Graduate Studies in Industrial Mathematics

Workshop at the University of Minnesota, November 17–19, 1995

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Over the past several years the University of Minnesota, in conjunction with the Institute for Mathematics and its Applications (IMA), has developed programs in industrial mathematics (both undergraduate and graduate) with strong links to local industries. The IMA has had a program of two-year industrial postdoctoral positions, with about four in residence each year. The duties of these postdocs include internship in an industrial setting, working under a specific mentor on specific problems, as well as participation in the regular IMA program. This relationship has deepened ties between the IMA and industry, as well as brought important problems from industry into the fabric of the regular program. These have been featured in a biweekly seminar in industrial mathematics, presented largely by these postdocs and their mentors. The success of this interaction led, last year, to the creation at the University of Minnesota of the Minnesota Center for Industrial Mathematics (MCIM), which now oversees all aspects of the involvement of academic mathematics in the industrial setting at Minnesota.

One of the central concerns of the MCIM is to see this successful interaction broadened so as to include many universities which have developed, or which have interest in, similar programs. Thus, one of its first projects was to sponsor this workshop. It brought together over fifty mathematics faculty from such universities and industry researchers who have been working together with mathematics faculty and students. The goals of the workshop were:

1) To gain better understanding of the opportunities in industry for students who wish to pursue careers in industry.

2) To develop a document explaining to departments that wish to consider a graduate program in industrial mathematics what steps need to be taken.

The publication of the National Research Council report "Graduate Education in Science and Engineering" in the fall of 1995 recommended, among other things, the broadening of the intellectual content of the Ph.D. by introducing more interdisciplinary courses and an increase in the diversity of skills acquired during this training by adding a minor to the course of study. These objectives were also perceived in a "Workshop on Graduate Student and Postdoctoral Education and Training" (June 5–6, 1995; World Wide Web: `www.nsf.gov/mps/workshop.htm`), sponsored by the NSF Directorate for Mathematical and Physical Sciences, which also suggested increased use of off-campus internships and other real-world experiences.

The concept of industrial mathematics is not new. An extensive article published in the *American Mathematical Monthly* (1941), entitled "In-

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dustrial Mathematics", by T. C. Fry, details the qualities and challenges of mathematicians in industry. This article, of course, anticipated extensive involvement of mathematicians in government and industry during the war. It states that, "the mathematician in industry, to the extent that he functions as a mathematician, is a consultant, not a project man. The successful mathematician must not only be competent as a mathematician; he must also have the other qualities which a consultant requires." More recently, a booklet by A. Friedman and J. Lavery published by SIAM in 1993, "How to Start an Industrial Mathematics Program in the University", provides detailed guidelines in this direction.

In October 1995 SIAM published a three-year study on "Mathematics in Industry". This study, based on interviews with five hundred mathematicians in industry and their managers, explores the environment for mathematicians and mathematics in industry and the skills these mathematicians require in their work; the report also provides guidelines for curriculum that will help develop the required skills.

Starting with all of this activity as background, the intent of the MCIM workshop was to look for future opportunities to develop programs in industrial mathematics. The MCIM workshop included talks by industrial mentors of students and postdocs, as well as talks by the students and postdocs who have worked with industrial mentors. A panel of mathematicians from six universities described their ongoing industrial mathematics programs.

At the end of the workshop, the participants were divided into three discussion groups, each centering on one of the following topics:

I. How to develop skills needed in industry.

II. What is the desired curriculum for industry-oriented students?

III. How can university faculty develop contacts with industry?

We summarize the content of these discussions.

I. Developing Skills

The first group focused on three sets of specific skills needed by a mathematician in industry.

Depth and Breadth

It is very important, at each level of achievement, to maintain an appropriate balance of depth and breadth. It was felt that for the B. S. degree, breadth—that is, familiarity with a wide range of mathematical tools—is more important than depth in a specific area. On the other hand, it is crucial for the student seeking the Ph.D. to develop significant depth in several related areas, sufficient to begin to do research in those areas. The recommendation for the M.S. lies somewhere in the middle of these extremes. At the same time, M.S. and Ph.D. students should be broad-based enough to be facile with the entire undergraduate curriculum.

It was the opinion of this group that there should be an M.S. thesis requirement. This would show evidence of deep work and commitment, thereby strengthening the student's résumé. It was also felt that industrial experience develops the student's breadth. Written reports and/or thesis on an industrial project will be important in demonstrating the student's interest and ability in industrial applications of mathematics. Students with industrial experience have an edge over those who do not when approaching a company for employment.

The group also recommended that students be exposed to courses in statistics and in fields outside of mathematics.

Computation

Computational skills are essential in an industrial setting. The group recommended that students be exposed to basic mathematics before their computational toolbox is filled with the proper tools, so that they can learn the use of these tools in the context of advanced mathematics.

Communication

Students must acquire skills to communicate with nonmathematical and nontechnical coworkers. This is crucial when talking to managers. In industry one is often put in a position to "sell" to a manager who is most likely from another discipline and possibly not a scientist.

A way to develop these skills is to demand uniform expectation throughout the student's courses, project, and thesis. Students should be taught to write and present well. Formal train-

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ing in speech giving and presentation is highly recommended.

II. Curriculum

The second group accepted as a guiding principle the recommendation of the NRC report: that a student should finish a graduate program in industrial mathematics in two years for an M.S. degree and in five years for the Ph.D.

It was agreed that broad and thorough undergraduate preparation is important for students wishing to pursue advanced degrees in mathematics with emphasis in industrial applications. Every undergraduate should take, among other subjects, (i) rigorous advanced calculus, (ii) linear algebra, (iii) complex variables, and (iv) courses in the sciences. Students who did not have (i)-(iii) as undergraduates must take them in their first year of graduate studies. The group believes that the curriculum should be lively and that a highly stimulating environment be provided. In particular, courses should focus on the understanding and solution of concrete, complex problems rather than on succeeding in examinations. (Traditionally, many students tend to take courses to get through qualifying examinations rather than for learning's sake.)

In the graduate program the following core subject areas are recommended.

- Scientific computing with numerical analysis and visualization
- Modeling (with exposure to real data and experiments)
- Analysis
- Probability and statistics
- Discrete mathematics
- Differential equations
- Optimization

This group recommended that computing and visualization be integrated throughout the curriculum. At the appropriate time in their careers, students should be exposed, in a meaningful way, to real data, experiments, and processes. Some form of practical experience is also recommended. This could be in the form of internship or clinical experience. This is critical in developing the student's "soft skills", such as communication, teamwork, etc.

It is important that students be exposed to courses in areas of application. In conjunction with faculty from those areas, the department should develop a list of suggested courses, and students should make their choices with the assistance of an advisor in order to form a coherent body of knowledge.

III. Developing Contacts with Industry

While it is generally agreed that a relationship with industry is crucial for the success of an industrial mathematics graduate program, it is also acknowledged that this is a very difficult task to accomplish. To start, it is imperative that the department and the university administration make a serious commitment to developing these relationships. In the department, a faculty member must be designated as point-of-contact. Collaboration with faculty in other departments who already have industrial contacts is a viable approach. Also, graduates with jobs in industry should be approached.

The level of contact with industry will vary depending on the structure of the company. Start with an engineer or a scientist who is a potential customer of what the department is offering. Managers could be contacted to identify the proper personnel to approach.

A suggestion to initiate collaboration is to organize a workshop that focuses on specific topics to entice industry interest. The workshop should provide a forum for the industry speaker to describe problems. In this way, new contacts can be developed. It is important to follow up each contact with some tangible effort on the part of the mathematician in order to show interest and seriousness in helping industry solve its problems. The collaborative effort with industry should produce deliverables such as computer codes and technical reports.

Finally, the group recommended that mathematics departments look seriously into the issue of industry job placement for their graduates. They should approach the job placement office at their university and coordinate efforts to place students in jobs in industry.

IV. General Questions

These issues arose during the discussions at the group sessions:

(i) What are the benefits for a department?

(ii) Does the present reward system in the university provide adequate encouragement for faculty who wish to make contact with industry and develop industrial mathematics programs?

(iii) Should nontenured faculty be encouraged to "take the risk" of embarking in this direction?

The feeling of the industrial participants was that the students who have gone through an industrial mathematics program, including internship in industry, have a much better chance of finding jobs in industry. For the faculty the interaction with industry offers not only intellectual enrichment but also opportunities of consulting, as well as funds to support their students and their own research. Developing an industrial mathematics program requires a large time investment on the part of the faculty. The reward system must be very clear on the expectations. This is particularly important in the case of nontenured faculty.

In concluding remarks it was said that, "the future of the mathematical sciences in the United States will depend on our ability to broaden the educational base by preparing more students for careers in nonacademic environments."

—Avner Friedman

The Program at the University of Minnesota

In the fall of 1993 the mathematics department at the University of Minnesota started a program in industrial mathematics, offering the M.S. degree. This idea seemed to recommend itself from the success of the postdoctoral program in industrial mathematics at the Institute for Mathematics and its Applications and the deep and broad interest in that program exhibited by scientists and mathematicians in the industrial setting. In the fall of 1996 this program will begin to offer the Ph.D. as well.

The philosophy of the program is that a student considering a career in industry should be provided with the skills, background, and experience necessary to succeed. This is surely the mission of any graduate program, but in this case it refers especially to the characteristics, and the methods to attain them, of successful mathematicians in industry, some of them elusive, some of which are not typical concerns of a mathematics program. These characteristics include ability to formulate problems, ability to assimilate techniques to solve an unusual problem, communication skills, teamwork attitude, knowledge of application areas, and industry research experience.

The cornerstone of the graduate program is the industry internship. Students are placed in an industrial setting working on a problem relevant to the industry. The work is co-supervised by an industrial mentor and a faculty member. M.S. students are asked to write a thesis based on the research results. Ph.D. students will normally continue the work and develop it into a dissertation. The Minnesota Center for Industrial Mathematics (MCIM) plays a substantial role in this program: to find industry problems and pair them with a student and a faculty member. In 1996 twelve students will be sent for summer internships to various industries.

In addition to the internship and its thesis, the two-year master's degree program requires 8 credits in related fields, as well as courses in advanced mathematics, including at least one sequence in core subjects. Courses in mathematical modeling, numerical methods, and mathematics in industry have been crafted specifically to this program (see the course descriptions below).

At present the form of the Ph.D. program is still on the drawing board. The driving philosophy is that it should be a mainstream mathematics degree, not a degree through an interdisciplinary program, typically taking five years to complete. As previously mentioned, the dissertation is expected to consist of original research, in the optimal case to be developed from the internship experience.

The University of Minnesota has done very well in developing industry relationships and the structure under which collaboration with industry will take place. At least ten companies have agreed to be involved in collaborative research through our students. It seems that through time and the hard work of IMA and MCIM and their industry contacts, potential and existing industry partners are beginning to appreciate the value mathematicians can bring to industry. Students have also become very excited about their education because their mathematical journey is immersed in real-world experience, and it is expected that between the preparation this program provides and the direct contacts developed students will find appropriate positions in industry upon graduation.

Course Descriptions

Mathematical Modeling Course (developed by Fadil Santosa)

The goals of the course are to develop ability to formulate problems from a nonmathematical description and to identify features relevant to a model, to be able to analyze the model using basic methods. A central feature of the course is the project, worked out in teams with the instructor acting as tutor/facilitator. An oral presentation, as well as a written report, is essential to develop communication skills and teamwork spirit. This year's projects were: forest fire models, multilane traffic, and spread of Ebola virus. Another feature of the course is the case study presentations: for example, on various problems from thermal imaging, cooking, and solidification processes.

Industrial Mathematics

(developed by Avner Friedman and Walter Littman)

This is based on the series of books about industrial problems taken from the presentations at the industrial seminar at IMA. The purposes of the course are to expose students to how mathematics is used in industry and to motivate them in learning mathematics through interesting applications. A common thread in mathematical modeling of industrial problems is to gain better understanding of industrial models and processes through mathematical ideas and computations. The resulting knowledge can be used in design and development of products.

By the end of the course, students will have been exposed to several real-world applications of mathematics and will have learned basic material in theory and computations of ordinary and partial differential equations, integral equations, calculus of variations, and control theory. A somewhat unusual feature about many of the problems treated is that one needs a combination of several techniques to obtain solutions to the problems.

Numerical Methods

(developed by Bernardo Cockburn, John Lowengrub, and Mitchell Luskin)

This course has a hands-on approach to learning numerical methods and the context in which they are used. Students are taught concepts of stability, convergence rate, and discretization errors in order to have the ability to assess the accuracy and reliability of computational methods. The link between the theory of numerical analysis and the practice of scientific computing is emphasized. In addition to writing computer codes that implement methods, students are exposed to visualization techniques, to writing "drivers" for scientific computing packages, and to developing graphical interfaces for scientific software. Topics covered include methods for ordinary and partial differential equations, numerical linear algebra, and optimization. Students are assigned projects which are worked out in groups and are asked to make a presentation on their work, as well as to submit a written report. The projects, which involve the use of several computing methods to solve a problem, are typical in industrial, scientific, and business applications.

—Fadil Santosa