XV

Logic of Replacement Variate Generators

Latent variable "models" are not detectors of unobservable properties/attributes (causal sources). They are of a kind with component "models", and, while both possess model-like features, neither is a model in the classical sense of the term. Both component and latent variable models are *replacement variate generators*, quantitative recipes for the construction of random variates that *replace* a set of variates $\mathbf{X}_j$, j=1..p, in some particular, optimal fashion. Replacement variate generators involve two types of (random) variates, *input random variates* and *replacement variates*. Let the equations of a given replacement variate generator, R, be symbolized as f($\underline{\mathbf{X}}$;$\underline{\mathbf{r}}$), in which $\underline{\mathbf{X}}$ is a p-element random vector of input variates and $\underline{\mathbf{r}}$, a vector of random replacement variates. Let any additional distributional and moment constraints inherent to R be symbolized as D, and let $\underline{\Pi}$ contain the parameters of the generator. A given replacement variate generator will then be symbolized as R:[f($\underline{\mathbf{X}}$;$\mathbf{r}$),D,$\underline{\Pi}$]. In the special case of a unidimensional generator, $\underline{\mathbf{r}}$ is a single random variate **r**. The majority of the component and latent variate generators considered herein are unidimensional, and, hence, the discussion to follow is phrased in terms of these generators.

1. *Input and replacement variates*

1a. Input random variates

The input random variates are a set of p random variates appearing in the equations of a replacement variate generator and symbolized therein as $\mathbf{X}_j$, j=1..p (or, alternatively, as the random p-vector $\underline{\mathbf{X}}$). In a particular context of employment of the generator, the scores that comprise the distributions of the $\mathbf{X}_j$, j=1..p, can be produced prior to analysis because, prior to analysis, the researcher is in possession of rules, r_j, j=1..p, for the production of such scores.

Example: A score from the distribution of $\mathbf{X}_1$ is produced in accord with rule r_1: "Take an individual, p_i, from population *P* under study [details required as to how such an individual should be chosen], have him answer the first item of the Beck Depression Inventory [details required as to the test taking protocol], and code his response as per the instructions provided in the BDI test manual."

The input random variates of latent variate generators have traditionally been called "manifest variates", this name arising from the bogus observability/unobservability distinction of the Central Account. More will later be said on this dichotomy. No special name has been, to date, invented for the input variates of component generators.

The following should be noted:

a) That a rule of score production for each of the input variates is known prior to analysis, explains why a sample of realizations on $\underline{\mathbf{X}}$ (drawn from *P*), these organized as an N by p matrix, comprises the data to be analyzed.

b) To know the set of rules $\{r_1,r_2,...,r_p\}$ settles the issue as to the meaning of the symbols $\mathbf{X}_j$, j=1..p, that appear in the equations of replacement variate generators, because $\mathbf{X}_j$ stands for the set (distribution) of scores that would be produced by applying r_j to each member of the population *P* under study, i.e., by taking as the argument of r_j each member, p_i, of *P* in turn: $x_{ji}=(r_j(p_i), p_i \in P)$.[1]

c) The rules $\{r_1,r_2,...,r_p\}$ define the events to which random vector $\underline{\mathbf{X}}$ refers. The fact that $\underline{\mathbf{X}}$ is *random* simply means that the events generated by application of $\{r_1,r_2,...,r_p\}$ to the members of *P* occur with probability described by density function of $\underline{\mathbf{X}}$.

d) Under typical circumstances, knowledge of r_j allows the researcher to take for granted that each $\mathbf{X}_j$ *uniquely* symbolizes: The fact that $\mathbf{X}_2$ is uniquely associated with rule r_2, in that r_2, and *not* r_1, nor r_3, is the rule by which scores on $\mathbf{X}_2$ are produced, establishes scores constructed according to r_2 as uniquely X_2-scores.

e) Latent and component variate generators make claims about the distribution of $\underline{\mathbf{X}}$ in *P*, but *what* the $\mathbf{X}_j$ stand for is settled, not by anything indigenous to statistical theory, but by antecedent knowledge of $\{r_1,r_2,...,r_p\}$.

f) There exist certain rules of score production that happen to be rules for the production of measurements. If concept "φ" is embedded in a normative practice of measurement, and rule r_φ happens to be a rule for the production of measurements of the φ of the members of some population *P* of objects, the application of r_φ to member t of *P* yields a measurement of the φ of t in some particular units. In this special case, the scores that comprise the distribution of $\mathbf{X}_\varphi$ are signified by concept "φ", i.e., they are φ-scores.

<u>Example</u>

P is a population of objects, p_k, each having a mass, m_k. A measurement of the mass of an object is produced in accordance with the following rule:

$r_m \equiv$ the mass of object p_k is $-\dfrac{a_{ref/p_k}}{a_{p_k/ref}}$, in which $a_{m/n}$ is the acceleration of object m induced by object n, and ref stands for an antecedently chosen reference object .

Application of this rule to object p_k drawn from *P* yields a measurement, m_k, of the mass of p_k, i.e., $r_m(p_k)=m_k$. Mach put forth this definition of *mass* on the basis of a posited representation relation and evidence from a gedanken experiment that this relation actually did hold (see Falmagne, 1992). This particular definition of mass produces a measurement scale, $r_m(p_k)$, at the ratio level of measurement, in the sense that $t \times r_m(p_k)$ is also a mass scale, in which $t>0$.

1 Knowledge of $\{r_1,r_2,...,r_p\}$ cannot, of course, settle every possible nuance of score production, but, then, $\{r_1,r_2,...,r_p\}$ need only settle what constitutes correct score production behaviour in those contexts that they were designed to govern.

Let $\mathbf{X}_k$ (a random variate) stand for the population of scores, masses, produced by the application of rule r_m to each of the members of *P*, and let $f_{\mathbf{X}}$ be the density function of $\mathbf{X}_k$. Then one can rightly say that the concept *mass* signifies the scores that comprise the distribution of $\mathbf{X}_k$.

Note, however, that such conceptual signification is in no way guaranteed by the mere fact that there exists a rule r_j for the production of scores that comprise the distribution of an $\mathbf{X}_j$. A rule is simply a standard of correctness, and a rule for score production, a standard of correctness in regard the production of a certain type of score. Hence, a rule can always be invented that yields scores that are *not* signified by a concept from ordinary language. Consider the following scenario:

i. *P* is a population of humans, p_k.

ii. A score for individual p_l is produced in accord with the following rule:

$r_t \equiv$ Have individual p_l, a member of population *P*, respond to each of the 21 items of the Beck Depression Inventory (BDI), code his responses according to the instructions provided by the BDI test manual, sum these coded responses and add to this sum his shoe size.

Now, application of r_t to any object p_k contained in *P* yields a score, $r_t(p_k)=s_k$. Let $\mathbf{X}_t$ be the random variate whose density function, $f_{\mathbf{X}t}(s)$, describes the distribution of the scores, s_k, in population *P*. With a suitable choice of $f_{\mathbf{X}t}(s)$ statistical analyses can proceed as per usual (if a few more variates were constructed, a factor analysis could be carried out on the resulting set). However, *the scores s_k are not signified by a concept, i.e., they are not measurements*.

1b. Replacement variate

A replacement variate is a random variate that is symbolized in the equations of a replacement variate generator as **r**.[2] In contrast to the input random variates, there does not exist, prior to analysis, a rule by which the scores that comprise the distribution of **r** can be produced. The absence of an antecedently available rule of score production for **r** has two immediate consequences:

a) The N by p matrix containing the data to be analyzed in the employment of R:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$] does not include scores that are realizations of **r**.

b) The meaning of the symbol **r** is settled by R:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$] itself. In particular, the symbol **r** is a place-holder for any random variate constructed so as to satisfy R:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$], when, in fact, $\underline{\mathbf{X}}$ satisfies it. In the event that particular $\underline{\mathbf{X}}$ satisfies the requirements stipulated by R:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$], it will be said that R:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$] describes $\underline{\mathbf{X}}$. The variates **r** that satisfy R:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$] will be called *replacement variates to $\underline{\mathbf{X}}$ (under R:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$])*, because they optimally replace, stand in place of, or approximate $\underline{\mathbf{X}}$, in senses particular to R:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$].

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2 The symbol employed to represent the replacement variate will, in what follows, depend upon the generator considered.

2. *Properties of replacements*

Assume that, for particular replacement variate generator $R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]$, population P_T, and set of input variates $\underline{\mathbf{X}}^*$, $\underline{\mathbf{X}}^*$ is described by $R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]$. In the case of a latent variate generator, this means that there exists a numerical realization of $\underline{\Pi}$, say $\underline{\Pi}_o$, such that $\Omega_T \subset R_1$, i.e., $\Omega_T=\Omega(\underline{\Pi}_o)$ (see Chapter 2). Then:

a) *Existence*: $\underline{\mathbf{X}}^*$ will be said to be replaceable under $R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]$, meaning that there exists at least one replacement variate to $\underline{\mathbf{X}}^*$ (under $R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]$).

b) *Cardinality of replacement*: Since **r** is a place-holder for any random variate constructed so as to satisfy the requirements inherent to $R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]$, the *number* of replacement variates to $\underline{\mathbf{X}}^*$ (under $R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]$) is jointly determined by $R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]$ and features of the distribution of $\underline{\mathbf{X}}^*$. Let the set *C* contain all replacement variates to $\underline{\mathbf{X}}^*$ (under $R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]$), and let Card(*C*) be the cardinality of this set of constructed random variates.

comment: Whereas knowledge of $\{r_1,r_2,...,r_p\}$ fixes the meaning of the symbols $\mathbf{X}_j$ by explaining how scores that comprise the distributions of these variates come to be, and, under the applications most familiar to the behavioural scientist, ensures that this reference is unique, such meaning fixing rules are not available, prior to analysis, for the replacement variate (latent or component variate). The Central Account urged the latent variable modeler to believe that the scores that comprise the distribution of the latent variate to $\underline{\mathbf{X}}$ are measurements of a single existing cause (property), these scores signified by an ordinary langauge concept whose identity is in need of revealing. This belief was the result of commitment to an incoherent account of concept meaning and signification. Regardless, the number of latent variates to $\underline{\mathbf{X}}$ (replacement variates to $\underline{\mathbf{X}}$) is not a matter for superstition, but is, instead, determined by the interaction of data and replacement variate generator $R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]$. Moreover, the values that Card(*C*) can assume in an employment of $R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]$ is a key feature of a description of $R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]$.

c) Any statistical statements into which the symbol **r** enters are statements about each of the elements of *C*. For example, $E(\underline{\mathbf{X}}^*\mathbf{r})$ is a set function defined over *C*.

d) *Construction formulas*: Let T represent the totality of requirements imposed on **r** by $R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]$ when it describes $\underline{\mathbf{X}}^*$, i.e., the totality of requirements that a random variate must satisfy in order for inclusion in *C*. These requirements may be expressed in the form of a construction formula, $\mathbf{r}=T[R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]]$, according to which each of the elements contained in *C* can be produced.

comment: For particular generator $R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]$, the construction formula $\mathbf{r}=T[R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]]$ might, or might not, call for the elements of *C* to be functions of the input variates. When the elements of *C* are required to be functions of the input variates, they might, or might not, be linear functions. The construction formula $\mathbf{r}=T[R_1:[f(\underline{\mathbf{X}};\mathbf{r}),D,\underline{\Pi}]]$ settles such matters.

e) *Optimality*: The replacement of a set of input variates brought about under particular generator R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$] is optimal in a sense characteristic of R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$]. Specifically, replacement variates to $\underline{\mathbf{X}}^*$ (under R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$]) are, at least, optimal with regard the requirements laid down by R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$], for these are precisely the variates constructed to satisfy these requirements. This sense of optimality is the primary sense of optimality delivered in the replacement of $\underline{\mathbf{X}}$ by **r** under R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$]. Senses of optimality delivered by particular generators encountered in practice include various brands of covariance and variance explanation, generalizability, invariance, and rank reduction. In addition to the primary sense of optimality delivered by a given replacement variate generator, a number of secondary (consequent) senses may also obtain. Secondary senses of optimality are not explicit in the requirements that **r** must satisfy, but, rather, are contingent upon these requirements. Both primary and secondary senses of optimality will be discussed in the contexts of discussing particular generators.

comment: A comparison of two or more replacement variate generators, e.g., the linear factor and principal component generators, is a comparison of the brands of optimality, in relation to the costs entailed, of the replacements delivered by each. That is, it is a comparison of the optimality/cost profile characteristic of each type of replacement.

f) *Characteristics of set C*: A key feature in a description of both the replacement of a particular set of input variates by a particular generator, and the generator itself, is a description of the characteristics of the set *C* of replacement variates thus produced. Two notable features of interest are:

i) Card(*C*), i.e., the number of replacement variates to $\underline{\mathbf{X}}^*$ that exist given that R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$] describes $\underline{\mathbf{X}}^*$;

ii) In the event that Card(*C*)>1, the "similarity" of the replacement variates contained in *C*. This issue involves a consideration of the distinctness of those variates that are replacement variates to $\underline{\mathbf{X}}^*$ under R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}_o$], or what might be called the "breadth" of set *C*. It is, of course, possible to quantify "similarity" and "distinctness" in many different ways. Two classes of measure are generated by considering either pairwise functions of the elements of *C* (examples being the Pearson Product Moment Correlation between two members of *C* and the minimum, over *C*, of this correlation, i.e., ρ^*) or the relationships between members of *C* and a set of "external variates" (variates, $\mathbf{Z}_i$, not included among the input variates analyzed). An example of the latter would be a comparison of the vectors $[\rho_{\theta i,Z1},\rho_{\theta i,Z2,...}]$ and $[\rho_{\theta j,Z1},\rho_{\theta j,Z2,...}]$, in which $\boldsymbol{\theta}_i$ and $\boldsymbol{\theta}_j$ are two members of set *C*.

g) *Additional restrictions to achieve uniqueness of replacement*: If particular set of input variates $\underline{\mathbf{X}}^*$ is replaceable under particular generator R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$], and it so happens that Card($C_{\underline{\mathbf{X}}^*}$)>1, it is sometimes possible, through the imposition of *additional* restrictions, to yet achieve a unique replacement.

comment: Various types of additional restriction have been discussed explicitly by psychometricians, or have arisen as pragmatic features of analysis. These include distributional requirements placed on **r**, additional analytic activities undertaken in the hope that Card(*C*) will, at the least, be reduced, and conditions placed on the number of input variates. As was seen in Chapter 5, Mulaik, for example, suggested that the indeterminacy inherent to linear factor analytic replacements (the fact that Card(C)>1) can, potentially, be overcome by carrying out research following an initial factor analysis, this research aimed at selecting from the set C of common factors to $\underline{\mathbf{X}}$, a preferred variate. There is also the much discussed "behaviour domain" response to indeterminacy. The issue considered there pertains to the conditions under which allowing p, the number of input variates, to become large, brings about uniqueness of replacement.

h) *Equivalence of replacements under distinct generators*: The replacement variates generated under distinct generators, say, R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$] and R_2:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$], will, generally speaking, answer to different optimality requirements. Should a particular set of input variates $\underline{\mathbf{X}}^*$ be replaceable under both generators, the resulting sets of replacement variates, say, C_1 and C_2 will not usually be equivalent sets. There might, however, exist special circumstances under which sets $C_1$and C_2 *are* equivalent. An example is the first principal component and ulcf replacements of an $\underline{\mathbf{X}}$, under the condition that $\underline{\mathbf{X}}$ remains ulcf replaceable as $p\to\infty$. This particular case has traditionally been called the asymptotic equivalence of the first principal component and common factor, and arises because Card(C_{ulcf})=Card(C_{pc1})=1, the single replacement variate contained in each set being the same variate.

i) *Testability.* It need not be the case that a particular set of input variates $\underline{\mathbf{X}}^*$, distributed in a particular population P, is representable by some particular generator R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$]. Generator R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$] might require the distribution of $\underline{\mathbf{X}}^*$ in P to satisfy conditions, F. In such a case, replacement variates to $\underline{\mathbf{X}}^*$ (under R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$]) exist only given that the distribution of $\underline{\mathbf{X}}^*$ does, in fact, satisfy these conditions. Whether the distribution of $\underline{\mathbf{X}}^*$ in P does, in fact, satisfy these conditions can, potentially, be tested on the basis of a sample of N realizations of $\underline{\mathbf{X}}^*$ drawn from P. But since $\underline{\mathbf{X}}^*$ is replaceable under R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$] just in case its distribution *does* satisfy F, such a test is then a test of an *hypothesis of replaceability* (an hypothesis of existence of replacement variates to $\underline{\mathbf{X}}^*$ (under R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$])):

H_o: Card($C_{\underline{\mathbf{X}}^*}$)=0 vs. H_1: Card($C_{\underline{\mathbf{X}}^*}$)>0.

If, on the basis of a particular sample from P (stored in N×p data matrix X^*), estimate of $\underline{\Pi}$, $\hat{\underline{\Pi}}$, based on X, and particular fit function, L, it is judged that $\underline{\mathbf{X}}^*$ is, within limits, described by R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$] in P, then it will be said that $\underline{\mathbf{X}}^*$ has been judged replaceable under R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$]. The estimated parameters, $\hat{\underline{\Pi}}$, are then employed to estimate the resulting construction formula according to which are produced the replacement variates **r** contained in $C_{\underline{\mathbf{X}}^*}$.

Not every replacement considered in this book places restrictions on the distribution of $\underline{\mathbf{X}}^*$, and, hence, not every replacement considered involves a test of replaceability.

j) *Replacement Loss*: It will be expected as a matter of course that hypotheses of replaceability are standardly false. For example, the ulcf (or even multidimensional factor analytic) replacement never truly holds. Let $\Sigma_{\underline{\mathbf{X}}^*}$ be the covariance matrix of $\underline{\mathbf{X}}^*$ in population P under study, $\Sigma_A = \underline{\Lambda}_A \underline{\Lambda}_A' + \Psi_A$ be the best ulcf approximation to $\Sigma_{\underline{\mathbf{X}}*}$, i.e., the choice of $\underline{\Lambda}_A$ and Ψ_A that makes Σ_A as "similar" as possible to $\Sigma_{\underline{\mathbf{X}}*}$, and $\Sigma_{\underline{\mathbf{X}}*} \neq \Sigma_A$. The discrepancy between $\Sigma_{\underline{\mathbf{X}}*}$ and Σ_A is quantified by some fit function $L(\Sigma_{\underline{\mathbf{X}}*};\Sigma_A)$, the *replacement loss*, and a decision is made as to whether $L(\Sigma_{\underline{\mathbf{X}}*};\Sigma_A)$ is small enough to view $\underline{\mathbf{X}}^*$ as *essentially* ulcf-replaceable. In practice, $L(\Sigma_{\underline{\mathbf{X}}*};\Sigma_A)$ is estimated on the basis of a sample of observations drawn from *P*, and, if the inference is made that $L(\Sigma_{\underline{\mathbf{X}}*};\Sigma_A)$ is acceptably small, the analyst proceeds as if $\underline{\mathbf{X}}^*$ is, in fact, ulcf-replaceable. This way of thinking is the essence of the RMSEA (root mean-square error of approximation) method of fit assessment (Steiger & Lind, 1980) in factor analysis and structural equation modeling.

3. *Latent variates, manifest variates, components, and constructed random variates*

Latent variable and component generators involve what have, herein, been called replacement variates. These are random variates constructed to replace, in some optimal sense, a set of input random variates. What then of the distinction between *manifestness* and *latency* that has been the traditional foundation of latent variable modeling? The source of the manifest/latent distinction is, of course, the Central Account: Latent variates, in contrast to manifest variates, are said to be "unobservable", "unmeasureable", "unknown", or "hypothetical". But, as was seen in Part II, this talk is nonsense, and is only given the ring of legitimacy through the illegitimate equating of it with the true perceptual unobservability that is a characteristic of certain constituents of natural reality (e.g., viruses at the turn of the century).

McDonald's 1974 article on indeterminacy attempted to define the concept *unobservable random variate*, but his definition presupposed the CA mythology. In a more recent exchange on these issues, McDonald (1996a, p.595) insisted that "...the history of psychometric theory is consistent with the notion that latent traits/common factors are defined by the principle of local independence." Now, this is one of those convenient sound-bites that psychometricians favour when the enticing fairy-tales they have spun, in this case, the Central Account, are under attack. McDonald's (1996a) definition evidently has not been taken seriously by the psychometrics community, for, if it had been, there would have been no need for the Central Account. Instead, psychometrics could have pursued one of the following paths:

i) As Mulaik has urged, latent variable analyses could have involved the search for a phenomenon ψ, represented by a variate $\mathbf{Y}$, with $\mathbf{Y}$ possessing the property that when the $\mathbf{X}_j$ are conditioned on it, the $\mathbf{X}_j$ are statistically (locally) independent.

To carry out such a search would require knowledge of the rules of employment of concept "ψ" that signifies phenomenon ψ, for phenomenon ψ is just a constituent of natural reality to which the rules of employment of concept "ψ" warrant application. One would also have to be able to justify the claim that the scores that comprise the distribution of variate $\mathbf{Y}$ are signified by concept "ψ".

If path (1) had been followed, reports of latent variable analyses would have sounded something like the following: "We tested to see if anxiety [scores with respect to which were produced by rule r_A (see appendix for details) and the distribution of variate $\mathbf{Y}$ containing these scores in population P_T] was a latent variate to the phenomena represented by $\underline{\mathbf{X}}$. That is, we tested whether $\mathbf{Y}$ renders the $\mathbf{X}_j$ conditionally independent (see McDonald, 1996a). However, $\mathbf{Y}$ did not render conditionally independent the $\mathbf{X}_j$, and, hence, was not a latent variate to the $\mathbf{X}_j$. We will soon be testing depression for latent variate-hood." And if it so happened that the scores that comprise the distribution of $\mathbf{Y}$ were shoe sizes, the $\mathbf{X}_j$, various measures of intellectual functioning, and, conditional on $\mathbf{Y}$, the $\mathbf{X}_j$ were conditionally independent, then shoe size would, according to McDonald's definition, be a latent variate to the $\mathbf{X}_j$. Regardless, neither the CA, nor latent variable models, would have been required if path (i) had been taken.

ii) The psychometrics community might have come to recognize that the expression "defined by the principle of local independence" was, in fact, construction talk. That is, it was an expression of a required ingredient in a *recipe* for the construction of random variates which can rightly be called latent variates to $\underline{\mathbf{X}}$.

Neither of paths (a), nor (b), were taken by the discipline of psychometrics. Path (a) does not, in fact, square with the unobservability property that McDonald (1972, 1974, 1975, 1977), and so many others, has urged is necessary for *latent variate*-hood. In what sense would "shoe size" or "weight" be unobservable should either render conditionally independent a set of variates? And, far from taking path (b), psychometrics, with McDonald contributing significantly, evolved the extra-statistical prop of the CA to explain what was meant by *latent variate*. Moreover, McDonald's requirement that *latent trait/common factor* be instantiated on the basis of the property of local indepence does not square with his other preferred requirement, to wit, that latent traits/common factors be common properties/attributes signified by ordinary language concepts. This is because the grammars of such concepts make no mention *whatsoever* of statistical properties, let alone the particular statistical property of local independence. For example, in teaching to another person the third person application of the dispositional term *dominant*, no explanation of the property of local independence is required. McDonald's 1996 claim that "latent traits/common factors are defined by the principle of local independence" merely licenses application of *latent trait/common factor* to *any* random *variate* that renders conditionally independent a set of input variates. But the practice of latent variable modeling cannot coherently involve the *search* for a variate, for variates are not constituents of natural reality, but are, rather, created by human beings. McDonald's definition features one in a list of requirements that must be satisfied by a constructed random variate in order that it replace a set of input variates in precisely the sense insisted upon by some particular latent variable model.

The analysis of *unobservability* contained in McDonald (1974) has McDonald stating that a true score is unobservable because "a researcher cannot replicate observations and so cannot obtain a random variate whose mean is the true score." But why does he believe that observations can't be replicated in psychology? More to the point, what is meant by the claim that observations *are* replicable in the case of certain physical measurement operations? Certainly, if one measures a property κ by following rules r_κ for the measurement of κ, then, as long as these rules are clear, one can replicate measurements of κ. For to replicate measurements of κ is not to produce *identical* numbers, but to produce multiple numbers each of which is a measurement of κ by virtue of the fact that it was produced by following r_κ. That identical measurements are not, in fact, produced is a very practical matter, resulting in part from the non-identicality of the behaviour humans produce when they follow rules. The proper target of McDonald's concern should not be the issue of the *replicability* of measurements, for replication within a practice of measurement means nothing essentially different from replication within any rule guided practice, namely, the repeated following of a set of rules.

McDonald fails to grasp that the reason that the observations that he is talking about, i.e., those on a "true score" random variate, cannot be replicated is not due to their unobservability, nor that they are not "physical quantities", but the fact that there are no antecedently existing rules for the production of such "measurements" (i.e., there exist no rules that a researcher *could* follow). A true score is a construction born from the marriage of a set of input variates and a set of mathematical equations. No definition is given of a property "φ" for which the referred to true-scores are to be measurements. Hence, there can be no rule r_φ for the production of any such scores. If by "observation taken with respect the true score of variate $\mathbf{X}_1$" McDonald means a measurement of some particular property, i.e., a score signified by some particular concept, then McDonald literally does not know what he means, because no definition is provided of any such concept that is supposed to signify the scores that comprise the distribution of $\mathbf{X}_1$. *Following analysis*, such scores may be taken as realizations on constructed random variates, but these scores are certainly not measurements. In the physical sciences, definition is the starting point. The fact that one can produce measurements of mass presupposes a criterion for the employment of the concept. There are well known rules for the production of measurements of the masses of objects, though repeated application of these rules in regard a particular object would, of course, result in a set of numerically non-identical measurements.

Component variates have traditionally been seen as unproblematic because they are functions, non-linear or linear, of a set of input variates. As was seen in Chapters III and V, the CA portrays component "models" as inferior to latent variable "models" because, according to the CA, component models are not detectors of unobservables. To sum up, received opinion on how to define *manifest*- and *latent*-variate, and on how to distinguish these concepts from that of *component variate*, has presupposed the nonsense of the CA. And while it can be dangerous and unpopular to cut loose from conventional usage, this is exactly what will be done in the current work, this departure a necessity if, finally, a reasonable account is to be given. With no further ado:

i) The relevant distinction with regard the employment of component and latent variate generators is that between *input variate* and *replacement variate* (instances of the latter class of variate being the component and latent variates). The *manifest variate/latent variate* dichotomy must be dispensed with as serving no other purpose than to propogate the CA mythology.

ii) The scores that comprise the distribution of an input variate (in a population *P* of objects under study) are *meaningful* in certain desirable senses. At the least, these scores are produced by following a rule of score production, and because it is not possible to follow a rule without knowing it, the rules for the production of the scores that comprise the distributions of input variates are known to the researcher prior to analysis. Moreover, under certain special circumstances, these scores can rightly be claimed to be measurements taken with respect a particular concept, "ψ" (e.g., *length of skull*), in which case they are signified by concept "ψ" (if {1.3,2,6,4.2} are measurements (in cm.) of the lengths of three caterpillars, these numbers are signified by the concept *length*).

iii) Prior to analysis of $\underline{\mathbf{X}}^*$, there exists no rule for the production of the scores that comprise the distribution of a replacement variate (i.e., a component or latent variate) to $\underline{\mathbf{X}}^*$. These types of variates are represented in the equations in which they appear by symbols that are place-holders for any variate that satisfies the requirements stipulated by the generator as required for replacement variate-hood, should $\underline{\mathbf{X}}^*$ be describable by the generator. Given that $\underline{\mathbf{X}}^*$ is describable by generator R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$], the rule for the production of scores on a resulting replacement variate is made available to the researcher in the form of a construction formula.

iv) However, even if $\underline{\mathbf{X}}^*$ should happen to be described by particular replacement variate generator R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$], and, hence, the $\mathbf{X}^*_j$'s be replaceable (under R_1:[f($\underline{\mathbf{X}}$;**r**),D,$\underline{\Pi}$]), the fact that one can offer up the resulting construction formula by which replacement variates to $\underline{\mathbf{X}}^*$ can be produced does not make the scores that comprise the distributions of these replacement variates *meaningful* in the way that scores on the input variates can be meaningful. For realizations on a constructed random variate, be it a component or a latent variate, are not signified by *any* concept from ordinary language. Moreover, while an input variate can be invented so as to represent a particular phenomenon of interest, this brought about through the laying down of a rule of correspondence prior to analysis, no rules of correspondence are layed down to assign replacement variates this representational role. Replacement variates are, rather, variates constructed so as to possess certain pre-specified properties. The sense in which scores on such variates are meaningful is just that they are precisely the scores that do, in fact, possess these pre-specified properties.

comment: Note that, as long as the issue is the "meaning" of the scores comprising the distribution of a random variate, the fact that a replacement variate is a function (e.g., a linear combination) of the input variates is irrelevant. For example, the fact that variate **c** is constructed as $\underline{v}_1$'$\underline{\mathbf{X}}$, in which $\underline{v}_1$ is the first eigenvector of the distribution of $\underline{\mathbf{X}}$, ensures that **c** is determinate (in that Card(C)=1), but does not bring about the signification of scores on **c** by an ordinary language concept. Regardless of the meanings of the input variates, $\mathbf{X}_j$, it is not as if the scores that comprise the distribution of **c** might turn out to be signified by, e.g., *anxiety*, for such functions of a set of input variates have no place in ordinary language. Conceptual signification does not come about via the statistical properties of random variates.

Now, (iv) suggests a key reason for the often frantic defense of the Central Account that was mounted by certain of the psychometricians who took part in the indeterminacy debate. The reader will recall that the CA promised that latent variable models can be used to detect unobservable properties/attributes (causal sources) that are, somehow, inherently meaningful, but whose identities must be guessed. In particular, when a particular $\underline{\mathbf{X}}$ is described by a particular latent variable model (generator), the CA explains the researcher's task as being to make an inference as to the (unknown) identity of the ordinary language concept presumed to signify a detected property/attribute (causal source). This process is called "interpreting the latent variate". As was seen in Chapter III, belief in this facet of the Central Account is rampant within psychometrics. When McDonald and Mulaik invoke notions such as "empirically meaningful" and "uniquely defined" (described in Chapter V), they are simply asserting the truth of the CA's promise that the scores that comprise the distribution of θ to $\underline{\mathbf{X}}$ are somehow inherently meaningful. That the concept *latent variate to* $\underline{\mathbf{X}}$ denotes the elements of a set of constructed random variates does not square with the CA, and brings psychometrics face-to-face with a reality it has never wanted to admit, to wit, that many of the variate types with which it deals lack certain prized forms of meaningfulness. Rozeboom said as much in 1988, correctly diagnosing a chief source of angst at the root of the classical indeterminacy debate to be, not the nonuniqueness of the referent of *common factor to* $\underline{\mathbf{X}}$, but, rather, the fact that, following a factor analysis, the *identities* of such referents were not determinable.

Constructed random variates simply do not *have* identities in this desired sense, because the scores that comprise their distributions are not signified by concepts from ordinary language. The reality is that the "models" of psychometrics are replacement variate generators, and replacement variates lack many of the senses of meaning rightly prized by the behavioural and social scientist. The discipline of psychometrics must either return to its CA induced bewitchment, or turn to a consideration of how to conceptualize and describe the constructed random variates that are the currency of the statistical machinery that it produces. The following is a brief synopsis of some of the facets that are important in describing constructed random variates (component and latent variates):

i) Fundamental to the description of, and differentiation between, the replacement variates constructed under distinct generators is the formula by means of which these variates are constructed. When one asks, "what is meant by the first principal component of $\underline{\mathbf{X}}^*$", a legitimate answer is to provide the formula by which this random variate can be constructed. So too for the question "what is meant by common factor to $\underline{\mathbf{X}}^*$."

ii) As was previously noted, replacement variates generated under a particular generator are optimal in a sense particular to the generator. Hence, in describing a particular generator, it is proper to describe these senses of optimality, as well as the costs they entail, and other general features of the replacment (e.g., Card(C)).

iii) The replacement variates generated under a particular generator to replace a particular set of input variates are random variates and thus are describable in terms of their statistical properties, both those that they must satisfy (e.g., any distributional requirements imposed by the generator), and those that are contingent features (e.g., correlations with variates not included in the analysis).

While the official word of psychometrics on the topic of latent variable modeling has always presupposed the Central Account, certain activities of psychometricians themselves betray the falsity of this account. For example, the random variates denoted by $\boldsymbol{\theta}$ in the equations of latent variable models are standardly *assigned* distributional properties. Thus one is told: "Since there is no "natural" scale in such cases we are at liberty to construct one to suit our convenience" (Bartholomew, 1980, p.296); "It is important to note that the common factor of the random variables y'_g, g=1,2,...,n, is itself not a uniquely defined variable. In fact any variable θ whose correlation with each y'_g is ρ'_g, g=1,2,...,n, can be called "the common factor of the y'_g". The reader can verify that if θ and E_g, g=1,2,...,n, are uncorrelated random variables, each with unit variance, then the correlation matrix between the Y_g defined by $Y_g=\rho_g\theta+E_g\sqrt{1-\rho^2_g}$ is of rank 1 when the ρ^2_g are placed in the diagonal. Thus θ is a common factor of Y" (Lord and Novick, 1968, p.373). One does not, however, *assign* characteristics to the distribution of an existing property in some population P. Such characteristics are the objects of empirical investigations and discoveries. Specifying antecedently the distributional features of $\boldsymbol{\theta}$ is equivalent to specifying some of the requirements that must be satisfied in its construction, even if this fact is often obscured by the dubious practice of calling such requirements "assumptions."

Similarly, when Cureton and D'Agostino (1983, p.8) describe Thurstone's principle of parsimony of linear factor representations, they are describing a pragmatic step that can be taken in order to overcome an obstacle, rotational indeterminacy, to the creation of replacement variates to $\underline{\mathbf{X}}$. In contrast to its *usual* portrayal, Thurstone's principle of parsimony was *not* a scientific principle for choosing between two "theories" or "empirical accounts." Science is concerned merely with arriving at the *true* theory or empirical account. It does not prescribe properties to the entities it studies, but, rather, discovers such properties. But factor analysis involves the construction of random variates, and Thurstone's pragmatic principle of parsimony is a recipe for the construction of random variates possessing certain attractive properties.

iv) Finally, the replacement variates constructed under a particular generator to replace a particular set of input variates, are optimal in senses particular to the generator. Variates that are optimal in one sense or another will often prove to be useful in particular scientific contexts, else there would have been little reason for their invention. To note these uses is, then, a fundamental means of describing a particular type of replacement variate (and its generator).