profitability depends on the relationship between the carry, which is determined largely by interest charges, and the upper arbitrage boundary provided by the Eurodollar rate. For gold, this relationship is illustrated in Exhibit 2.

Inspection of the graph reveals that when the gold C gets either "too close to" the Eurodollar boundary rate, as in early 1988, or "too far from it," as in mid-1989, a golden turtle trade can be established and held until the gold C comes back to a more normal relationship with the boundary. At that time the position is closed out and the profit on the trade calculated. This approach to defining a trading strategy differs from that in other studies that use techniques such as moving averages and standard deviations to generate trading decisions (see Monroe [1992], Monroe and Cohn [1986], Rentzler [1986]).

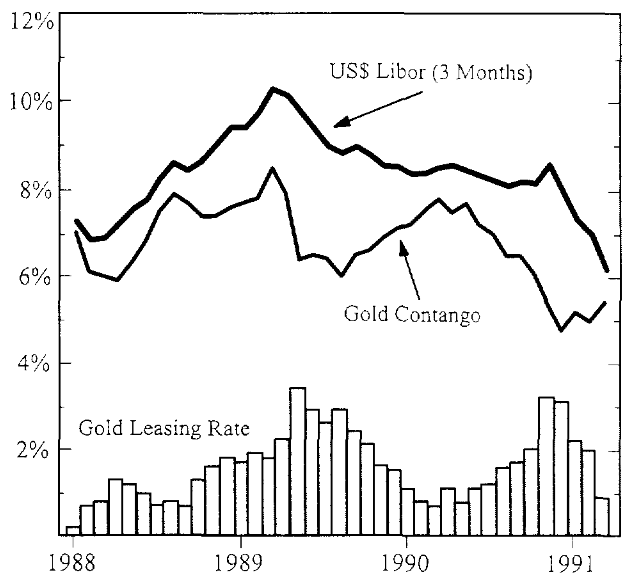
To determine the number of gold and Eurodollar contracts to be used in the golden turtle trade, it is necessary to specify the tailing procedure and the method of determining the spread hedge ratio between gold spreads and Eurodollar contracts. Recognizing that there are different methods of determining the tail, in the golden turtle the objective is to isolate C.

The tailed spread in this case can be specified so that for every long (short) deferred contract there will be F(0, T)/F(0, N) short (long) nearby contracts. For every unit of the deferred contract, there will be [1 + C(0)] units of the nearby contract. From (5), the profit function for a short-the-nearby, long-the-deferred, tailed gold spread is: $\Pi_{TGS}/(100\text{ oz.}) = G(1, N)\Delta C(N, T)$.

Assuming the trade is initiated on August 8, 1994, and taking the gold price G(1, N) to be equal to $379, the G(0, N) for the October 1994 delivery (Aug. 8/94), the hedge ratio follows by observing that one basis point equals 0.0001, that the gold contract is written for 100 ounces, and that the value of one basis point for a Eurodollar contract is $25. Hence, the hedge ratio for the number of gold spreads per Eurodollar contract is ($25/[($379)(100)(0.0001)] = 6.596.

To see this more precisely, observe that the basic problem is to derive the number of tailed gold spreads that, for a given basis point change, will (locally) have the same dollar value change as the corresponding dollar value change in the Eurodollar contract. The profit function for a golden "bear" turtle, long one Eurodollar contract and short Q* tailed gold spreads, is:

**EXHIBIT 2**
**GOLD CONTANGO AND LEASING RATE**




Source: Consolidated Gold Fields, *Gold 1991.*

$$\Pi_{gt}(1) = \$2{,}500\,[r_{EU}(0, T) - r_{EU}(1, T)] + 100Q^{*}G(1, N)[C(1) - C(0)]$$

$$= \$2{,}500\,[EU(1, T) - EU(0, T)] + 100Q^{*}G(1, N)[C(1) - C(0)]$$

where $r_{EU}(i, T)$ is an annual percentage interest rate calculated as 100 minus EU(i, T), the quoted Eurodollar contract price at time i.

When $Q^*$ is selected to be consistent with the dollar-equivalent hedge ratio, the golden bear turtle will be profitable when the differential between the annualized gold C and the Eurodollar rate narrows. The converse would hold for the golden bull turtle; the trade will be profitable when the differential between the annualized gold C and the Eurodollar rate widens. Correct calculation of $Q^*$ follows.

The dollar-equivalency hedge ratio is calculated by setting $\Delta C = 0.0001$ and solving the tailed gold spread profit function.[14] On a per contract basis this produces:

$$\Pi_{TGS} = (100)G(1, N)(0.0001) = (0.01)G(1, N)$$

As before, because G(1, N) is not known at t = 0 when the trade is initiated, a proxy is required. In the absence of a better value, we use G(0, N). If the August 8, 1994, price of $379 for the Oct 94 contract is used, then $3.79 is the value of one basis point (per contract) in a tailed gold spread. Relating this basis point value to $25 for a Eurodollar future provides the appropriate hedge ratio for the golden turtle:

$$Q^{*}HR = \$25/\$3.79 = 6.596$$

$$= \text{Number of tailed gold spreads per Eurodollar contract}$$

Together with the size of the gold tail, this number can now be used to construct the trade. Because we need to match the number of contracts in the tailed spread with the hedge ratio, the golden turtle is somewhat more complicated to implement than the tailed gold spread. Recall that in order to get a correct trade size for the tail, it is necessary to gross up [1 + C(N, T)] until an approximately integer relationship is established for the two legs of the spread. Observing that the Oct 95 price on August 8, 1994, was $399, then for the Oct 95/Oct 94 spread (1 + C) = 1.0528 to produce a tailed spread of nineteen to twenty. Since 3 × 6.59 is 19.77, a potential trade would involve nineteen deferred gold, twenty nearby gold, and three naked Eurodollar contracts. Because 6.59 × 3 is not exactly twenty, there may be some slippage between trading profits and the theoretical profit function.

Suppose the difference between the Eurodollar rate and the annualized gold C is expected to change — that it is expected to widen. This can happen a number of ways, but suppose the C stays constant and the Eurodollar rate increases. In this case, $r(1) - r(0) > 0$. Because the profit function for a long position in Eurodollars is $\Pi_{EU} = \$2{,}500\ \Delta EU = \$2{,}500\ (-\Delta r)$, when r is expected to rise, a short position in Eurodollars is profitable.

Similarly, if the widening occurs because the Eurodollar rate is unchanged $r(1) - r(0) = 0$, with the gold C falling, $C(1) - C(0) < 0$, it follows that a spread that is long the nearby and short the deferred is indicated. For the Oct 95/Oct 94 tailed gold spread and a Dec 94 Eurodollar contract example: When the Eurodollar/gold C interest rate spread is expected to widen, the appropriate trade involves short three Dec 94 Eurodollar contracts combined with a tailed spread that is long the nearby twenty Oct 94 contracts and short the deferred nineteen Oct 95 contracts.

Similarly, when the interest rate differential is expected to narrow, the appropriate trade is long the Eurodollar combined with a tailed gold spread that is short the nearby and long the deferred. The appropriate combination of these positions involves calculation of the size of the tailed spread, adjusted for the appropriate spread hedge ratio.

## V. SUMMARY AND CONCLUSION

Using cash-and-carry arbitrage restrictions, this article develops the profit functions for a number of important spread trading strategies: tandems, turtles, and stereos. As with any type of speculative trading, it is necessary to make judgments and predictions of unknown variables to use the strategies.

Yet these spreading techniques provide opportunities for speculating on changes in the variables that are embedded in the futures net cost-of-carry. For trades such as the golden turtle, arbitrage boundaries can be exploited to fine-tune trading opportunities. On balance, the universe of possible speculative trades is considerably expanded by spread trading approaches.

## ENDNOTES

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[1]Calendar spreads are also sometimes referred to as futures straddles, as in Peterson [1977]. Both these terms, however, also refer to options trading strategies. Schwager [1984] and Poitras [1994] provide a general overview of spread trading techniques.

[2]The soy crush spread involves trading the value of soybean contracts against the value of soybean meal and soybean oil contracts. The production relationship is

defined by the number of pounds of meal and oil that is obtained when one bushel of soybeans is crushed. The crack spread connects the value of a crude oil contract with the gasoline and heating oil contracts. This spread refers to the process of "cracking" or distilling a barrel of oil into various components, the most important of which are heating oil and gasoline. Other types of possible production relationship spreads are discussed in Tzang and Leuthold [1990] and Schap [1992].

[3]Because for every short, there is a long, the profit function for the short-nearby/long-deferred spread is the negative of the long-nearby/short-deferred spread. As a result of this symmetry, it is necessary to consider the profit function for only one of the positions; the other trade is treated implicitly as the negative.

[4]The cash-and-carry arbitrage interpretation of C(t, N, T) can be motivated by taking F(t, N) to be S(t), the price of the spot commodity, and examining the mechanics of the arbitrage connecting spot and futures prices. While somewhat more abstract, the futures-futures cash-and-carry arbitrage has the same logical mechanics as the spot-futures arbitrage. The functional determinants of the C(t, N, T) term will depend on the cash-and-carry arbitrage for a specific commodity. For example, gold will have a C that depends primarily on interest charges of carrying gold through time, while Treasury bonds will have a C dependent on both the interest charge for carrying T-bonds as well as a carry return arising from coupon interest earned on the underlying security.

[5]Consider the November 9, 1979–February 15, 1980, gold price example. At t = 0, the June 81/June 80 dollar equivalency tail is determined as [\$471.2/\$420.81] = 1.11977, which translates into a spread of twenty-eight June 80 and twenty-five June 81. At t = 1, [F(1, T)/F(1, N)] = 1.19829 for $\Delta C$ = 0.0785242 and F(1, N)$\Delta C$ = \$55.24. Similarly, profit on the nearby contracts is 28(703.5 – 420.8) = \$7915.6 and on the deferred contracts 25(843.0 – 471.2) = \$9295.0. Recognizing that profit has been defined per contract unit of the deferred gives (\$9295.0 – \$7915.6)/25 = \$55.18; the 0.06 difference in the two figures is due to rounding error associated with the tail.

Besides describing tail profitability, these calculations also reveal the limitations of using T-bill interest rates as a measure of change for the carrying charge interest rate reflected in gold prices. Even though T-bill interest rates did not change substantially between November 1979 and February 1980, the gold interest rate increased about 7.85%. Hence, in contrast to the impression given in the previous discussion, of the total spread change over the period of (\$139.50 – \$50.40) = \$89.10, only around \$34.00 of this change can be attributed to changes in price levels.

[6]With suitable modification, this type of profit function also applies to all other debt futures contracts, although it is not possible to deal adequately with a number of technical issues that are applicable here. For example, direct comparison of the IRR and the actual repo rate is distorted by the various seller's options to select the cheapest deliverable T-bond. In addition to illustrating how to derive the IRR from T-bond futures, Siegel and Siegel [1990] provide a more complete development of the IRR-R relationship.

[7]In other words, the spread hedge ratios involve variables defined at t = 1. These are obviously not known at t = 0 and, as a result, must be approximated. In the absence of information that might improve the estimate, the ratio of current values can be used. In certain cases, hedge ratio adjustment during the life of the trade may be required, and this will have to be incorporated into trade design. This practical substitution of current for future values occurs in virtually all the spread hedge ratio evaluation situations.

[8]The profit function is approximate because the exact profit function requires two tails to be specified for the trade. The precise method of specifying the two tails is similar to setting a hedge ratio in the stereo trades.

[9]Commodities where the butterfly may be a feasible trading strategy for traders with higher transaction costs are those with a significant seasonal factor in the term structure of futures prices, such as heating oil. For example, by forming the spread using fall-winter-summer contracts, changes in the butterfly could be used to speculate on, say, the advent of an unexpectedly cold winter, without being concerned with changes in the level of heating oil prices.

[10]The arbitrage is one-sided because of the difficulty of effecting the short-the-cash arbitrage. Gold is available for shorting from a number of sources, including central banks, at varying rates. The short fee combined with delay in taking possession of the gold for shorting purposes makes this trade much more complicated and expensive than the long-the-cash arbitrage, which involves only borrowing at or near LIBOR, using the funds to purchase gold, and simultaneously covering the position in the futures or forward market. Hence, while the upper bound provides an effective and well-defined upper boundary on the gold futures price, the lower boundary is significantly less effective and not as well-defined.

[11]Significantly, the IRR depends fundamentally on the cheapest deliverable commodity. For T-bond spreads, there may be numerous changes in the cheapest deliverable bond over longer trading horizons. This can give rise to variations in trade profitability.

[12]Even though the discussion is cast in terms of futures trading, the techniques have direct application to

currency forward contracts as well as extension to currency swap trading.

[13]In this case, the forward interest rate applies to covered interest arbitrages starting at t = N and ending at t = T. Hence, i(0, T − N) is the forward interest rate observed at t = 0 applicable to a covered interest arbitrage trade starting at t = N and ending at t = T.

[14]When the spread length, (T − N), is different from one year, a further adjustment is required to convert to annualized basis points. For example, if (T − N) is six months, then ΔC = 0.0001 actually refers to 2 annualized basis points. Because the Eurodollar has $25 per annualized basis point, it is necessary to gross up the number of gold spreads to correspond to the discussion related to Exhibit 2.

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# SUBMISSION OF MANUSCRIPTS

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Strict academic standards for the consistency and correctness of theoretical reasoning and empirical results do apply. A relatively high level of mathematical analysis is acceptable where appropriate — derivatives are inherently mathematical — but only what is necessary and appropriate to the material. The math should be expressed in as clear and simple a form as possible, and separable to the maximum extent possible from the logical flow of the discussion, so that articles can be understood by the more casual reader who does not wish to follow the math step by step.

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