7

State-descriptions and reductions

If he is not to appeal to knowledge that he cannot justify (Chapter 6), there seems to be only one way for the Necessitarian to try to rebut the Regularist's claim,

> *P10* *If a project is one of a kind and it does not succeed, then it is physically impossible, that is, it is doomed*

And that way is to meet this contention head-on; to argue that it is not logically true:

> That a unique project is undertaken and fails ought not to turn out to be a logically sufficient condition for attributing impossibility to projects of that sort (even if there are no other instances). Some of these unique projects are physically possible, which is to say, counterfactually, they could have succeeded. By "could have succeeded" I mean, using the possible worlds idiom, that in some other possible world very like this one, some of these projects *do* succeed.

The sentiments expressed in this last rebuttal are clear and doubtless strike a sympathetic resonance in many critics of the Regularity Theory. But if we are to pursue this line of criticism, we must be very careful as to what shall be reckoned "some other possible world *very like* this one."

Although it is easy enough to imagine another possible world that differs from this world in just one event, it is hard to see how that world could also be 'very like' this world if that world didn't also have – as a result of that different event – a somewhat altered future from that of the actual world. Consider again the swimmer who sets out to cross Lake Superior but who does not reach the far shore. Could another possible world W_1 share the same physical laws as the actual world and yet differ from the actual world only in the fact that in 1891 the attempt to swim Lake

Superior was successful? Surely we must assume that in W_1, if the swimmer had been successful, the subsequent newspaper accounts would have reported success, unlike the accounts in the actual world that reported failure. In the actual world, the swimmer failed to collect the prize money; but surely she would have in W_1. In the actual world, the swimmer had promised before her attempt that if she were successful she would endow her college with a scholarship; because she made the same promise in W_1, she would have had to carry through her promise in W_1 or would have had to do some explaining as to why she was reneging; etc.

Most of us believe that this world we find ourselves in (the actual world) is pretty uniform, and the idea that another possible world could differ from this world in some large-scale event – of the sort that the Necessitarian is wont to say (counterfactually) could have occurred – without that event having profoundly different consequences from those that occurred in this world, is a nonstarter.[1] If we are going to try to advance these investigations by invoking alternative possible worlds, Regularists and Necessitarians alike are going to have to impose some reasonable and realistic constraints on the manner and extent of the allowable variations between the possible worlds compared.

There are probably as many different ways to impose these constraints as there are philosophers who have tackled the problem, from Todd (1964) to D. Lewis (1973) to Kahneman (1982). Fortunately, we do not have to review this bewildering variety, for the very nature of the problem, as we have here been examining it, suggests the kinds of constraints appropriate for our purposes.

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[1] "There are such things in nature as parallel cases, that which happens once, will, under a sufficient degree of similarity of circumstances, happen again." (Mill 1965, p. 201). The lesser the degree of similarity required for repetition, the greater the uniformity of a world. A world, W_j, may be said to be more uniform than another world, W_k, if *fewer* properties must be instanced by an event for that event to be repeated (i.e., for there to be another event of that same event-kind). In the actual world, for example, the ceiling light turns on whenever I, my wife, my daughter, my son, a babysitter, a houseguest, a tradesman, or a friend flicks the wall switch (well, nearly always). But in a *less* uniform world, the ceiling light comes on only after the switch is flicked by a blue-eyed, forty-two-year-old, twice-published, deaf poet. There are, of course, worlds *more* uniform that the actual world. Such a world would be, e.g., one in which any two liquids mixed together always yielded a precipitate. This happens only for very special cases in the actual world.

I would hazard a guess that, on a scale of uniformity, the actual world is a middling one. But I must admit that I am very unsure about this. In any event, the topic of uniformity will constitute the entirety of the section titled "Seminar of April 13: Uniformity" in Chapter 11.

For the Regularist, *singular* statements are ontologically primary. Physical laws are logically derivative. Contingent universal material generalizations derive their 'legitimacy', as it were, from their being reducible to conjunctions of singular propositions, that is to propositions containing only individual constants, purely descriptive predicates and relations, and truth-functional connectives, for example: $\sim Ra \vee Ba$; etc.[2]

Extending Carnap's terminology of 1942 (Carnap 1961), we may say that a *state-description* of a possible world *W* is the class of (or the conjunction of) all the true propositions obtaining in *W*. Then, in terms of this extended concept of a state-description, the Regularity Theory may be expressed thus: All physical laws of a possible world *W* are reducible[3] to some subset (simplification) of the state-description of *W*. In other words, Regularists disallow that there are any actual or possible physical laws whose truth derives from anything other than a world's *singular* facts.

It was stated in Chapter 3 that Regularity is a species of Reductionism. Clearly, the Regularity Theory of Physical Laws is a necessary, but not a sufficient, condition for Absolute Reductionism, the theory that *all* general truths – whether physical laws, moral laws, aesthetic principles, etc. – are reducible to singular propositions, that is, to subsets of a world's state-description. Put another way: Absolute Reductionism holds that a possible world is specified *completely* by its state-description. But although a Regularist is free to maintain this latter, stronger thesis of Absolute Reductionism, it is not required by his own theory. One can promote the thesis that physical laws are reducible to singular truths without being logically required to make the same claim for other general truths.

The problem at issue – examining physically possible alternative worlds – becomes tractable against this background of making singular propositions ontologically basic, provided the state-description model is a viable one. If it is viable, we should then like to see how, in terms of it, the differences between Regularity Theory and Necessitarianism can be spelled out. Is the state-description model viable?

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2 Numerical laws will be discussed in Chapter 13. There it will be argued that numerical quantities are just members of infinite classes of purely descriptive predicates.

3 The use of the term "reducible" (and kindred terms, such as "reduction") here is perfectly apt, although some care ought to be taken so as not to confuse the current sense with another. Here we are talking of the possibility of a general proposition being 'reducible to' singular ones. This contrasts with the sense in which one theory or law may be said to be 'reducible to' another. E.g., phenomenological thermodynamics has often been said to be reducible to statistical mechanics, and optics to the theory of electromagnetic radiation.

There are four worries about the appositeness of this model. Each must be addressed.

The first problem arises concerning the account we are to give of the 'singular statements' that comprise the model. For if a state-description is thought to be composed of sentence-tokens, then in a finite universe there can never be enough sentence-tokens to describe *everything*. For every sentence-token itself would have to be described by some *further* sentence-token, and this condition cannot be satisfied among a finite number of things. In other words, we would have here an instance of Russell's 'Tristram Shandy' Paradox (1938, pp. 358–60). Since we hardly want the applicability of the state-description model to depend on an empirical claim as to the finitude of the material universe, we avoid the paradox by making the 'statements' in the state-description abstract entities that exist in unlimited number: sentence-types or propositions, according to one's preference.

The second problem arises from a discovery by Bar-Hillel: Once the primitive vocabulary of a formal language *L* is extended beyond monadic predicates to dyadic and *n*-adic relations, there is no *general* solution to the number of state-descriptions constructible (Bar-Hillel 1951). However, this formal barrier poses no problem for our particular use of the concept of a state-description. We are not constrained to regard the only acceptable sense of "state-description" as being one that pertains to a linguistic construction. We are free to define a sense of "state-description" that makes the term refer to a set (perhaps infinite) of propositions whose existence has nothing whatever to do with our specifying a vocabulary or indeed even with our ability to know their truth or to enumerate them. Rather than conceiving of worlds as constructed out of state-descriptions, we may regard the logical priority the other way around. We can conceive of a world (especially *this* world), as antecedently given; and a state-description as the set of *all* the singular propositions true in that world.[4]

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[4] To speak of the world, as I just have, as antecedently given must surely invite questions about the status of the physical world and raise the specters of incommensurable descriptive statements and alternative conceptual schemes. I happen to be a Realist about the external, physical world. I am convinced that the world is not of my making, nor of the making of scientists; that it existed temporally prior to our existence; that science – although it has essential creative elements – is ultimately directed to finding out how the world *is*; and that how the world is, apart is from the surface of this planet, *[footnote 4 continued on page 83]*

The third problem addresses the question of what is meant, precisely, in saying that a universal material generalization is 'reducible' to singular propositions. One might think that a 'reduction' would consist simply in the conjunction of all the (true) 'Nicod instantiations' or 'confirming instances' of that generalization. After all, "Sa & Pa" and "Sb & Pb" are supposed, each, to confirm "$(x)(Sx \supset Px)$"; and thus what could be more natural than to assume that "$(x)(Sx \supset Px)$" is reducible to "Sa & Pa" and "Sb & Pb" and the like?

One has practically only to write down the suggestion to see that it will not work. The proposition "$(x)(Sx \supset Px)$" is logically equivalent to "$(x)(\sim Px \supset \sim Sx)$" whose Nicod instantiations (e.g., "$\sim Pa$ & $\sim Sa$") are pairwise logically inconsistent with each of the Nicod instantiations of "$(x)(Sx \supset Px)$" (see Hempel 1965). This is a pretty unpromising tack to take: Logically equivalent universal propositions would end up having reductions logically inconsistent with one another.

The species of reduction that works is what is called the 'quantifier-free equivalent' (*QFE*, or more exactly QFE^k, i.e., the quantifier-free equivalent with respect to the universe *Uk*, the universe consisting of the *k* distinct individuals, *a*, *b*, … *k*). (See Faris 1964, pp. 115–24.)

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4 *[cont.]* is with rare exception not of our doing. But with this said, I am content to leave the matter. Not because I think I have given a persuasive argument for Realism (I haven't). But because I think that a proper argument deserves a separate treatment; and more importantly, because the issue is separable from the one at hand. One needn't be a Realist (although I think it is easier if one is) to conceive of there being a set of singular propositions – known or unknown – that are true of the world. One may wish ultimately to give a different analysis from the one I would venture as to how contingent propositions acquire their truth; but these differences need not affect the eventual answers to questions as to whether, and which, contingent propositions are nomologically necessary, physically possible, etc. In other words, to the extent that persons subscribe to the Laws of the Excluded Middle and of Noncontradiction, they need not agree absolutely as to the nature of truth to proceed to the questions of whether general truths are grounded in singular ones and whether any contingent statements are nomological.

The companion specters of incommensurable descriptive statements and alternative conceptual schemes are much more difficult to exorcise. Once again, the attempt will not be made here. A disbelief that there are, in the end, incommensurable descriptive statements and conceptual paradigms of sufficient difference as to preclude judgments of consistency, etc., makes the job at hand far easier. But, again, it is not essential for the task. Regularists and Necessitarians can share, or not, beliefs as to whether there is, or can be, only one conceptual scheme adequate for describing the world. Whatever their views on this issue, the fundamental debate will still remain: Do physical laws bear a nomic necessity, and, if so, what account is to be given of this necessity?

Intuitively, the *QFE* is simply the result of eliminating universal quantifiers in favor of conjunctive strings of singular propositions, and existential quantifiers in favor of disjunctive strings. Thus, for example, the *QFE* of $(x)(Sx \supset Px)$ is:

$$(Sa \supset Pa) \;\&\; (Sb \supset Pb) \;\&\; \ldots \;\&\; (Sk \supset Pk)$$

For convenience, I will call the converse of the relation of being-the-*QFE*-of a "packing of." Thus, "$(x)(Sx \supset Px)$" may be said to be a *packing of* "$(Sa \supset Pa)$" with "$(Sb \supset Pb)$," along with "$(Sk \supset Pk)$."

On this score, then, on defining a precise formal sense in which a universal proposition may be 'reduced' to singular propositions, the state-description model passes muster.

The fourth problem, however, concerns a challenge to the very coherence of the notion of such a reduction. For – this challenge would say – when it is further examined what the relation of reducibility is, it will be found that it presupposes an irreducible universal proposition and thus vitiates the larger theory in which it is embedded.

This last problem has been lurking since the earliest attempts to explicate the relation of reducibility. Russell, in his 1918 series of lectures, "The Philosophy of Logical Atomism" (in section 5, entitled "General Propositions and Existence"), had spoken to it directly. (In the following, when he says "general" and "particular," I am sure he means "universal" and "singular.")

> I do not think one can doubt that there are general [universal] facts. It is perfectly clear, I think, that when you have enumerated all the atomic facts in the world, it is a further fact about the world that those are all the atomic facts there are about the world, and that is just as much an objective fact about the world as any of them are. It is clear, I think, that you must admit general [universal] facts as distinct from and over and above particular [singular] facts. (Russell 1965, p. 235; parenthetical glosses added)

A few years later, in 1922, when Wittgenstein's *Tractatus* first appears in print, we find a brief allusion to the same issue:

> Propositions comprise all that follows from the totality of all elementary propositions (and, of course, from its being the *totality* of them *all*). (Thus, in a certain sense, it could be said that *all* propositions were a generalization of elementary propositions.) (Wittgenstein 1961, section 4.52)

Wittgenstein is not as expansive as Russell, and does not say – as does Russell – that there are universal facts "distinct from and over and above particular [singular] facts." Indeed, Wittgenstein may have been loath to say such a thing, given his larger reductionist program.[5] But whatever his reasons for not following Russell in positing irreducible universal propositions, Wittgenstein's remarks invite, as do Russell's, questions about the adequacy of the set of singular propositions that figures in the theory.

Russell speaks of a complete enumeration, and Wittgenstein of the totality of elementary propositions. What is the status of this requirement, which we may aptly call 'closure'? If it is to be stated by another proposition subjoined to the original set of singular propositions, then the resulting set contains a universal proposition, and thus no state-description – understood to consist wholly of singular statements – could possibly be an adequate description of this world or of any other. But if the closure condition is not a universal proposition to be added to the original set, what, then, might it be? Even if Wittgenstein did not address this question explicitly, it is not one a modern-day Regularist can ignore. For, the Regularist, like Wittgenstein, is pursuing a reductivist program that makes singular propositions ontologically basic and that disallows irreducible physical laws. If Wittgenstein's theory – the prose forerunner of Carnap's formal state-description theory – harbors, however implicitly, Russell's irreducible universal propositions, then the modern-day Regularity Theory is in danger of losing its keystone.

Notice how Wittgenstein talks of generalizations as 'following from' singular propositions.[6] Is this mere infelicity, or is it meant literally? If taken literally, the way is open to challenge the theory. For, it might be argued, no universal proposition 'follows from' a conjunction, however long, of singular propositions:

5 As a matter of fact, there is a problem as to which of these two philosophers originated the idea of enumerating singular facts. Russell, in his preface to his lectures, says that the lectures are "largely concerned with explaining certain ideas which I learnt [prior to 1914] from my friend and former pupil Ludwig Wittgenstein" (1956, p. 177, parenthetical gloss added). It may well be, then, that Wittgenstein has not dropped an inference that Russell earlier made (1918), but that Russell infers from Wittgenstein's theory (of 1914) something that Wittgenstein himself would not have.

6 Actually, Wittgenstein spoke of 'elementary' propositions, which were supposed to be a certain proper subset of the class of singular propositions. Whether or not a useful distinction of this sort can ever be drawn between 'elementary' singular propositions and 'nonelementary' ones, we have no need of such a distinction here. What the state-description model requires is that there be a set of all singular propositions true in a world. If some of these singular propositions happen not to be 'elementary' (i.e., are deducible from others in the set), no matter. Redundancy, in any amount, is perfectly tolerable in a state-description as described above on page 81. At most, redundancy will simply amount to logical overdetermination of the universal conditionals that are true in that world (i.e., that are generable from the state-description). The self-consistency of the model will be assured by the requirement that all its members be true.

> No set of singular propositions implies a universal proposition, for no matter how many true singular propositions of the form "$Sn \supset Pn$" one strings together, there is always the theoretical possibility of there being some constant q such that Sq & $\sim Pq$. In other words, packings are not cases of deducibility. Clearly, to ensure logical implication, a further premise is needed to the effect that all the relevant constants (individuals) have been enumerated. Suppose, for example, that we were considering a possible world of only three individuals, a, b, and c; and suppose that the following is true:
>
> (**S**) $Ga \supset Ha$
> $Gb \supset Hb$
> $Gc \supset Hc$
>
> In such a world, the following proposition would also be true:
>
> *P12* $(x)(Gx \supset Hx)$
>
> But this latter universal proposition, this packing of the set S of singular propositions, is not *deducible* from S. To ensure the relation of logical implication, a further premise is needed, namely:
>
> *P13* $(x)(x{=}a \vee x{=}b \vee x{=}c)$
>
> But *this* premise is itself universal, and it is indispensable to the validity of the inference. Thus it is impossible, in principle, to reduce a universal generalization to singular propositions. QED.

This argument about closure has convinced many persons that no case of reduction of universal propositions – whether physical laws, moral laws, aesthetic principles, or what have you – to singular propositions can be valid.

But Regularists have a rebuttal: The argument for closure, attractive as it is, rests on a confusion. To show this, we begin by examining the inference the other way around.

> Suppose we have the universal generalization "All human beings are mortal," that is, "$(x)(Hx \supset Mx)$." Is the following deducible from this universal proposition?
>
> *If Swartz is a human being, then Swartz is mortal (alternatively, $Hs \supset Ms$)*
>
> On virtually all accounts, the latter singular proposition does logically follow (is deducible) from the former universal one. But how can this be? Shouldn't we have to conjoin to the former a 'domain'-condition? For example, something of this sort:
>
> $(x)(x{=}\text{Swartz} \vee x{=}\text{Hume} \vee \text{x=Lili Pons} \vee \text{x=Terry Fox} \ldots \vee \ldots \vee \ldots)$
>
> Such an additional premise is not required, and it is easy to see why. It is a presupposition in the use of the quantifier "(x)" that it range over all and only those items in a possible world. But because this is so, we hardly need "$(x)(x{=}a \vee x{=}b \vee x{=}c)$" to get from the complete set of singular conditionals (of the form "$Sn \supset Pn$"), for example, the set **S**, to a proposition of the form "$(x)(Sx \supset Px)$," for example, *P12.*

What we logically require is that the set of singular conditionals *be* complete; we do not require an additional premise *stating* that the set is complete. An explicit closure condition is required for inferences about unspecified worlds; but when the world is specified, the closure condition, contra Russell, is otiose.

P12 and **S** are not logically equivalent (like-valued in all possible worlds), and Wittgenstein stumbled slightly when he described all propositions (including, presumably, universal generalizations) as 'following from' singular propositions. But neither are *P12* and S *merely* materially equivalent, that is, merely like-valued in the actual world. They are an intermediate case: like-valued in all possible worlds in which $(x)(x{=}a \vee x{=}b \vee x{=}c)$. $P12 \leftrightarrow \mathbf{S}$ is false, that is,

$$\Box\,(P12 \equiv \mathbf{S})$$

is false. But

$$\Box\,[(x)(x{=}a \vee x{=}b \vee x{=}c) \supset (P12 \equiv \mathbf{S})]$$

is true, that is, $P12 \equiv \mathbf{S}$ is a relative necessity in all those possible worlds in which $(x)(x{=}a \vee x{=}b \vee x{=}c)$. Provided we are talking about one such of these worlds, we may say unproblematically that *P12* is a packing of the set **S**, and thus that *P12* is reducible to **S**. The packing relation, although not the relation of logical implication, is nonetheless far stronger than mere material implication, is perfectly intelligible, and is free of internal incoherence. Wittgenstein was right, then, not to take Russell's step of positing irreducible universal propositions. A state-description model, either nascent (Wittgenstein's) or developed (Carnap's), need not be encumbered with irreducible general propositions.

In the end, the state-description model emerges intact, and it can be used to illuminate the differences between the Regularist's and the Necessitarian's respective views about physical possibility.

By insisting that the mere failure of a unique project ought not to constitute logically sufficient grounds for all projects of that type being physically impossible, the Necessitarian finds himself claiming:

> Two possible worlds, W' and W'', have identical physical laws. But in W', project X is undertaken and fails; in W'', X is undertaken and succeeds.

Is this point of view compatible with the state-description model, or does it require that the Necessitarian adopt a different model?

The state-description model is a very uncompromising view of reality. For one, it ties the universal generalizations that are true in a world very closely to that world's singular facts, far more so than one might at first suppose. Naive intuitions tell us that there should be considerable leeway possible in constructing a world to fit a given set of universal generalizations, that singular facts in a world can vary greatly while the universal generalizations remain constant. But the state-description model allows for little of such flexibility. A god setting out to design a world different from this one but in which all the same generalizations were to hold true, would find himself limited to a far greater extent than we would have initially surmised.

For example, one might suppose that merely increasing the number of individuals in a possible world is compatible with there being no concomitant change in the universal propositions true in that world. But this is not so. Consider a world, W_1, in which there is only one thing, *a*, and it has the properties *G* and *H*; and a world W_2, in which there are exactly two things, *a* and *b*, and both have the properties *G* and *H*. For example, W_1 consists solely of a single green hexagon, whereas W_2 consists of two green hexagons and nothing more. One might assume that these two worlds share exactly the same universal generalizations, for example, "Everything that is green is hexagonal." But surprisingly, there is a universal generalization that is true in W_2 that is false in W_1:

$$(x)([Gx \ \& \ Hx] \supset (\exists y)[(Gy \ \& \ Hy) \ \& \ (x \neq y)])$$

'Mere duplication' of any one or of several items, events, or states in a possible world will change the set of universal generalizations obtaining in that world. In the state-description model, possible worlds cannot differ from one another merely in their numbers of constants without also differing in their universal generalizations.

Suppose M stands for a kind of project (for example – recalling our earlier discussion – an attempt to swim across Lake Superior) whose successful outcome we will designate N. Let two possible worlds, World′ and World″, be worlds in which M has exactly one instance a in each; that is, M is a unique project in World′ and is a unique project in World″. Suppose further that a fails in World′; that is, $\sim Na$ is true in World′. In World″, however, a succeeds; that is, Na is true in World″. Can World′ and World″ share identical physical laws in a state-description model? We can see in Table 2 that they cannot.

Table 2. *Worlds differing in the outcomes of counterpart unique projects*

	World′	World″
I. State-descriptions (*partial listing*)		
	$Ma \ \& \sim NA$	$Ma \ \& \ Na$
	$Sb \ \& \ Yb$	$Sb \ \& \ Yb$
	•	•
	•	•
	•	•
II. Consequential universal generalizations (*UG*) (*partial listing*)		
	UG'_1: $(x)(Mx \supset \sim Nx)$	UG''_1: $(x)(Mx \supset Nx)$
	•	•
	•	•
	•	•

For if a is the only instance of M in each of these worlds, and a is N in one and $\sim N$ in the other, then the respective (or counterpart) universal generalizations, *UG*, pertaining to events of kind M, that is, UG'_1 and UG''_1, must differ from World′ to World″. Clearly, unique projects confer special constraints on the set of universal generalizations true in a world. If being a true contingent universal generalization having unrestricted terms is a sufficient condition for being a physical law, then two possible worlds that differ in the outcomes of their respective unique projects must differ in their physical laws.[7] This conclusion is perfectly general. It has nothing to do with the specifics of our examples (swimming Lake Superior, making a river of Coca-Cola), nor with the number or variety of predicates in the state-description, nor with the number of individuals.

The Necessitarian must, then, eschew the classical state-description model. So long as physical laws are identified with universal propositions reducible to the singular propositions true in a world, the Necessitarian's desire that counterpart unique projects should have differing outcomes in possible worlds sharing the same physical laws cannot be realized. Two choices are open to the Necessitarian. He can either (1) modify the state-description model or (2) reject that model altogether, arguing in this latter instance that physical laws are irreducible to singular propositions.

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[7] Note that, even though UG'_1 and UG''_1 (see Table 2) are consistent with one another (there is some possible world in which they are both true, viz., any world in which nothing is M), these two generalizations are not both true either in W' or in W''.