

**Part 3:**  
**Replacement Variate Generators**

## XIV

### Introduction

As was seen in Chapter IV, when a given  $\underline{X}$  is ulcf representable, the set  $C$  of common factors to  $\underline{X}$  contains an infinity of constructed random variates. This fact does not square with the Central Account and so, as documented in Chapter V, in an attempt to protect the CA, the nature and implications of indeterminacy were frequently misportrayed by members of the psychometrics community. However, the Central Account is a mythology. That a mythology has been erected as the foundation on which to base the practice of latent variable modeling serves to not only distort what can be achieved in the employment of a latent variable model, but to taint the very impressive mathematical innovations realized by the discipline of psychometrics. The question, then, is how should latent variable models and modeling be conceptualized. What can the latent variable modeller expect to achieve when he employs a latent variable model? Because it has spent virtually its entire existence bewitched by a mythology, the discipline of psychometrics has failed to provide a workable answer to this question. And just because it has attempted to quash efforts to examine implications of the indeterminacy property of the linear factor model, it has failed to recognize that indeterminacy is the very door that must be opened in order to achieve a convincing characterization of latent variable modeling.

As will be remembered from Chapter V, a number of prominent authors have claimed that the indeterminacy property of the linear factor model (the fact that, when a given  $\underline{X}$  is ulcf representable, set  $C$  contains an infinity of constructed random variates), seems to lack practical implications for the employment of the linear factor model in research. It has been said, for example, that:

- i) "The indeterminacy of factor scores has been held by some writers (Guttman, 1955; Schonemann & Wang, 1972, for example) to create problems about the common factor model. A curious feature of these problems is that they do not seem to have any obvious bearing upon the use of the model in practice" (McDonald, 1977, p.165).
- ii) "...many psychologists who use factor analysis would not see any importance in the questions that have been raised about the indeterminacy of factor scores. Essentially, they know very well how seldom they require the assessment of factor scores, and they see no logical connection between questions about factor scores and the widely accepted aim of factor analysis, namely, the interpretation of a common factor in terms of the common attribute of the tests that have high factor loadings on it" (McDonald & Mulaik, 1979, p.298)
- iii) "We notice a difficulty in coming to grips with these interpretations of factor-score indeterminacy. As stated above, given at least the existence of errors of measurement, no test of finite length will be perfectly correlated with the factor, hence no test of finite length will have the properties of a common factor variable. It is not enough to be told that attaching meanings to factors on the basis of factor loadings is "clearly" rendered suspect by the existence of alternative possible factor variables. One needs also to be told how to find at least two distinct possible factor variables and on what grounds one might place distinct interpretations on them. Instead, Guttman (1955) showed only that one can construct alternative factor variables in infinitely ways

by adding to a regression estimate  $\hat{X}$  a value of a random variable  $D^*$  ...defined "by throwing dice or turning a roulette wheel" (McDonald & Mulaik, 1979, p.300).

iv) "With the exception of remarks in Mulaik (1976), what all of these discussions have in common is a failure to relate the problem described to any discernible realities of factor analytic practice" (McDonald & Mulaik, 1979, p.301)

v) "We view the theoretical aspects of having an explicitly indeterminate model as very serious, but there is little empirical evidence for a direct impact of the problem on well defined data sets" (Velicer & Jackson, 1990b, p.98)

However, these comments, representing, as they do, a general view that "indeterminacy lacks teeth", necessarily miss the point, because they *presuppose* the mythology of the Central Account. The fact that indeterminacy has had little impact on the practices of psychometricians and applied researchers is merely testimony to the straight-jacket that the CA has placed on the thinking of these individuals. The question is not whether the indeterminacy property of the linear factor model *has* had an impact, but whether it *should* have an impact. Similarly, McDonald and Mulaik (1979, p.300) couldn't have come up with a better example of question begging than when they claim that "It is not enough to be told that attaching meanings to factors on the basis of factor loadings is "clearly" rendered suspect by the existence of alternative possible factor variables. One needs also to be told how to find at least two distinct possible factor variables and on what grounds one might place distinct interpretations on them." For the practice of "interpreting the latent variate" is a key element of the fallacious portrayal of latent variable models as detectors of unobservable properties/attributes (causal sources), and rests on an incoherent account of the nature of concept meaning and signification. An urbild such as the Central Account can only flourish in fertile soil. Sadly, the fertile soil in this case is comprised of basic confusions over the terms of scientific investigation, confusions that were discussed in Chapter XII of the current work. Not surprisingly, these confusions have militated against psychometricians seeing latent variable models for what they are. Thus, Mulaik (2005) claims that random variates constructed in accord with (4.4)-(4.6) are merely *candidates* for the role of common factor to  $\underline{X}$ . However, he also speaks of there being "variates in the world" and believes that the equations of linear factor analysis make no mention of constructed random variates. In fact, there is no such thing as "variates in the world", there are just variates. Variates are not elements of natural reality, but are, rather, constructed by humans. Thus, if the variates referred to in the equations of the linear factor model exist, then they are constructed, the only question being "how are they constructed." Mulaik has also suggested that the output of a linear factor analysis could be employed in an extra-factor analytic search for a variate (whose values are produced in accord with an antecedently specified rule of score production) that renders conditionally independent the manifest variates. But is this the aim, then one does not require latent variable models: One can screen variates directly to see whether they render conditionally independent a set of variates.

Nothing about the indeterminacy property requires explanation in terms of this practice. On the contrary, the fakery inherent to this practice is seen through an honest interpretation of the indeterminacy property. The Central Account is nonsense, while an accurate characterization of the indeterminacy property, a property having to do with the cardinality of a set of constructed random variates, points the way towards a logical characterization of both latent variable and

component modeling. In particular, neither component, nor latent variable, models are detectors, but are, rather, *replacement variate generators*, quantitative recipes for the construction of random variates that *replace* a set of variates  $\mathbf{X}_j, j=1..p$ , in some particular, optimal fashion. The task of the current chapter will be to provide a sketch, rather than a fully worked account, of this reality. While the fact that latent variable models are, like component models, replacement variate generators has for some time been implicit in the writing of psychometricians, the omnipresence of the Central Account has militated against correct characterization. McDonald's 1975 article *Descriptive Axioms for Common Factor Theory, Image Theory, and Component Theory* (reviewed in Chapter V) is a case in point. As will be remembered, McDonald generated a defining axiom set for each of the linear factor, image, and principal component "model". Each of these axiom sets spell out the requirements that must be satisfied in order that a constructed variate or variates may legitimately be called common (and unique) factors to  $\underline{\mathbf{X}}$ , partial- and anti-images to  $\underline{\mathbf{X}}$ , or principal components to  $\underline{\mathbf{X}}$ . Based on these requirements, McDonald generated construction rules for the production of partial- and anti-images to  $\underline{\mathbf{X}}$  and principal components to  $\underline{\mathbf{X}}$ . However, after observing that variates that satisfy the axioms of the model cannot be constructed as linear combinations of the manifest variates, he simply ignores formulas (4.4)-(4.6) that *do* in fact yield the variates that answer to these axioms, and instead invokes the metaphysical property of unobservability to explain why a linear combination cannot be a common or unique factor.