VII

The myth of the latent variable model as detector

1. *Latent variable models are not tools of detection*

As described in Chapter III, the latent variable model is portrayed in the Central Account as a detector of properties/attributes (causal sources)[1] of the phenomena represented by the sets of variates, the manifest variates, that are input into latent variable analyses. Once again, let there be a particular latent variable model, lvm, a particular set of manifest variates, $\mathbf{X}_j$, j=1..p, distributed in a particular population, P_T, let $\underline{\Pi}$ contain admissible values of the parameters of lvm (contained in subspace π), $\underline{\Omega}_T$ contain the values, in P_T, of the parameters of which lvm makes claims, and M_{lvm} be the manifold traced out by lvm as $\underline{\Pi}$ ranges over π. The Central Account claims, then, that

If $\underline{\Omega}_T \subset M_{lvm}$, i.e., $\underline{\mathbf{X}}$ is described by lvm,

then this is evidence that:

i. There has been detected a property/attribute common to (causal source of) the phenomena represented by the $\mathbf{X}_j$;

ii. The concept *latent variate to* $\underline{\mathbf{X}}$ signifies this property/attribute (causal source);

iii. The scores that comprise the distribution of the random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ are measurements of the detected property/attribute (causal source).

The latent variable model is taken to be, essentially, a litmus test for the existence of members of a class of "entities"[2], either properties/attributes or causal sources, depending on whether one subscribes to the CAM or CAC. It will, herein, be argued that this portrayal of latent variable modeling is illegitimate.

To begin, consider several examples involving tools of detection that are employed elsewhere in science.

Scenario i: A semiconductor detector takes advantage of the fact that the electron-hole pairs that elementary particles produce in a semiconductor junction momentarily increase the electrical conduction across the junction. The Cherenkov detector, on the other hand, makes use of the fact that a particle emits light when it passes through a nonconducting medium at a velocity higher than the velocity of light in that medium (the velocity of light in glass, for example, is lower than the velocity of light in a vacuum). In a Cherenkov detector, a material such as glass, plastic, water, or carbon dioxide serves as the medium in which the light flashes are produced. Hence,

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1 In fact, belief in the need for special tools of detection within this context arises from the belief that properties/attributes and causal sources are unobservable. The unobservability issue will later be considered in its own right.

2 The term *entities* is shown in quotations because the CA is built on the model of a material entity detector.

when a sub-atomic particle passes through such a medium, a light flash is produced, and is, in turn, registered by photomultiplier tubes.

Scenario ii: Physics employs track detectors (including cloud, bubble, and spark chambers) to detect, and "observe" the action of, high-energy subatomic particles released by radioactive substances. The cloud chamber, for example, consists of a vessel several centimeters or more in diameter, with a glass window on one side and a movable piston on the other. The piston can be dropped rapidly to expand the volume of the chamber. The chamber is usually filled with dust-free air saturated with water vapor. Dropping the piston causes the gas to expand rapidly and causes its temperature to fall. This renders the air supersaturated with water vapor, but the excess vapor cannot condense unless ions are present. Charged nuclear or atomic particles produce such ions. Any such a particle passing through the chamber leaves behind it a trail of ionized particles upon which the excess water vapor will condense, thus making visible the presence of, and path taken by, the particle. A combination of knowledge and theory has made it possible to correlate particle type with characteristics of the paths left by particles. Knowledge of these correlations may then be used to identify the type of particle that, in a given instance, left a path, and provide information on other of its features. For example, by situating a cloud chamber inside a magnetic field, it can be determined whether a given particle is positively, negatively, or neutrally charged, and its mass and energy inferred (Bueche, 1972).

Scenario iii: The first cold virus detection protocol devised by researchers was rudimentary: Secretions from the nose of an individual with a cold were diluted in a salt solution, filtered, and then placed in the nose of a volunteer who then lived for a few days in isolation. If the volunteer caught a cold, it was inferred that the solution given him contained the agents responsible for colds. The evolution of molecular biology, and, in particular, nucleic acid hybridization, eventually saw the invention of highly sensitive detectors such as the polymerase chain reaction, which could be used to detect cold viruses through the presence of even very small amounts of their nucleic acids (Tyrrell & Fielder, 2002)

Scenario iv: In chemistry, an "indicator" is a natural or synthetic substance that changes color in response to the nature of its chemical environment. Indicators are used to provide information about the degree of acidity of a substance (its pH) or the state of some chemical reaction within a solution being tested or analyzed. One of the oldest indicators is litmus, a vegetable dye that turns red in acid solutions and blue in basic ones. Other indicators include alizarin, methyl red, and phenolphthalein, each one useful in the detection of acidity levels within a particular range or of certain types of chemical reaction.

Scenario v: Neutral particles such as neutrons and neutrinos can be detected via the nuclear reactions that occur when these particles collide with the nuclei of certain atoms. Slow moving neutrons produce easily detectable alpha particles when they collide with boron nuclei in borontrifluoride. Neutrinos, on the other hand, barely interact with matter, and are detected through the use of tanks of perchloroethylene (C2CI4, a fluid used in dry-cleaning). Neutrinos that happen to collide with chlorine nuclei produce radioactive argon nuclei. The perchloroethylene tank is flushed at regular intervals, and the newly formed argon atoms, present in minute amounts, are then counted. This type of neutrino detector, placed deep underground to shield it from cosmic radiation, is currently used to measure the neutrino flux from the sun.

Neutrino detectors may also take the form of scintillation counters, the tanks in this case filled with an organic liquid that emits light flashes when penetrated by the electrically charged particles that are produced by the interaction of neutrinos with the liquid's molecules.

Now, while the details of operation of particular detectors are hugely complicated, the logical principles of their operation are as follows:

Development of ψ-detector
Based on a combination of theory and knowledge about ψ-entities (i.e., entities signified by concept "ψ") it is known that, under the conditions that define detection scenario S, x will produce result $\{p_1,p_2,...,p_t\}$ if and only if x is a ψ-entity.

Application of ψ-detector:
If, when introduced into detection scenario S, particular entity z produces result $\{p_1,p_2,...,p_t\}$, i.e., $S(z)\rightarrow\{p_1,p_2,...,p_t\}$, the decision is made that z is a ψ-entity.

The creation of a tool of detection requires that some of the entries on the right side of $S(\psi)\rightarrow\{\_,\_,...,\_\}$ can be inferred. For example, in regard the use of a track detector situated within a magnetic field, theory and experience establish that the path of a non-neutrally charged particle will bend toward the magnetic pole opposite to its charge. The task of inferring some of the entities on the right side of $S(\psi)\rightarrow\{\_,\_,...,\_\}$ is a task for scientific, empirical investigation. However, such inferences can only be made given the satisfaction of a number of pre-empirical requirements, most essential among these the capacity to identify which entities are, and are not, ψ-entities. One must be able to distinguish ψ-entities from other types of entities whose detection is not of interest. To demonstrate, for example, that any member, x, of the class of ψ-entitities produces result $\{p_1,p_2,...,p_t\}$, under some set of background conditions, one must be able to *recognize* x, the entity producing $\{p_1,p_2,...,p_t\}$, *as* a ψ-entitity. *To be able to identify ψ-entities as such is nothing other than to grasp the correct employment of the concept "ψ" that signifies ψ-entities: ψ-entities are those entities to which the concept "ψ" can correctly be applied.* However, the correct employment of any concept is fixed by linguistic rules. Thus, to possess the capacity to identify ψ-entities, and distinguish these entities from other entities whose detection is not of interest, is to grasp the linguistic rules $r_{"\psi"}$ that fix the correct employment of concept "ψ". The linguistic rules $r_{"\psi"}$ settle the issue as to which entities concept "ψ" can, and cannot, be correctly applied.

Example.

Detection scenario, S: Bubble chamber situated within magnetic field, etc., etc.

Entities to be detected: Alpha particles (Definition: *alpha particle*. A positively charged nuclear particle, consisting of two protons bound to two neutrons).

If particle x is an alpha particle (an entity denoted by concept *alpha particle*, as defined above), then, according to theory and knowledge, the path that it will leave in the media of the bubble chamber will have the properties $\{p_1$=thick and dense, p_2=curvature towards negative pole of magnetic field,...,p_t}. That is, S(x)→{p_1=thick and dense, p_2=curvature towards negative pole of magnetic field,...,p_t}. Further details would typically be added, including properties characteristic of other types of particle that are *not* characteristic of an alpha particle, and, hence, that would be useful in eliminating as candidates other particle types.

The result, say {p_1, p_2,...,p_t}, produced by a particle y is *not* the result produced by an alpha particle if a failure to grasp the rules of application of the concept *alpha particle* has led to a misapplication of the concept *alpha particle* to y (say, if one is under the mistaken impression that the concept *alpha particle* is applied to a particle with one neutron). The result {p_1, p_2,...,p_t} is an empirical fact about a member of the class of alpha particles only if the concept *alpha particle* was correctly applied to y, the particle that produced {p_1, p_2,...,p_t}. Issues pertaining to the empirical behaviour of alpha particles are empirical issues, but the grounds for calling a given particle an alpha particle (i.e., the grounds of application of the concept *alpha particle*) is a grammatical issue. One must grasp the rule that "the concept *alpha particle* is applied to (denotes) positively charged nuclear particles consisting of two protons bound to two neutrons". In the absence of an understanding of the normative, rule-governed employment of the concept *alpha particle*, claims such as "this path was produced by an alpha particle" and "this experimental setup allows for the detection of alpha particles" would be no more intelligible than the claims "this track was produced by a grek particle" and "this experimental setup allows for the detection of galpha-5 particles".

The point of discussing scenarios (i)-(v) is not to suggest that what is found in the natural sciences should be copied in the social and behavioural sciences. In fact, the scenarios of detection found in the natural sciences may not be transferable, or be only partially transferable, to many areas of investigation within the social and behavioural sciences. The point is rather to note that in scenarios of detection, definitional issues are foundational, and that the natural sciences have engaged in hard-fought battles over the conceptual issues whose resolution is a precondition of fruitful empirical investigation. At the root of the development of any detector S of ψ-entities is the capacity to articulate the rules of correct employment of concept "ψ" that signifies ψ-entities. For in the absence of this capacity, one literally does not know what one is to detect, and, hence, cannot employ science to discover how ψ-entities behave within the proposed scenario of detection. There would, for example, be no possibility to detect alpha particles, nor, for that matter, of generating theory pertaining to alpha particles, in the absence of a definition of the concept *alpha particle*. For an understanding of how this concept is employed is, inter alia, an understanding of *which* constituents of natural reality are rightly called alpha particles and, hence, an understanding of *what* it is that is to be detected or theorized about.

Consider, now, what is involved in the derivation and employment of a particular latent variable model, say, model lvm:

<u>Derivation</u>

i. It can be shown that if a random variate $\boldsymbol{\theta}$ is statistically related to a second set of random variates $\mathbf{Z}_i$, i=1..p, by equations and distributional requirements that, in large part, define lvm, then the joint distribution of the $\mathbf{Z}_i$ possesses properties {c_1,c_2,...,c_r}. The possession of {c_1,c_2,...,c_r} by the joint distribution of the $\mathbf{Z}_i$ is equivalent to $\underline{\Omega}\subset M_{lvm}$, i.e., is equivalent to the $\mathbf{Z}_i$ being described by lvm;

Application

ii. The scores that belong to the distributions of each of a set of (manifest) variates $\mathbf{X}_j$, j=1..p, are produced by following rules of score production, $\{r_1,r_2,...,r_p\}$, known to the latent variable modeller prior to his analysis of the $\mathbf{X}_j$, j=1..p, by lvm. The data to be analyzed are, as a result, contained in N by p score matrix X, created by the application of the rules $\{r_1,r_2,...,r_p\}$ to a subset of the members of population P_T under study;

iii. Based on the data contained in X, the judgment is made as to whether $\underline{\Omega}_T \subset M_{lvm}$, i.e., whether the joint distribution of the $\mathbf{X}_j$, j=1..p, in population P_T, possesses properties $\{c_1,c_2,...,c_r\}$, i.e., whether $\underline{\mathbf{X}}$ is described by lvm;

iv. If the judgment is in the affirmative, this is taken as evidence in favor of one of the following:

> CAM: i. There has been detected a property/attribute common to the phenomena represented by the $\mathbf{X}_j$;
> ii. The concept *latent variate to* $\underline{\mathbf{X}}$ signifies this property/attribute;
> iii. The (unobservable) scores that comprise the distribution of random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ are measurements with respect this property/attribute;
> iv. The identity of this detected property/attribute is unknown, and must be inferred by "interpreting the latent variate."
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> CAC: i. There has been detected a cause of the phenomena represented by the $\mathbf{X}_j$;
> ii. The concept *latent variate to* $\underline{\mathbf{X}}$ signifies this cause;
> iii. The (unobservable) scores that comprise the distribution of random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ are measurements with respect this detected cause;
> iv. The identity of this detected cause is unknown, and must be inferred by "interpreting the latent variate."

Despite superficial similarities, what is involved in the employment of a latent variable model is not at all akin to what is involved in a proper scenario of detection such as those of (i)-(v). The portrayal of the latent variable model as a detector is illegitimate, for the derivation and employment of a latent variable model lack key ingredients necessary for it to play this role. The creator of a tool of detection is in possession of a concept "γ" that denotes members of the class of γ-things that are to be detected. This means not merely that he can utter the concept name γ, but that he grasps the linguistic rules that fix the correct employment of "γ". For he must actually know what he is, and is not, trying to detect, and this is equivalent to knowing to which constituents of natural reality concept "γ" is, and is not, correctly applied. Knowing to which constituents of natural reality concept "γ" is correctly applied is to grasp a feature of the meaning of concept "γ". This conceptual mastery is required because, to construct a detector of γ-things, the creator of such a tool must possess knowledge and theory about how γ-things do, in fact, behave under the conditions of the detection scenario. He must come to know that, under the conditions of the detection scenario, γ-things behave as $\{p_1,p_2,...,p_t\}$. And to generate such knowledge in regard the behaviour of γ-things requires that he can *identify* an entity x *as* a γ-thing, when, in fact, it *is* a γ-thing. But to be able to identify an x *as* a γ-thing, and distinguish γ-things from entities that are not γ-things, is no more than to grasp the linguistic rules that fix the correct employment of concept "γ", for these rules fix to which constituents of natural reality application of concept "γ" is warranted:

That entity x just produced, under the conditions of the detection scenario, the result $\{p_1,p_2,...,p_t\}$. The rules of application of "γ" warrant application of "γ" to x, i.e., x is, in fact, a γ-thing. Hence, a γ-thing just produced $\{p_1,p_2,...,p_t\}$

Within the context of a latent variable analysis, the latent variable modeller *does* possess rules, $\{r_1,r_2,...,r_p\}$, for the production of the scores that comprise the distributions of the manifest variates, $\mathbf{X}_j$, j=1..p. Disregarding any other claims that he wishes to make about these scores (e.g., that they are measurements of some particular property), antecedent knowledge of rules of score production fixes the meanings of the symbols $\mathbf{X}_j$, j=1..p, that appear in the equations of lvm: The symbols $\mathbf{X}_j$, j=1..p, stand for idealized populations of scores that would be produced by applying the rules $\{r_1,r_2,...,r_p\}$ to the members of population P_T under study. The antecedent laying down of such rules is what justifies, in certain cases, the claim that the manifest variates represent phenomena, e.g., behavioural phenomena, of interest. However, *neither the creator, nor the applier, of the latent variable model possesses a concept, i.e., is able to cite rules that fix the correct employment of a concept, that denotes some particular constituent of natural reality (a property, behavioural phenomenon, material (e.g., biological) entity, etc.) that is to be detected.* He specifies nothing *to* detect, and, therefore, cannot, and does not, proceed to establish scientifically that some particular type of constituent of natural reality has certain effects on the phenomena represented by the $\mathbf{X}_j$, these effects encoded as properties $\{c_1,c_2,...,c_r\}$ claimed by his latent variable model to be properties of the joint distribution of the $\mathbf{X}_j$. The portrayal of the latent variable model as a detector of *anything*, let alone the Central Account's portrayal of the latent variable model as a detector of properties/attributes (causes) of the phenomena represented by sets of manifest variates, is wholly illegitimate.

The latent variable modeller *is* in possession of a concept *latent variate to* $\underline{\mathbf{X}}$, whose rule of employment is of the form:

Definition: *latent variate to* $\underline{\mathbf{X}}$: A random *variate* $\boldsymbol{\theta}$ possessing the (model specified) properties $\{p_1,p_2,\ldots,p_t\}$, e.g., p_1: $E(\boldsymbol{\theta})=0$, p_2: $V(\boldsymbol{\theta})=1$, and p_3: [conditional on $\boldsymbol{\theta}$, the manifest variates contained in $\underline{\mathbf{X}}$ are statistically independent].

Unlike behavioural phenomena, properties of substances (e.g., radioactivity), and material entities (e.g., alpha particles), random variates are not constituents of natural reality. They are not "out there" awaiting detection. They are, rather, functions that are *created* by humans for certain purposes. And, indeed, the creation of random variates is precisely what is involved in the employment of a latent variable model: A given latent variable model is a specification of the properties that a random variate must possess in order to be a latent variate to $\underline{\mathbf{X}}$. To correctly claim that a given $\underline{\mathbf{X}}$ is described by a given latent variable model is to correctly claim that there can be constructed at least one such latent variate to $\underline{\mathbf{X}}$, i.e., at least one random variate satisfying the properties required by the model for latent variate-hood. Following analysis, the issue arises as to the means by which to create such latent variates, an issue that involves the derivation of a construction formula.

The Central Account's portrayal of the latent variable model as a detector of properties/attributes (causes) of the phenomena represented by sets of manifest variates, and the concept *latent variate to* ***X*** as signifying a property/attribute (cause) of the phenomena represented by the manifest variates contained in **X**, is a mythology. And this is obvious from a consideration of what is absent from the context in which latent variable models are created and employed. In the absence of a means to identify members of a class of constituents of natural reality to be detected, the latent variable modeller cannot possess knowledge and theory about any such entities that *could* be used in the construction of a tool of detection. It is, then, not surprising that, in contrast to the creator of a proper tool of detection, the latent variable modeller cannot, and does not, state what *kind* of thing he is trying to detect (is it, e.g., a material entity or a force?), nor the *medium* in which it resides (e.g., air, water, blood plasma, atoms of some element, brain tissue), nor the spatio/temporal *location* at which it is likely to be found (but, then, perhaps he believes that a workable answer is that it is located in an unobservable "domain" or "realm"), nor the mechanism by which it is supposed to bring about the properties $\{c_1,c_2,...,c_r\}$ of the distribution of the $\mathbf{X}_j$. Theory predicts that a sub-atomic particle, a material entity, will produce tracks through particular types of media. Scientific practice bears this out. Material entities *do things* to other material entities. In contrast, latent variable analyses are not about constituents of natural reality, denoted by the concept *latent variate to* ***X***, that could potentially be studied, theorized about, and, gradually understood, but, rather, the post-analysis creation of random variates possessing of properties required to warrant their signification by concept *latent variate to* ***X***.

If, as claimed in the Central Account, the latent variable modeller *were* truly in the business of detecting the properties/attributes (causes) of the phenomena represented by sets of manifest variates, he would begin his investigations by laying down rules that fixed the sense of the concept *property/attribute (cause) of the phenomena represented by the manifest variates*, and these rules would make clear the constituents of natural reality, say φ-entities, that this concept signified, and were the constituents to be detected. He would then proceed to establish scientifically that these constituents have certain effects on the phenomena represented by the $\mathbf{X}_j$, these effects encoded as properties $\{c_1,c_2,...,c_r\}$ of the joint distribution of the $\mathbf{X}_j$. But, of course, if this *were* his program, *he would have no need to employ a latent variable model*. For, instead of concocting pseudo-cases about "latent variates", he would state that φ-entities (entities signified by concept *property/attribute (cause) of the phenomena represented by the manifest variates, X_j, j=1..p*, this concept defined as such and such...) do such and such to the phenomena represented by the manifest variates. Given that he actually had a class of entities to investigate, he would, as do scientists, conduct research into the natures of φ-entities. He would develop theory about φ-entities, from which predictions could be generated.

Let it be emphasized that this is not a matter of possible maturational differences between the theories and practices of the natural sciences, and those of the social and behavioural sciences. The latent variable modeller does not begin with rules of employment for a concept that signifies the members of a class of entities to be detected. He thus lacks the *capacity* to identify a class of constituents of natural reality whose detection might be of interest. And without any such a class of constituents, he has nothing about which to theorize. The Central Account, which convinces him otherwise, is mere mythology. The derivation of a latent variable model begins, not with a statement (theory) about the relationships between various types of natural phenomena, but with a statement about relationships amongst *variates*: If a random variate, $\boldsymbol{\theta}$, has certain joint moment relations with a second set of variates, $\mathbf{Z}_i$, i=1..p, then such and such is the case. The claim that "**θ** renders conditionally uncorrelated the $\mathbf{Z}_i$" is not of the same *kind* of claim as "a *sub-atomic particle* (definition) will behave as $\{p_1,p_2,...,p_t\}$ under conditions $\{c_1,c_2,...,p_q\}$ in a bubble chamber". *The latent variable model is not a tool of detection, but, rather, a tool for the production of random variates with prescribed properties*, a case that will be developed in Part 3 of the book.

That latent variable models cannot function as tools of detection should be of no surprise, since the very reason Spearman invented factor analysis was to side-step the difficult task of clarifying the grounds of application for the concept *intelligence* (its cognates and related terms), the concept that denoted the phenomena of interest to him. Popular lore propogated within psychometrics has Spearman providing a "quantitative definition" for *general intelligence*. But, unlike, for example, Mach's definition of *mass*, in which the meanings of the terms on which the definition is based are carefully explained, Spearman did not bother to clarify the meanings of the terms of his "quantitative definition." Rather than wrestle with the explication of the correct employments of the concepts that informed his work, a common-place activity in other sciences, he passed the buck to a set of equations, the equations of his factor "model". As with virtually all of his successors, he failed to comprehend both the nature of psychological concepts and the role of definition within science. In particular, Spearman did not provide rules of employment for a concept "γ" that signified γ-things (some material entity, force, property, or natural phenomenon) that he then studied, eventually coming to an understanding that, in certain media, γ-things had particular effects on the phenomena of interest to him (performance on various intellectual tasks). He did not work out, even approximately, the rules of employment for the concept *general intelligence*, nor did he explain in what sense such a concept might denote constituents of natural reality. His vague allusions to mental energy notwithstanding, he did not explain whether, if the concept *general intelligence* denoted, its referents were material entities, forces, or properties (recall E.B. Wilson's original concerns). If they were material entities, in what medium were they to be found? If he believed them to be located within the bodies of humans, as some have suggested, there were already in existence theory and procedures for the detection of material entities residing in human bodies. If this had been his belief, there would have been no need for the invention of the latent variable model.

Spearman at times implied that his aim was to *measure* general intelligence, but without rules of employment for the concept *general intelligence*, this claim is simply vacuous. In the absence of such rules for the employment of the concept *general intelligence*, Spearman possessed no grounds for application of the concept to any constituents of natural reality. He, therefore, could not, and did not, produce a theory about the nature and action of any referents of the concept *general intelligence*, and certainly did not develop, in his linear factor model, a detector for the presence/existence of any such alleged referents. Nor, then, did the equations of Spearman's model represent what he *knew* about the relationship between the phenomena of which he had an interest, and a (p+1)th constituent of natural reality, general intelligence.[3] In truth, it was never clear what the results that he generated were *about*, a point made incisively by Wilson in his original reviews of Spearman's work.

2. *Affirming the consequent*

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3 As will later be discussed, latent variable "models" are not models in the classical sense of representers of known states of affairs.

Latent variable modellers are well versed in the dangers of the logical fallacy of affirming the consequent. They might, therefore, find it tempting to interpret the case presented herein, that latent variable models cannot be used as detectors, as simply a particular realization of this fallacy: "yes, it is wrong to conclude, based on the fact that a set of variates is undimensional in a linear factor analytic sense, that an attribute (cause) of the phenomena represented by these variates has been detected, because this would be affirming the consequent. One can only *disprove* the existence of such an attribute (cause) when the variates are *not* unidimensional." In fact, the fallacy of affirming the consequent is wholly unrelated to the case herein argued. Recall that the logical fallacy of affirming the consequent results when an argument of the following structure is made:

if p, then q
q
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therefore p

This is a fallacy because, when both q and the material implication are true, one can consistently assert both p and ~p. Given a true material implication, it is, therefore, fallacious to conclude p from the truth of q.

However, the fundamental material implication of linear factor analysis does not take as p, "there exists an attribute (cause) of the phenomena represented by the manifest variates", and as q, "the manifest variates are unidimensional in a linear factor analytic sense." If this *were* the type of material implication in play, and it were true, then it would be called a law. Note that such laws can be generated only given the usual requirements: i) the capacity to identify in nature the relata of the law (a conceptual capacity), in this case either attributes or causes of the phenomena represented by particular sets of manifest variates; ii) intensive empirical investigation of the relata (the capacity to investigate the relata presupposing the existence of each of the relata), this yielding knowledge of their properties, locations, relationships, etc. If material implications of this sort *were* known to be true, then, of course, they would have nothing to do with latent variable modelling: p would not be a set of prescribed characteristics of a random (latent) variate, but, rather, some existing constituent of natural reality. That is, to establish laws relating the "attributes" or "causes" of the phenomena represented by the manifest variates to the dimensionality of these variates would be the product of ordinary scientific research into such attributes and causes, not latent variable modelling. However, within the context of latent variable modelling, there does not even exist the possibility of working toward the formulation of such laws, because there have not been layed down rules that fix the correct employments of the terms *attribute* (*cause*) *of the phenomena represented by the manifest variates*. As a result, it is wholly unspecified *what* phenomena should be the objects of study, and science cannot proceed to study features of natural reality that it cannot identify.

The material implications found within latent variable modelling are of the type exemplified by linear factor analysis. The fundamental material implication of (unidimensional) linear factor analysis takes as p,

"there exists a continuous random variate $\boldsymbol{\theta}$ with $E(\boldsymbol{\theta})=0$, $V(\boldsymbol{\theta})=E\boldsymbol{\theta}^2=1$, and such that the p residuals, $\underline{\mathbf{l}}$, of the linear regressions of the manifest variates, $\mathbf{X}_j$, j=1..p, on $\boldsymbol{\theta}$ have a covariance matrix that is diagonal and positive definite", and, as q:

"the covariance matrix of the manifest variates is $\Sigma=\underline{\Lambda\Lambda}'+\Psi$, in which Ψ is diagonal and positive definite"

The false equating of a random variate, $\boldsymbol{\theta}$, with some particular constituent of natural reality, e.g., attributes or causes, is at the very root of the Central Account. True scenarios of detection require the establishment of law-like material implications relating two constituents of natural reality: If an individual hears sound t then the shape of the resulting brain wave is f; If the particle passing through a bubble chamber is of type t, then it leaves a track of type s. True, discovered if-then material implications involving classes of natural phenomena p and q are indeed the input into the development of "litmus tests" for the presence of p. But the place of p in linear factor analysis is not taken by a natural phenomenon that has been shown to be related to some q in accord with the relation of material implication, but, rather, by a random variate. Thus, the fundamental if-then relations of latent variable models are not laws relating constituents of natural reality, but, rather, defining *recipes* for the construction of random variates that can rightly be called latent variates to some particular set of manifest variates (the case taken up in Part 3 of the book).

3. *Variations on the theme*

The key Central Account claim that a latent variable model is a tool of detection can be dressed up in many different sets of clothes. The superficially different appearances of certain of these instantiations of this claim can give the appearance that at least some of these alternative statements escape indictment by the arguments offered to this point. To complete the case, several of these variations on the core theme are considered individually.

i. Over the years, experts in latent variable modeling have implied that the application of these models can be used to detect "...quantitative aptitude, verbal aptitude, mathematical ability, subject-matter knowledge in psychology..perseverence and creativity" (Lord & Novick, 1968, p.537), traits and abilities of various other sorts, and a wide variety of other psychological phenomena, not to mention phenomena arising in other disciplines. Perhaps then the latent variable model is an open-ended tool of observation that can be used in a manner analogous to the way an astronomer uses a telescope. The astronomer can train his telescope on a segment of space, and report, in an observation language particular to his trade, on the astronomical objects he sees. In analogous fashion, perhaps, when the latent variable modeller employs latent variable model lvm, and $\underline{\Omega}_T\subset M_{lvm}$, *some* "psychological object" has been discovered. When the latent variable modeller then "interprets the latent variate", he is perhaps describing this discovery in an observation language particular to *his* trade. The latent variable model is, as it were, a telescope for psychological "entities", these entities residing within an unobservable "domain" (just as planets reside in space).

Consider a context in which such open-ended discovery can occur. Under a particular set of background conditions, the physicist collides electrons with protons and "observes", through the use of a track detector, what he comes up with, and, in particular, whether he has produced any new types of sub-atomic particle. Notice what is involved.

a. The physicist knows, at least roughly, the kinds of thing that he is observing and, in particular, can justify the claim that he is looking at particles (Definition: *particle*. {...}), that what will be produced in his experiment are known and, possibly, currently unknown (undiscovered), types of particle. He grasps the rules of correct employment of the concept *particle*, and so long as he has employed the term correctly in making such claims, he is, in fact, making claims about particles (claims which may yet turn out to be *factually* incorrect). The physicist has, at his disposal, a language of observation appropriate for the description of his discoveries. The entire context of observation is saturated with concepts whose employments the physicist has come to master. Physicists discovered mesons while studying cosmic rays. Mesons were recognized as constituting a new discovery because no concept in use at the time was applicable to these particles. But, certainly, the newly discovered particle type was recognizable as a material entity (Definition: {...}), and, in particular, a sub-atomic particle (Definition: {...}) containing one quark (Definition: {...}) and one antiquark (Definition: {...}). It was not, then, for example, a baryon (i.e., the concept *baryon* {Definition: *baryon*. Sub-atomic particle that contains three quarks} could not correctly be applied to these newly discovered particles). The existence of mesons *could* have been hypothesized in advance of their discovery because physicists were in possession of the required conceptual ingredients to formulate such an hypothesis. The point is that in the search for new sub-atomic particles, there exists a language that can be legitimately employed to conceptualize what has been, and might be, found.[4]

b. The physicist can explain that the class of entities, particles, about which he seeks to make discoveries, are material entities. A material entity occupies space and is characterized by a gravity and inertia. Since a material entity occupies space, it has a size, spatio/temporal location, etc., and these properties determine, in part, senses in which it is observable and unobservable, and the ways in which it *can* be observed. The physicist can state *where* the sub-atomic particles of interest to him are to be found, and, hence, the rough spatial coordinates on which his observations should be trained. A difficulty he faces in his study of sub-atomic particles is that they are very small and cannot be observed with the naked eye. These considerations are central to his creation of his tools of detection (e.g., track detectors), tools which are created to detect specific types of constituents of natural reality. The way in which his observational setup functions he knows through theory and fact, painstakingly developed within his discipline, *about* sub-atomic particles.

In contrast, the latent variable modeller does not have at his disposal an observation language, the concepts contained therein denoting constituents of natural reality that might be revealed in the employment of a latent variable model. He speaks of "properties/attributes (causes) of the phenomena represented by the manifest variates", but does not provide rules that fix the correct employment of the concept *property/attribute (cause) of the phenomena represented by the manifest variates*, $\boldsymbol{X}_j$, $j=1..p$. In his hands, the concept literally has no sense, and, thus, certainly does not signify particular constituents of natural reality. The same unclarity

4 This is not to say that there are never discovered features of natural reality that threaten to defy conceptualization, but only that physics and other areas of science possess extensive and legitimate observation languages in which discoveries can be described.

attends the other terms, e.g., "...quantitative aptitude, verbal aptitude, mathematical ability, subject-matter knowledge in psychology..perseverence and creativity", which he attempts to name his alleged discoveries. The latent variable modeller cannot say what *kind* of thing he is interested in detecting, nor *where* it might be located. In detecting a stellar body, the astronomer observes (visually) a, heretofore unknown, material entity. He can thus employ visual perceptual predicates (brightness, colour terms, spatial organizational terms) to describe what he sees. In latent variable modeling, there exists no "stage of discovery", only the make-believe one created through the invokation of the Central Account. This is why the results of latent variable analyses are claims expressed in terms of *variates* ("the latent variate was judged to be agreeableness"), with the Central Account invoked to "explain" to the reader what has been found (to wit, something unobservable, perhaps located in a "latent domain"), rather than in terms of features of natural reality. Latent variable models were simply not *designed* to be open-ended tools of discovery. It was never the case, for example, that: a) There was a realization, at some point in the history of the behavioural sciences, that there existed a domain of entities that were perceptually unobservable; b) A diagnosis was made as to why these entities were, in fact, perceptually unobservable; c) There was a need to make observable the entities contained within this domain because of their scientific importance; d) The latent variable model was developed as a tool of observation to overcome the diagnosed brand of unobservability.

The physicist can describe his discoveries in terms of a legitimate observation language: "*This* sub-atomic particle contains one quark (definition:___) and one antiquark (definition:___)." His tools of detection and observation are *designed* to facilitate the detection and observation of the constituents of natural reality signified by these concepts. In contrast, the latent variable modeller simply chooses an ordinary language concept label on the basis of estimated parameter values, and attaches this label to his alleged discovery, calling this practice "latent variate interpretation". This without ever having explained what *kind* of thing he believes to have detected (is it a material entity?), where, in natural reality, it is located, what brand of unobservability explains why it can't be observed, and how it is that he is justified in applying an ordinary language concept in this manner. In fact, as will later be argued, his belief that it makes *sense* to attempt to ascribe psychological predicates to alleged unobservable "entities", on the basis of a set of parameter estimates, betrays profound misunderstandings of the nature of concept meaning and signification.

ii. When $\underline{\Omega}_T \subset M_{lvm}$, the scores that comprise the distribution of the random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ are measurements with respect to *some* psychological property/attribute common to the phenomena represented by the $\mathbf{X}_j$, j=1..p. The identity of this property/attribute must be inferred and this is the task undertaken in "latent variate interpretation." Due to the unobservability of these measurements, one must estimate them with a function, t($\underline{\mathbf{X}}$), of the $\mathbf{X}_j$, and this means that, in effect, t($\underline{\mathbf{X}}$) is an error laden measure of the unobservable property/attribute.

This version of the CAM perhaps appears somewhat different than the basic CAM, but, in all essential details, it is not. The claim inherent to this version still portrays the employment of a latent variable model as a case of the if-then reasoning characteristic of a scenario of detection: If $\underline{\Omega}_T \subset M_{lvm}$, then the scores that comprise the distribution of the random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ are measurements with respect to *some* psychological property/attribute common to the phenomena represented by the $\mathbf{X}_j$, j=1..p. The justification for this interpretation of the employment of lvm would then be of the same form as that required of any scenario of detection.

In particular, to justify the claim that "these scores are measurements of property γ common to the phenomena represented by the $\mathbf{X}_j$, j=1..p", the latent variable modeller would have to establish that the rules that fix the correct employment of concept "γ" warrant application of "γ" to these scores. But the latent variable modeller does not make any such case. He offers up no conceptual argument to the effect that the rules that fix the correct employment of the concept "γ" that he employs, warrant the employment of "γ" as he employs it. Nor does he offer up a rule for the production of the scores that comprise the distribution of $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$, i.e., he does not know how these scores come about. In fact, on his *own* account, such scores are not observable, and, hence, are not created in a manner that *could* be known to him. Hence, his claim of conceptual signification, that "these scores are measurements of property γ common to the phenomena represented by the $\mathbf{X}_j$, j=1..p", is vacuous.

McDonald's abstractive property interpretation of latent variable modeling is a variant of this version of the CAM, with the further stipulation that the measurements with respect the detected property/attribute, i.e., the scores that comprise the distribution of random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$, are a probability limit defined in a behaviour domain. It will be argued in Chapter XV, however, that while this treatment can be considered a possible solution to the indeterminacy problems inherent to latent variable representations, it in no way justifies the Central Account that is presupposed in McDonald's treatment. In particular, employing a behaviour domain foundation of latent variable modeling does not justify the CAM, nor the liberties McDonald takes with the logic of conceptual signification in tagging the abstractive properties he claims can be detected, with ordinary language concept names.

The latent variable modeller might believe that (ii) can be saved by claiming that it is merely an *hypothesis* or *assumption* that the scores that comprise the distribution of random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ are measurements with respect to *some* particular psychological property/attribute common to the phenomena represented by the $\mathbf{X}_j$, j=1..p, and acknowledging that one can never be sure whether this hypothesis/assumption is true. He might even insist that when he engages in a latent variable analysis he constructs his variates to be measures of a particular property, and so "has a pretty good idea as to what is likely to turn up in his analysis." He might even console his readership by noting that uncertainties abound in science and this is just such an example. As was seen in Chapter V, this is the kind of argument of which Mulaik is fond. However, this line of reasoning is equally mistaken. Whether something is signified by some particular concept is not an empirical issue, and, hence, cannot be supported inductively through the accumulation of evidence. It is not the subject of empirical hypothesis. One supports the claim that θ is signified by concept "θ", whose correct employment is fixed by rules $r_{"\theta"}$, by demonstrating that, in fact, $r_{"\theta"}$ warrants application of "θ" to θ. This presupposes a grasp of $r_{"\theta"}$, for it requires the citing of relevant aspects of these rules, to wit, those that establish that "θ" can be applied to θ. The case that must be made is grammatical, not empirical. Scientific cases about empirical subject matter are of a different nature, and are characterized by entirely different types of support and uncertainty.

iii. If the scores that comprise the distributions of the variates, $\mathbf{X}_j$, j=1..p, are all signified by the same concept, say, "γ", then, if $\underline{\Omega}_T \subset M_{lvm}$, the scores that comprise the distribution of the random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ are also signified by "γ", and, hence, are measurements with respect concept "γ". Due to the unobservability of these measurements, one must estimate them with a function, t($\underline{\mathbf{X}}$), of the $\mathbf{X}_j$, and this means that, in effect, t($\underline{\mathbf{X}}$) is an error laden measure of the unobservable property/attribute.

The scores that comprise the distributions of the variates $\mathbf{X}_j$, j=1..p, are produced in accord with rules of score production, $\{r_1,r_2,...,r_p\}$, known by the latent variable modeller prior to the running of the latent variable analysis for which they are input. This is why the data to be analyzed are a sample of realizations on these variates. Now, imagine that it were the case that the scores on each variate, $\mathbf{X}_j$, j=1..p, happened to be signified by the same concept, "γ". That is to say, the X_1-scores (the scores that comprise the distribution of $\mathbf{X}_1$) were, in fact, γ-scores, so too were the X_2-scores, etc., etc.[5] Also, imagine that $\underline{\mathbf{X}}$ happened to be described by latent variable model lvm. The claim inherent to alternative (iii), then, would be that the scores that comprise the distribution of random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ are also signified by concept "γ". For example, if a set of variates, $\mathbf{X}_1$ to $\mathbf{X}_{10}$, were seen as various "dominance indicators", and happened to be ulcf representable, then the claim would be that the scores that comprise the distribution of random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ were also signified by the concept *dominance*. The belief inherent to (iii) underpinned Spearman's work, he taking the variates on which his work centered to be "measures of intellectual functioning", and the scores that comprised the distribution of $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ to be *general intelligence*(*g*)-scores. And it has been part of the fabric of latent variable modeling ever since. McDonald and Mulaik (1979, p.305), for example, state that "If the variables have been selected on the basis of certain common attributes defined in advance, there is no uncertainty as to the interpretation of a factor found common to those variables..."

Somehow, it just sounds more "likely" that the scores that comprise the distribution of $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ will turn out to be scores signified by concept "γ", if the scores that comprise the distributions of each of the manifest variates are signified by concept "γ". But while it might sound so, it is not so. No case has ever been put forward to support (iii), and no case *could* be put forward. For, despite the naive appeal of (iii), it has no basis in logic. The only issue that matters, here, is whether the scores that comprise the distribution of random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ are, in fact, signified by concept "γ". To justify the claim that they are, the latent variable modeller would, in the first place, have to be able to articulate the rules that fix the correct employment of concept "γ". But the latent variable modeller is in no position to do so. Under the influence of the CA, his analyses involve no such conceptual clarification, he wrongly believing that what is meant by "γ" can be sorted out through more latent variable modeling. But even if he did possess an understanding of the rules of correct employment of concept "γ", he would still, as in version (ii), have to establish that these rules warranted application of "γ" to the scores that comprise the distribution of random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$. And he cannot do this, because he possesses no rule for the production of these scores, and so cannot say how they came about. He believes, in fact, that these scores are unobservable.

iv. A particular latent variable model, lvm, is a "defining theory" for an hypothesized entity, φ, signified by theoretical concept *latent variate to* $\underline{\mathbf{X}}$. If it exists, φ is perceptually unavailable, but detectable through its causal effects on the phenomena represented by the $\mathbf{X}_j$, these effects encoded as properties $\{c_1,c_2,...,c_r\}$ in the joint distribution of the $\mathbf{X}_j$ (the possession of these properties being equivalent to $\underline{\Omega}_T \subset M_{lvm}$). To test $\underline{\Omega}_T \subset M_{lvm}$ is to test the hypothesis that φ exists.

5 *How* such signification is brought about has not been grasped by psychometricians judging by the existence of the practice of factor interpretation, in which an ordinary language concept-name is simply attached to the scores of a variate. The fundamental confusion over conceptual signification is handled as a separate topic later in the chapter.

This idea has found its way into latent variable modeling via empirical realist philosophy of science, and its popularization in such papers as Cronbach and Meehl's 1955 installment on construct validity. As will be recalled from Chapter III, the empirical realist claims that a theoretical term, e.g., *electron*, *phlogiston*, *neutrino*, is a term that is introduced by a scientific theory. Such a term designates a putative unobservable cause of observed phenomena, and, hence, cannot be defined in terms of observational terms. Theoretical terms are open concepts that "...get their meanings from the data-language contexts in which they are used [but] semantically *designate*...causal features of natural reality generally concealed from perception but knowable through their data consequences" (Rozeboom, 1984, p.212) The empirical realist claims that,

> "...one or more theoretical terms 'τ_1',...,'τ_n' are implicitly defined by a theory '$T(o_1,...,o_m,'\tau_1',...,'\tau_n')$' in which are conjoined all the sentences we take to be criterially true of the τ_i, including in particular statements telling how these are related to observational entities $o_1,...,o_m$. Substituting placeholders for all the terms introduced by the theory leaves us with a complex sentence schema '$T(o_1,...,o_m,\_,...,\_)$' that constitutes the meaning of the terms 'τ_1',...,'τ_n' by specifying their conceptual roles. But for theory '$T(o_1,...,o_m,'\tau_1',...,'\tau_n')$' to be *true*, '$\tau_1$',...,'$\tau_n$' must designate an *n*-tuple of entities that satisfy *n*-adic predicate '$T(o_1,...,o_m,\_,...,\_)$'. Empirical realism insists that so long as reality provides such entities this theory establishes them as the referents of its open terms and thereby does indeed attain full-blooded semantic truth...empirical realism's claim is that theoretically (implicitly) defined concepts semantically function much like denotative descriptions whereby we fashion reference to a not necessarily observable entity *e* out of our conception of *e*'s salient attributes: Theoretical terms are *about* whatever features of the world have the observationally describable character that their defining theory says they have." (Rozeboom, 1984, p.212).

Exporting this thinking to the domain of latent variable modeling, one takes:

a) The term *latent variate to* $\underline{\mathbf{X}}$ to be a theoretical term which denotes an unobservable entity, φ, hypothesized to exit;
b) φ to have causal effects on the phenomena represented by the $\mathbf{X}_j$, these effects encoded as properties $\{c_1,c_2,...,c_r\}$ in the joint distribution of the $\mathbf{X}_j$ (and equivalent to $\underline{\Omega}\subset M_{lvm}$);
c) The equations and distributional specifications that constitute lvm to be part of a theory about φ and its relationship with the observable phenomena represented by the $\mathbf{X}_j$.

Then, if, for given P_T and $\underline{\mathbf{X}}$, it happens to be the case that $\underline{\Omega}_T\subset M_{lvm}$, the part of the theory represented by lvm is, provisionally, taken to be true, φ has been shown, provisionally, to exist, and the term *latent variate to* $\underline{\mathbf{X}}$ is taken as semantically designating the existing, but unobservable cause φ.

Variation (iv) is really no more than a standard scenario of detection dressed up in fancy clothing, and, hence, is not a proper characterization of the role of the latent variable model in science. Now, there certainly do exist within science scenarios of the type described by the empirical realist. To see that the employment of a latent variable model is *not* among these, consider two such scenarios.

Scenario i. With regard the population of humans, let there be a set of "observables" **c**:{c_1=nasal congestion,c_2=nasal discharge,c_3=sore throat,c_4=persistent coughing}. Let there be a theory, T, in which is introduced a theoretical term "σ" which designates what are believed to be existing, but currently undiscovered, members of a class of material entities, σ. T claims that σs, if they exist, are causally responsible for **c**, and are additionally: i) acutely infectious; ii) affect the upper respiratory tract, and, in particular, the mucous membranes of the nose and throat; iii) not allergens or toxins; iv) produce no fever; v) produce **c** for a relatively short duration (roughly 7 days). The theoretical term "σ" designates material entities that, if they exist, i.e., if theory T(c_1,...,c_m,"σ") is *true*, have empirical natures that include the properties claimed for them by T. Something along these lines might have been the defining theory for the class of entities known as common cold viruses, prior to their discovery. However, the development of the theory of viruses, in general, and of the class of viruses called the common cold viruses, in particular, was complicated and circuitous (see, e.g., Fielder, 2002). Many problems had to be addressed in order for scientists to come to grips with the nature of the hypothesized causal agent σ. For example, σ had to be distinguished from distinct agents capable of generating effects similar to **c** and, as it turned out, was, until the development of highly sophisticated observational aids, truly perceptually unavailable due to its very small size. A rough chronology would include the following important developments:

By the late 1700s: The microbial world was known to consist of protozoa, fungi, and bacteria, all visible with a light microscope.

1840: The idea of living, microscopic, infectious agents was unpopular (Fielder, 2002). Jacob Henle posited that there existed infectious agents too small to be seen with a light microscope, and set out postulates according to which, in his opinion, the existence of these agents could be proven.

1850-: Louis Pasteur developes general theory of germs, invents notion of a vaccine, and creates vaccines against vaccines for sheep anthrax and chiken cholera. Worked to develop a vaccine for rabies in the 1880s, but conceptualized his work in terms of the concept *virus*, latin for poison.

Late 1800s: The idea that microbial organisms produce new organisms discredited the previous notion of spontaneous generation. Gradually, science came to see that it was likely that σs were likely capable of producing new versions of themselves.

Robert Koch, a student of Henle, and the British surgeon Joseph Lister developed techniques for growing cultures of single microbial organisms that aided in the assignment of responsibility for specific diseases to specific bacteria types.

1880: Adolf Mayer engineered the first experimental transmission of a viral infection when he demonstrated that extracts from infected tobacco leaves could be used to produce tobacco mosaic disease in a new plant. However, Mayer was unaware that he was dealing with a viral agent. Because he was unable to isolate a bacterium or fungus from the tobacco leaf extracts, he considered the idea that tobacco mosaic disease might be caused by a soluble agent, eventually concluding (incorrectly) that a new type of bacteria was likely to be the cause.

1892: The Russian scientist Dimitri Ivanofsky discovered that the tobacco mosaic causal agent was small enough to pass through a porcelain filter known to block the passage of bacteria. He too failed to isolate bacteria or fungi from the filtered material, and concluded in 1903 that the filter might be defective or that the disease agent was a toxin rather than a reproducing microbial organism.

1898: Also using porcelain filters, Martinus Beijerinck demonstrated that the filtered material was not a toxin because it could grow and reproduce in the cells of the plant tissues. Beijerinck referred to this new disease agent as a contagious living liquid—contagium vivum fluid—initiating a 20-year controversy over whether viruses were liquids or particles.

1898: Viruses were refered to as "filterable agents". Hence, Friedrich August Johannes Löffler and Paul F. Frosch (both trained by Robert Koch) described foot-and-mouth disease virus as "the first filterable agent of animals".

1900: Walter Reed and colleagues described yellow fever virus as "the first human filterable agent". For several decades viruses were referred to as filterable agents, but gradually the term virus came to be employed to designate this new class of infectious agents.

1917: A major difficulty science faced in coming to understand what it was dealing with in viral agents arose from the fact that viruses are twenty to one-hundred times smaller than bacteria, and so cannot be seen with the aid of light microscopy. That viruses were material particles was deduced on the basis of several observations. Félix d'Hérelle discovered that viruses of bacteria, which he named bacteriophages, could make holes in a culture of bacteria. Because each hole, or plaque, developed from a single bacteriophage, this fact provided the first method for counting viruses (the plaque assay). Then, in 1935, Wendell Meredith Stanley succeeded in crystallizing tobacco mosaic virus to demonstrate that viruses had regular shapes. Finally, in 1939, the tobacco mosaic virus was rendered observable with the aid of the electron microscope.

1940s and 1950s: Because of the ease with which bacteria could be grown in the laboratory, many discoveries were made about the natures of viruses through the study of bacteriophages. In a number of different laboratories, cold research involved the taking of secretions from the noses of individuals suffering from colds, treating these secretions, and introducing these treated secretions into the noses of volunteers, to see if they caught colds. Attempts were made to culture the entities believed responsible for colds on the model of bacteria, the result being a longterm string of failures (Fielder, 2002).

Cold viruses rendered observable with aid of electron microscope.

mid 1900s: Scientists came to realize that the sought-after causal agent σ of **c** was actually a class of distinct classes of agents. It came to be realized that these agents, unlike bacteria, would only grow in the presence of living cells.

1948 to 1955: Scientists at the National Institutes of Health (NIH) and at Johns Hopkins Medical Institutions revolutionized the study of animal viruses by developing cell culture systems that permitted the growth and study of many animal viruses in laboratory dishes.

1960: Advances in biochemistry allowed for discoveries to be made about the structure and methods of growth of viruses (Fielder, 2002).

1985: Researchers using advanced X-ray crystallography techniques produced a three-dimensional, atomic scale model of one of the most common cold viruses.

Scenario ii. Roughly the same schema can be used to describe the case of tuberculosis. It is now known that TB has existed since at least 2000 bc, as is evident from tubercles found in mummified bodies.

Early 19th century: Based on 900 autopsies, Gaspard Bayle described a set of "observables" that co-occurred in human populations. This symptom cluster was as follows: **c**:{c_1=coughing,c_2=shortness of breath,c_3=chest pain,c_4=loss of appetite,c_5=weight loss,c_6=fever,c_7=chills,c_8=fatigue}. Now, **c** turned out to be the consequence of a single, non-observable causal agent, σ. The discovery of the existence and nature of σ included the following landmarks:

René-Théophile-Hyacinthe Laënnec described the progression of TB from the initial tubercle through its final stages. Half of all untreated TB cases are fatal. J. A. Villemin showed that TB could be transmitted from humans to animals. However, in the absence of a convincing causal case involving material agents, the cause of TB was attributed to a person's lifestyle.

1882: Robert Koch discovered the bacteria that caused TB, mycobacterium tuberculosis, a rod-shaped bacterium. Using simple but precise observations and experiments, Koch demonstrated the presence of the bacteria and how it was transmitted. Hence, "σ", in this case, signifies the mycobacterium tuberculosis bacterium.

In these scenarios, scientists, employing theory, knowledge, ampliative generalizations, and any other tools they see as relevant, engage in a search for a constituent of natural reality that they believe to exist and be causally responsible for some set of phenomena. Theory guides this search and is modified in the face of discovery. Candidate constituents are screened to determine if they fit the bill of causal agent, and this presupposes that, within the particular domain of investigation, rules have been layed down to fix the employment of the concept *causal agent* (else such causal claims would be vacuous). Consituents of natural reality (e.g., material entities) can have causal effects on other constituents, and, hence, the search for material causal agents makes sense. In many such cases, the notion of an unobservable causal agent also makes sense. A material entity, for example, can be unobservable for many different reasons, including its small size or great distance from the observer.

The story of the search for the cause of colds involves all of the ingredients that employments of latent variable models would have to involve, but do not involve, in order for (iv) to be a legitimate description of what goes on in latent variable modeling. There existed the concept *cold*, a concept identifiable in language at least as far back as the hieroglyphics of the

Egyptians (Fielder, 2002). This concept, whose employment is fixed by linguistic rules, is instantiated by a set of criteria, this set including nasal congestion, nasal discharge, sore throat, and persistent coughing. That is to say, one is grammatically justified in applying the term *cold* to an individual when this individual manifests these criteria. The application of the concept to another is defeated if his manifestations of these criteria are caused by an allergen, are accompanied by fever, or continue for longer than about one week. It was unknown what caused colds, i.e., it was not known what caused an individual to manifest the criteria of the concept *cold*. However, researchers were able to embark on a program of causal research because they were clear about *what* such a program entailed. They were after some *thing* that was responsible for colds, i.e., responsible for an individual manifesting the criteria of the concept *cold*.

Cold researchers layed down rules that fixed the meaning of the concept *cause of cold*, for, if this concept had not been assigned a sense, it would have been unclear as to what was *meant* by attempts to discover the cause of colds, and claims of causal discovery would have been vacuous. Within this particular line of research, a number of distinct, and progressively more sophisticated, technical definitions of causality were employed. The earliest criterion of causality rested on whether an individual whose olfactory epithilium was exposed to secretions from the nose of a sick individual, would catch a cold himself (i.e., would manifest the criteria for application of the concept *cold*) within a few days of being exposed (Fielder, 2002). Later definitions rested on whether a given cold could be paired with a specific viral agent, this requiring the capacity to detect a wide range of viruses through detection of their nucleic acids in the sick individual.

A cold is comprised of various physiological and behavioural changes, and, hence, theory was developed in regard what *kind* of thing might cause such changes. Early researchers debated whether there existed a single, or multiple, causes, and in the early 1900s poisons and bacteria were considered as possible candidates for the causal agent. Bacteria were eliminated as candidates because the causal agents responsible for colds could not be filtered with filters appropriate for bacteria (the causes of colds turned out to be much smaller than bacteria), and were not culturable in non-living media. Through vigorous and lengthy research, it was discovered that colds were caused by a particular class of classes of viruses. It was unproblematic for the employment of the concepts *cold* and *cause of cold* that the referents of the latter were perceptually unobservable prior to electron microscopy, although their unobservability posed a variety of *empirical* problems (including the initial inability of scientists to describe the structures of these causal agents).

Now, a theory that introduces a theoretical term may be incorrect, and, hence, make an existential claim that is false. Phlogiston was a substance hypothesized to exist by chemists Johann Becher and Georg Stahl. According to phlogiston theory, any substance, s, capable of undergoing combustion contained an, as yet undiscovered, substance, phlogiston, and the process of combustion was the process in which s lost its phlogiston. Because it was known that a substance such as mercury becomes heavier during combustion, it was believed that phlogiston had negative weight, i.e., that s would become heavier as it lost phlogiston. Substances such as coal and sulfur were believed to be composed almost entirely of phlogiston. In experiments with the gas now known as oxygen, Joseph Priestley discovered its combustion supporting property, but described the gas as "dephlogisticated air". The phlogiston theory was disproved by the French chemist Antoine Lavoisier, who demonstrated through his quantitative experiments that combustion is a process in which oxygen combines with another substance. By 1900 virtually all chemists had recognized the validity of Lavoisier's work, and the phlogiston theory was discredited. Hence, a phlogiston theory posited the existence of a substance, γ, claimed by the theory to be responsible for certain of the characteristics **c** of combustion. But the theory was incorrect, and no such substance existed.

Variant (iv) is a mischaracterization of the employment of latent variable models in scientific work. The latent variable modeller doesn't have any idea as to what it would even mean to claim that there existed a causal agent that has effects on the phenomena under study, these effects then encoded in the joint distribution of the manifest variates as latent variable model implied properties $\{c_1,c_2,...,c_r\}$. He has no theory which suggests, in a given research context, what type of thing such an agent might be, where it would be located, what its causal action would be like. *And if he did, he could gain nothing from the employment of latent variable models*. He would just posit that "there exists a currently undiscovered γ (material entity/force/property/etc.) that brings about consequences $\mathbf{c}$:$\{c_1,c_2,...,c_t\}$ in humans (e.g., certain of their behaviours) under study. If it exists, it is hypothesized that γ behaves in the following way { ...} to cause $\mathbf{c}$:$\{c_1,c_2,...,c_t\}$, the location of the causal action being {...}." He would be hot on the trail of this causal agent (material entity/force/property/etc.), eliminating candidate entities that turned out not to fit the bill, and progressively narrowing the range of possibilities until he had *it*. If the entity was perceptually unobservable, a problem the diagnosis of which requires an understanding of the medium in which the entity resides, he would be able to say *why* it was so, and commence work on the creation of instruments to render it observable. None of this happens within the context of latent variable modeling, because latent variable models cannot be employed in the manner described by (iv). The concept *latent variate to* $\underline{\boldsymbol{X}}$ is not a theoretical term that signifies an hypothesized, but unobservable, causal agent residing in natural reality, but rather a concept that signifies random variates (which, they being functions created by humans, do not reside in natural reality, and have no causal powers) possessing particular model-specified properties.

v. If a particular latent variable model, lvm, describes a particular set of variates, then the equations and associated parameter values form part of the nomological network linking the observable manifest variates to the unobservable entity (construct) signified by *latent variate to* $\underline{\boldsymbol{X}}$. The unobservable entity might be agreeableness, dominance, etc. Adding variates to be modelled reduces the latitude inherent to a given network, and, in the limit, identifies the unobservable entity in question.

As will later be discussed, this idea is the essence of construct validation theory, and is badly confused. For, once again, neither the creator, nor the applier, of latent variable models possesses a concept, i.e., is able to cite rules that fix the correct employment of a concept, that denotes some particular type, κ, of unobservable constituents of natural reality, whose relationships with a set of observable phenomena *could* then be described through the creation of a latent variable model. The talk inherent to (v) is vacuous. The situation encountered in the employment of a latent variable model is not analogous to that of the biologist who sets up a net in the jungle and makes the netting progressively finer, until he finally catches specimens of the small bird he is after. For in this latter case, the biologist is after various types of *birds*, and this aim has meaning because he possesses a set of concepts in terms of which his work is organized and conceptualized. What he is after are living material entities signified by the concept *bird*, a concept whose correct employment is fixed by linguistic rules. If, after months in the jungle, he offers up a tarantula as an example of the birds he was after, others can rightly reject his scientific claims on the grounds that he has misemployed the concept *bird*. He is confused as to the correct employments of the concepts *bird* and *spider*, and, as a result, is unclear about *what* he is searching for. Finally, the idea that ordinary language psychological concepts denote unobservables whose essences can only be inferred via knowledge of "observables" is badly mistaken, a case that will be argued in a later section.

4. *Four illusion generators*

The task of seeing latent variable models for what they are, and are not, has been made especially difficult by the existence, within the psychometrics, of a dense thickett of conceptual confusions, bad analogies, and false identifications. These have played the role of illusion generators and have helped to maintain the Central Account despite its erroneousness. In this section, a small sampling of the more potent of these illusion generators are considered.

Ii. The descriptions of their work by Spearman, and other latent variable modellers, does, undeniably, have the *feel* of intellegibility. Spearman speaks of his interest in intelligence, his research makes many references to intelligence, and it is likely that most who have read Spearman have come away with the belief that they have a pretty good idea as to what the Spearman program of research was about. But the intellegibility of the work of Spearman, and of the many who have followed in his foot-steps in employing latent variable models is research, trades on the false identification of the concept *latent variate to* ***X*** with ordinary language psychological concepts (psychological concepts that are not technical concepts particular to the social and behavioural sciences, but, rather, are a part of ordinary language). Spearman talks about "general intelligence", factors, and *g*, all technical concepts whose rules of employment he is responsible for laying down. He also invokes the concept *intelligence* (its cognates and related terminology) that belongs to the English language, and whose correct employment is fixed by linguistic rules.

The employments of these ordinary language concepts are taught to, and learned by, children when they learn to speak. These concepts find their ways into countless products of language, from novels to television. Their existence in language long predates the formation of any of the social and behavioural sciences. While Spearman does not provide rules of employment for his technical concepts, leaving their meanings wholly unsettled, the meaning of the ordinary language concept *intelligence* is not unsettled.[6] Speakers of English well know how to employ this concept. A man who reasons well, or perhaps displays a range of intellectual proficiencies, is an intelligent man. A brilliant intellect is an intellect capable of illuminating complex issues. Unwittingly or otherwise, Spearman generated the illusion that his technical concepts are likewise intelligible by simply employing them interchangeably with ordinary language concepts, without ever establishing his right to do so. Recall that E.B. Wilson had made roughly this point in his original review of Spearman:

> A fundamental question which I can hardly know enough to ask aright is: What are these general factors, Verbal, Speed, Spatial 2, etc.? Are they like Spearman's *g*- intelligence-something intrinsically defined by the analysis? Recall that he says that he does not define *g* but measures it (and, I believe, attempts to interpret

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6 It is, however, a concept which, like so many psychological concepts, is, to paraphrase Baker and Hacker (1982), widely ramifying and lacking in uniform employment.

> it). Or are these general factors certain known traits definable otherwise than by our analysis? And is our problem really to explain the observed correlation coefficients in terms of these known traits? If the problem is this, we do not need any discussion of the general problem of resolution of variables into unknown general and unknown specific factors. We can proceed at once to select our known general factors and see how well we can reproduce the observed correlations by regarding only the intensities as unknown... (Wilson, 1929, p.161)

To justify his equating of ordinary language English concepts and the technical concepts of his factor analytic work, Spearman would have had to have shown that the rules of employment of, e.g., *intelligence*, are identical to those of his technical concepts, e.g, *g* (*general intelligence*). But, since, by his own admission, he was unable to come to terms with the employment of the concept *intelligence*, and, additionally, failed to provide rules of employment for any of the technical concepts he introduced, he clearly lacked the capacity to do so. The false identification of *latent variate to* ***X*** with various ordinary language psychological concepts (trait-terms, disposition-terms, abilities, etc.) very nicely produces the illusion that, in carrying out a latent variable analysis, the latent variable modeller is in the hunt for something *in particular*. In the absence of the real goods of a detection scenario (the ability to specify the class of entities to be detected, provide an account of the behaviour of these entities within the context of the detection scenario, etc.), the latent variable modeller speaks of his results as being about "balance", "arm strength", or "speed of bodily movement" (Thurstone), or other "consistent and stable human characteristics" such as "...quantitative aptitude, verbal aptitude, mathematical ability, subject-matter knowledge in psychology..perseverence and creativity" (Lord & Novick, 1968, p.537). He states that "The latent variable may be "real" in the sense that it could, in principle, be measured directly. An example would be some sensitive quantity like personal wealth" (Bartholomew, 1980, p.295). He states that "This latent variable analysis, employing eleven indicators of dominance, revealed a general "dominance" factor", and banks on the fact that his readers, being, as they are, speakers of the language, will possess the concept *dominance*, and, hence, conceptualize his claim in these terms. Yet, never does he establish that the rules of employment of *these* ordinary language terms (*personal wealth*, *balance*, *creativity*, *dominance*, etc.) warrant such employments.

The false identification of distinct concepts is endemic to general discussions of latent variable modeling and is the essence of the practice of "latent variable interpretation" (to be discussed later as a separate topic). It can be observed, for example, in the seminal work of Lord and Novick: "The factor analytic model is one of a number of models that give concrete form to the concepts used in theoretical explanations of human behavior in terms of *latent traits*. In any theory of latent traits, one supposes that human behavior can be accounted for, to a substantial degree, by isolating certain consistent and stable human characteristics, or *traits*, and by using a person's values on those traits to predict or explain his performance in relevant situations" (1968, p.537). Now, Lord and Novick do not provide special technical senses for particular trait terms, and, throughout their discussion, it is clear that they are using the term *trait* in its ordinary language sense (as a rough synonym for *disposition*). Needless to say, they do not establish, nor *could* they establish, that the technical concept *latent trait* has the same grounds of application as do particular ordinary language *trait*-terms. It would be no less justifiable, but just as wrong-headed, to claim that, in a particular application of a latent variable model, the concept *latent* *variate to* $\underline{\boldsymbol{X}}$ denotes alpha particles. It is as if to incant the term *ability* just prior to running a latent variable analysis will convert the latent variable model into an ability-detector.

Iii. How could properties/attributes and causes, the kinds of arguments that might feature in a scenario of detection, be confused with the random variate $\boldsymbol{\theta}$ that is featured in the equations of latent variable models? How is it that latent variable modellers seem not to grasp the marked differences between true contexts of detection and the application of a latent variable model? This is perhaps a result of the mistaken belief, prevalent within the social and behavioural sciences, that the objects of scientific investigation are variates. Science involves the study of processes, forces, entities, and other natural phenomena. Variates are not entities, properties, forces, or phenomena of any sort, but rather functions, and, hence, do not *do* anything. Variates are created and put to various uses by human beings. The physicist speaks of studying sub-atomic particles, material entities that are constituents of natural reality, and whose properties can be discovered, studied, and catalogued. Is a sub-atomic particle a variate? No, it is not. The causes of behavioural phenomena are not resident in the joint distributions of variates created by humans to investigate these phenomena, but, rather, in natural reality. Possible ingredients of the causal story of behavioural phenomena include biological structures (one cannot possess and exercise intellectual capacities without possessing a brain) and other behavioural phenomena (certain brands of nagging can cause great unhappiness).

Variates can, of course, be created to represent phenomena of interest to the scientist. This representational role is achieved by laying down rules of correspondence between the phenomena of interest and the values assumed by a variate, and to lay down such rules of correspondence presupposes the capacity to antecedently identify the relata of correspondence relations, which, in turn, presupposes a grasp of the rules of employment of the concepts that denote the phenomena to be represented. The skilfull creation of such correspondence relations allows the scientist to employ variates to make legitimate claims about the *phenomena* they represent. If the claim that "variates are the objects of scientific study" were merely shorthand for the claim that "phenomena represented by variates are the objects of study", then the former claim would be innocuous. But there is evidence aplenty that this way of speaking is not merely shorthand, but is indicative of profound confusion.

The latent variable modeller could argue, with some justification, that he studies behavioural phenomena, and that he represents this phenomena through the creation of a set of variates (functions), $\mathbf{X}_j$, which he calls the manifest variates, the scores on which are produced in accord with rules of score production. Unlike the manifest variates of latent variable analyses, the latent variate to $\underline{\mathbf{X}}$ does not *represent* any phenomenon of interest. No rules of correspondence are layed down to link this variate with a particular constituent of natural reality. The latent variable modeller's talk of variates being the focus of study betrays a profound truth about latent variable modeling, namely, that, while science is not in the business of studying variates, variates are, indeed, what the latent variable analyst's analyses are about. In particular, if, in a given application, $\underline{\mathbf{X}}$ is described by a given latent variable model, the yield is not a discovery of some constituent of natural reality (something that might well be the outcome of an ordinary scientific investigation), but, rather, the ability to create a variate (function) possessing precisely the properties prescribed for it by the latent variable model, a truth that will be explored in Part 3 of the current work.

Confusion over the place of variates in the scientific enterprise spelled doom for comments recently made by Stanley Mulaik on the issue of factor indeterminacy. Mulaik (2005, p.200) claimed that "In the original common factor model there is no such thing as a constructed common factor", implying that the construction of random variates that satisfy the requirements for factor-hood imposed by the linear factor model (i.e., those variates constructed in accord with (4.4)-(4.6)) is external to the model. This is mistaken and is a direct result of Mulaik's belief that there is a distinction to be drawn between "variates in the world" and constructed variates (see Chapter V of the current work for examples). In fact, there are no grounds for such a distinction, and there exist no variates that are *not* constructed. Variates are not constituents of natural reality, and, hence, are not "in the world." They are, rather, humans creations. Thus, when Mulaik looks into the equations of the linear factor model and sees the random variate $\boldsymbol{\theta}$, the common factor, he is, of course, seeing a constructed variate. With respect a given variate, the question to be asked is *how* was it constructed. Was it constructed through the antecedent laying down of a rule of score production or was it the product of a data analytically derived formula? As has been noted in Part II of the current work, latent variable modeling neither involves the antecedent laying down of rules for the production of θ-scores, nor rules that establish correspondence relations between the symbol $\boldsymbol{\theta}$ and particular constituents of natural reality. As will be argued in Part III of the current work, the random variate $\boldsymbol{\theta}$ plays no different a role in the equations of latent variable models than does the symbol $\mathbf{c}_1$ that stands for the first principal component variate. Both stand for random variates that must be built to satisfy particular sets of model imposed requirements.

Iiii. Latent variable models are not detectors of causes, and, yet, the CAC, the received account, portrays them as such. A primary prop in the propogation of the CAC illusion has been a variant of (Ii), namely, the false identification of the technical concept of *common factor to* $\underline{\boldsymbol{X}}$ with general senses of *factor responsible for....* This false identification has been used extensively as a tool to give the CAC the feel of intelligibility. What is meant by *common factor to* $\underline{\boldsymbol{X}}$? Well, it denotes the "factor responsible for the behaviour under study", "important factor", "determiner", etc. Cyril Burt had previously warned of the danger of this false identification: "It is, however, somewhat unfortunate that the term 'factor' is used for both conceptions-the statistical factors that we are discussing here and the genetic factors responsible for hereditary resemblances: the common name tempts the lay reader and the students to identify the two" (Cyril Burt, 1940, p.8). One, however, need not be concerned solely with the "lay reader and student", for the source of this problem lies with the experts whose chosen line of work is to invent and propogate latent variable modeling technology. Anderson (1959, p.11), for example, states that "Apart from any mathematical reason for such an assumption there are psychological or substantive reasons. The proposition is that latent quantities are the only important factors and that once these are determined behaviour is random (in the sense of statistical independence). In another terminology, the set of individuals with specified latent characteristics are "homogeneous"." And here one sees the implicit equating of technical and ordinary language senses of the term *factor*. Anderson begins as if to define the technical concept of *factor/latent quantity/latent variate*, this task requiring reference to the property of conditional independence. But in the midst of this endeavour he offers the opinion that "latent quantities are the only important factors" and that "once these are determined behaviour is random"? If *latent quantity* is a synonym for the technical concept *latent variate/factor*, in what sense can such quantities be the "only important factors". Does it follow then that there exist types of factors that are less important than latent quantities? If so, how can *latent quantity* and *factor* be synonomous technical concepts? In fact, Anderson has left the domain of technical definition, and is playing on the reader's intuitive grasp of ordinary causal stories, and the various constituents of natural reality, "important factors", that are a part of these stories. The problem is that he neither acknowledges, nor attempts to square, the two distinct senses.

The work of psychometrician Paul Meehl (e.g., 1977; Waller & Meehl, 1998) presents a seamless intermingling of technical senses of *factor/latent variate* as found within latent variable modeling and various non-technical senses of causality. Thus, in providing motivation for the development of his taxometrics suite of latent variable techniques, Meehl speaks of "an intrinsically dichotomous qualitative causal factor which is both necessary and sufficient for the disease", "a sine qua non but not sufficient factor", and "a quantitative factor of which disease probability is a quasi-step function ..." He does not explain the basis for his equating of these various senses of *causal factor* with the concept *latent variate* that is featured in his taxometric techniques.

The factor responsible for (the cause of) tuberculosis is the bacterium mycobacterium tuberculosis. More particularly, the cause of the tuberculosis disease, with its characteristic symptom cluster, and progression, is the invasion of the tissue of the lungs, and reproduction therein, by the mycobacterium tuberculosis bacteria. Frequently the rapidly mutiplying bacteria spread to the rest of the body via the bloodstream, thus causing further damage. The causal story of tuberculosis involves the behaviour of various material entities. The action of these material entities occurs in the human body, and, notably, the lungs. It is indeed the mycobacterium tuberculosis bacterium that is the unobservable (to the naked eye) cause. However, this causal agent can now be seen with the aid of a high-power microscope. A case that *p* is a causal determinant of *q* is a claim that *p* and *q* co-occur in a certain way. But such a claim, if it is to have a sense, presupposes the capacity to antecedently identify *p* and *q*. Certainly, variates do not cause each other. If the referent of the concept *latent variate to* $\underline{\boldsymbol{X}}$ were to be legitimately portrayed as a cause of the phenomena represented by the $\mathbf{X}_j$, then rules would have to be layed down to establish a correspondence between the latent variate to $\underline{\mathbf{X}}$ and some particular constituent of natural reality believed to be the cause. Certainly, it does nothing to stare at the symbol $\boldsymbol{\theta}$ and mouth "causal source". But no such rules are layed down in a latent variable analysis. And, if such rules *were* normally layed down as a part of analysis, then latent variable models, and the associated mythology of the Central Account, could be dispensed with. Moreover, there would be no need for the term *latent variate to* $\underline{\boldsymbol{X}}$. As do other scientists, the researcher could simply speak of the hypothesized cause of the phenomena under study.

Iiv. Mathematical and material *existence*.

> "I have the same feeling when a word like 'exist' is singled out from language and, without fixing its sense, people start racking their brains as to what exists and what does not" (Boltzmann, 1974, p.67).

Consider the following statement from Eysenck (1953, p.109): "In a sense, therefore, the concepts and laws to which factor analysis gives rise are "statistical artifacts"; they are so in the same way that all other scientific concepts and laws are "artifacts." Spearman's *g* (general intelligence) is a statistical artifact to precisely the same extent, and for the same reasons, that Newton's *g* (gravitational force) was a mathematical artifact. Neither has any actual existence, in the sense that a falling stone or an individual who is acting intelligently can be said to exist" . Now, in this quote, Eysenck employs a number of different senses of the concept *existence*, but neither acknowledges these differences, nor the very different types of evidence that is relevant to the support of each. One could, for example, inquire as to the existence of some particular material entity. The concept *meson* is defined as follows: "A sub-atomic particle containing one quark and one antiquark." One could ask if there existed any mesons, meaning, somewhat pretentiously, that one would like to know whether there existed at least one material entity that is rightly called a meson (i.e., to which the concept *meson* can correctly be applied). If the answer to this question were not known, then science could set out to provide an answer. Support for a claim of existence of a material entity is provided by various types of empirical evidence. On the other hand, there are various mathematical senses of *existence*. One could inquire as to whether there exists a solution to the equation x+5=9? Mathematical and empirical existence are very different issues. Support for a claim that the solution to an equation exists is provided by formal proof, or by providing a candidate solution and showing that it does, in fact, satisfy the equation.

The literature on latent variable modeling contains no shortage of examples of equivocation over the distinction between mathematical and empirical existence, not to mention outright conflation of the two. Frequently, discussions of the existence of solutions to various equations encountered within the context of latent variable modeling come to sound as if what is being discussed is the existence of entities believed to be denoted by the symbols contained in these equations. An example is the equivocation that plagued the use of the phrase "when the tetrad difference criterion holds, *g* has been shown to exist" (cf. Steiger and Schonemann, 1978, p.145). In the quote from Eysenck, on the other hand, the question concerning the existence of "an individual who is acting intelligently" is a question about the existence of a certain kind of human, and the existence of Newton's *g* is a question about the existence of a force. There is nothing *artifactual* about Newton's *g*. The concept denotes a force, and a force is not a material entity. The question that must be asked is, "what are the grounds for the support of a claim that a force exists". Neither is Spearman's *g* an *artifact*, but rather a wholly unexplicated concept. Is it to be taken as refering to a set of scores, to wit, the solutions to the set of equations that define the linear factor model? If so, then Wilson proved that *g* (in fact, an infinity of such *g*s) exists if, in fact, a set of variates is described by the linear factor model. Is it to be taken as signifying an entity of some sort? Certainly, Spearman and many others have acted as if it is to be taken in this way, and the Central Account enshrines it as such. If this is the idea, then the grounds for supporting this claim of existence certainly is not mathematical proof, nor factor analytic results, but, rather, an entirely different approach! In particular, what is needed is a criterion of application for the concept *g* (*general intelligence*) and some particulars regarding the nature of its alleged referents (are they material entities?), for then a search for these hypothesized referents could begin, and the question of their existence addressed. This is not to suggest that modern day psychometricians are confused in regard the relation between tetrad differences and the existence of mental energy. The CAC and, especially, the CAM, are, however, the modern counterparts to this early confusion.