III

The Central Account

Simplex sigillum veri: The charm of the picture is in its simplicity and naïvety.

1. *Introduction*

The psychometrician's grasp of the mathematics inherent to latent variable modeling has reached an impressive level of sophistication. However, there can exist a wide gap between mathematical and other types of understanding. In particular, knowledge of the mathematical details of latent variable models does not settle issues pertaining to the interpretation of the results generated in their employment. In their introductory remarks on latent variable models, Bartholomew and Knott (1999, p.12) state that

> A model serves to clarify the conceptual basis of the subject and provides a framework for analysis and interpretation. In a subject which has been criticized as arbitrary and hence too subjective, it is especially necessary to clarify in as rigorous a way as possible what is being assumed and what can be legitimately inferred.

But this is mistaken. Statistical models do not clarify the conceptual basis of the subjects for which they are models. On the contrary, in order to successfully create a model, and use it fruitfully in scientific work, the scientist must be able to articulate the rules of correspondence that link terms of the model to that which is to be modelled. The conceptual bases of latent variable modeling remain obscure, and statistical theory can say nothing when the issue is, for example, what, if anything, is signified by the symbol $\boldsymbol{\theta}$. The calculus of probability theory cannot itself be used to explain what are the referrents of the symbols on which it operates.

Many experts are at pains to point out that latent variable models are still *useful* regardless of the meaning of the concept *latent variate*. Latent variable models are, after all, testable claims about features of the joint distributions of sets of variates. The reader is told that

> Much of the philosophical debate which takes place on latent variable models centres on *reification*...However, the usefulness and validity of the methods to be described in this book do not depend primarily on whether one adopts a realist or an instrumentalist view of latent variables (Bartholomew and Knott, 1999, p.2)

and that

> One of the main controversies regarding FA is whether or not the factors have any real existence and have causal rather than just statistical implications. Some researchers in psychology maintain the reality of factors by giving them psychological names. However, the question of existence need not be established before a model can be used, though care is needed in interpreting such a model (Seber, 1984, p.222)

These claims, however, are perhaps somewhat disingenuous. Exactly what sense of "validity" and "usefulness" do Bartholomew and Knott have in mind? In regard Seber, is the fact that one can "use" a model truly an impressive fact? A more honest appraisal is provided by De Leeuw (1996, p.viii):

> Factor analysis, for example, can be formulated as a structural model for observed covariances, without even mentioning latent variables. They are useful, maybe even necessary, for the interpretation of the results.

In fact, it is not at all clear how a latent variable model could be "useful" in the absence of a convincing explanation of what is meant by concept *latent variate to **X***, and what can be discovered through the employment of latent variable models in general. Despite enormous efforts invested in the development of statistical inference for latent variable models, mere statistical testability is clearly *not* what is important, but, rather, the natures of the empirical propositions that can, in fact, be tested when latent variable models are tested. Issues pertaining to the meaning of *latent variate to **X*** are not just important, but, rather, are the essence of latent variable modeling. This is why

> Psychologists, who were still the main users of factor analysis, continued on their separate way developing the language and mystique of the subject...
> (Bartholomew & Knott, 1995, p.216).

DeLeeuw (1996, p.vii) too mentions psychology as a source of the mystification of latent variable models: "Statisticians were intially very critical of these models. This was caused, to some extent, by grandiose claims from psychologists such as Cattell." But whether psychologists are solely to blame for the mystification of latent variable models is open to debate, for statisticians who speak of "unobservability" or portray latent variable models as detectors of causes are invoking a facet of the same extra-statistical picture favoured by psychologists.

Latent variable models are perceived as being *useful*, and have gained their enormous popularity, precisely because their use, over time, has come to be conceptualized in terms of a compelling picture. This picture is, in this book, called the *Central Account.* It's core theses are that:

1) When a set of manifest variates **X** are described by a latent variable model, a property/attribute or cause of the phenomena represented by the manifest variates on which analysis centres has been detected

2) The concept *latent variate to **X*** denotes this detected entity;

3) This property/attribute (causal source) is unobservable, or unmeasurable, or underlying, or hypothetical (or some combination of these);

4) Latent variable models are testable, but not merely in the ordinary sense of statistical testability. To test whether **X** is described by a latent variable model is not merely to test a claim made about a parameter of the distribution of **X**, but to test an hypothesis about the existence of a property/attribute (causal source) of the phenomena represented by **X**;

5) Because that which is signified by *latent variate to **X*** is an *unobservable* property/attribute (causal source), its identity (the concept-name by which it should rightly be

called) and properties can only be inferred or estimated; 5) Latent variable models are, on epistemological grounds, special, when compared with component and other statistical models. They may be used to detect unobservable properties/attributes and causal sources.

There can be found in the extant literature on latent variable modeling countless variations on these core themes.

The linear factor model was the first of what are now called latent variable models. Many early factor analysts were convinced that linear factor analysis was a tool for behavioural research of unprecedented power. They believed it to be, on epistemological grounds, special. This view was a product of semantic commitments regarding the technical concept of *factor*, and ontological commitments regarding the putative referents of the concept. One can detect three significant streams of thought in early writings on linear factor analysis. First, when the linear factor model describes a set of manifest variates, **X**, the referent of the concept *factor to* ***X*** is an entity or force of some sort. Second, this entity or force has ties, in some way, to biology or genetics. Third, this entity or force is "responsible for", perhaps causally, the phenomena represented by the manifest variates. The related conception of the factor as an error free version of the manifest variates had not yet come into prominence, even though Spearman had developed the linear factor model at the same time as classical true-score theory. Over the years, a great many latent variable models were invented and put to use in reasearch. Gradually, less mention was made of putative biological and genetic bases to latent variates, and discussions of what latent variable modeling was about came to be expressed in subtler commitments to the Central Account. A measurement-based interpretation of the latent variate/manifest variate relationship, joined, and then gradually came to surpass the causality picture of latent variable modeling as the dominant picture. Most strikingly, an extensive array of terms, "unobservability talk", evolved to "explain" the nature of the referent of *latent variate to* ***X***.

The CA is an *ürbild* or *proto-picture*. It informs thought on the topic of latent variable modeling, but its theses are not the fruits of careful analysis. For example, the motivated reader will be hard-pressed to find an explanation of the concept *unobservable*, as employed within the context of latent variable modeling, that goes beyond a one sentence cliche. More sophisticated attempts might cite an empirical realist philosopher. Advocates of latent variable modeling will undoubtedly attempt to portray handling of the concepts of *latent variate*, *unobservability*, etc., as progressive, arguing that, if a definitive account has not yet been achieved, there has at least been some movement towards this ideal. In fact, there has been almost no scholarly thought on these issues, but rather the mindless repitition of elements of the Central Account, these turned into slogans for easy consumption (consider, for example, "factors cannot be determined, but only estimated" or "manifest variates are observable"). The various components of the CA are tacitly endorsed. Yet, because, by now, the account is deeply familiar to those who use and study latent variable models, it is taken to be *fact*, or received empirical *theory*, rather than world view. In some hazy, poorly specified way, it is just obvious that latent variable models and modeling are "about" that which the CA describes.

It would undoubtedly be incorrect to suggest that *all* who work within the area of latent variable modeling commit, tacitly or otherwise, to this proto-picture. It is always risky to attribute to an individual such a view, unless, of course, he admits to holding it. Moreover, virtually all of the "heavy-weights" of latent variable modeling have, at one point or another, disclaimed one or another of the features of the Central Account. However, it is true that such disclaimers are often accompanied by multiple claims which betray adherence to the Central

Account. At least fragments of the account are detectable in the discourse of a *great many* applied researchers and experts in the field of latent variable modeling. Many experts will, no doubt, claim that the linguistic props of the CA are merely "helpful" in conceptualizing work that employs, or is on, latent variable models, and, in any event,"are not taken too seriously." But their own actions (the course they have set for psychometrics) contradict this spin time and time again.

The aim of this chapter is to provide a relatively detailed sketch of the core features of the CA, and to demonstrate the commitment to it of experts and applied researchers alike[1]. A detailed explication is first given of each of the core theses of the Central Account. Next, a number of influential sources, these having shaped the practice of latent variable modeling, are shown to be predicated on the CA. Finally, examples of the CA taken from a variety of theoretical and applied secondary sources are given.

2. *The central account*

Let there be a particular latent variable model, lvm, a particular set of manifest variates, $\mathbf{X}_j$, j=1..p, distributed in a particular population, P_T, let $\underline{\Pi}$ contain the parameters of lvm (contained in subspace π), let $\underline{\Omega}_T$ contain the values, in P_T, of the parameters of which lvm makes claims, and M_{lvm} be the manifold traced out by lvm as $\underline{\Pi}$ ranges over all possible values in π.

The measurement and causality pictures

(i) In any latent variable analysis, a firm distinction must be drawn between the manifest variates, $\underline{\mathbf{X}}$, and the latent variate, $\boldsymbol{\theta}$. The scores that comprise the distribution of manifest random variate $\mathbf{X}_j$ are produced in accord with an antecedently specified rule r_j of score production. That is, for each manifest variate, one can provide, at the least, a rule by which the scores on the variate were produced, in advance of conducting the analysis[2]. One cannot do this for the scores that comprise the distribution of the latent variate. Realizations taken on $\boldsymbol{\theta}$ are not included in the data to be analyzed.

(ii) If $\underline{\Omega}_T \subset M_{lvm}$, i.e., $\underline{\mathbf{X}}$ is described by lvm, then:

a. This constitutes evidence of the existence of a property/attribute common to (causal source of) the phenomena represented by the $\mathbf{X}_j$. That is to say, the researcher has evidence that a property/attribute (cause) has been detected; b. The concept *latent variate to* $\underline{\mathbf{X}}$ signifies this property/attribute (causal source); c. The scores that comprise the distribution of the random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ are measurements with respect this property/attribute (causal source).

(iii) There are two primary elaborations of (ii).

Causality picture (CAC): i) The realm of psychological phenomena includes, among other things, "objects" and their unobservable causal sources; ii) These unobservable causal

1 The latter task is continued in Chapter 5, Responses to Indeterminacy.
2 For example, scores on manifest variate 1 are produced by having individuals answer the questions of the Brown Anxiety Inventory, coding these responses as per the test's manual, and summing these coded responses.

sources exist independently of whether they are perceived by humans, and possess identities independent of human conceptualization; iii) The job of science is to detect these sources and correctly identify them (i.e., call them what they really are, by the concept-name by which they should rightly be called); iv) The conditional statistical independence (or conditional uncorrelatedness) of $\underline{\mathbf{X}}$ given $\boldsymbol{\theta}$, a defining feature of lvm, is a definition of causality. Hence, lvm can be used to detect, identify, and study these unobservable causal sources operating within the realm of psychological phenomena; v) If $\underline{\Omega}_T \subset M_{lvm}$, i.e., $\underline{\mathbf{X}}$ is described by lvm, then: a) a causal source of the phenomena represented by the manifest variates has been detected; b) The concept *latent variate to* $\underline{\boldsymbol{X}}$ signifies this unobservable causal source; c) The scores that comprise the distribution of the random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ are the scores of individuals in P_T with respect this detected causal source; vi) Because the causal source is unobservable, even following its detection, its identity and empirical properties remain unknown. Model-based inferences must, therefore, be made. In particular, the researcher must *identify* the detected causal source, meaning that he must infer the concept-name by which the detected causal source should rightly be called. In so doing, he replaces the generic *latent variate to* $\underline{\boldsymbol{X}}$ with the inferred concept-name. This inference is called "latent variate interpretation" (in factor analysis, "factor interpretation"); vii) The score of an individual with respect the causal source is unobservable, and, hence, unknowable. It must, therefore, be estimated (cf. "factor score estimation). There will always be present error in an estimate of an individuals *true* score on the latent variate; viii) It may be said that the manifest variates *manifest* the effects of their causal source, and, hence, are "indicators" of its existence.

Measurement picture (CAM): i) The realm of psychological phenomena includes, among other things, objects (e.g., humans) and their properties/attributes; ii) In contrast to physical properties/attributes, many psychological properties/attributes are unobservable; iii) These unobservable properties/attributes exist independently of whether they are perceived by humans, and possess identities independent of human conceptualization; iv) The job of science is to detect these properties/attributes, identify them (i.e., call them what they really are, by the concept-name according to which they should rightly be called), and catalogue them; v) Latent variable models may be used to detect, identify, and study these unobservable psychological properties/attributes; vi) If $\underline{\Omega}_T \subset M_{lvm}$, i.e., $\underline{\mathbf{X}}$ is described by lvm, then: a) a property/attribute common to the phenomena represented by the manifest variates has been detected; b) The concept *latent variate to* $\underline{\boldsymbol{X}}$ signifies this unobservable property/attribute; c) The scores that comprise the distribution of the random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$ are the values of individuals with respect this property/attribute; viii) Because the property/attribute is unobservable, even following detection, its identity and empirical properties are unknown. Model-based inferences must, therefore, be made. In particular, the researcher must *identify* the detected attribute/property, meaning that he must provide the concept-name by which the property/attribute should rightly be called. In so doing, he replaces the generic *latent variate to* $\underline{\boldsymbol{X}}$ with the inferred concept-name. This inference is called "latent variate interpretation"; ix) An individual's measurement with respect the detected property/attribute is unobservable, and, hence, unknowable. It must, therefore, be estimated (cf. "factor score estimation). The estimate of the unobservable measurement of an individual is an appropriately chosen function, $t(\underline{\mathbf{X}})$, of the manifest variates. This function is then an error laden measure of the *true* property/attribute; x) The latent variable modeller must estimate the error of measurement inherent to the employment of $t(\underline{\mathbf{X}})$; x) It may be said that the manifest variates jointly measure the detected property/attribute, albeit with error. Or, one may, depending on one's preferences, state that: a) The manifest variates are fallible measures of the latent variate, which is a "trait", "ability", "disposition", etc.; b) The latent variate is a signal, and the manifest variates, noise corrupted "receivers" of this signal; c) The latent variate is "truth", and the manifest variates, "truth" plus "error"; d) The latent variate is a fundamental or principal variate, or dimension, which "underlies" the empirical domain of interest; e) The latent variate is an abstractive property.[3]

Latent variable models are tools which may be employed by the researcher to get past the "mere appearance" offered by the manifest variates and make inferences about "true" properties/attributes (causal sources) existing in the unobservable (latent) domain underlying observable psychological phenomena, these properties/attributes (causal sources), rather than their manifest counterparts, being of chief interest to the researcher. Theses (CA1) to (CA5) are elaborations of the causality and measurement pictures.

CA1: Identity and empirical nature

(i) The unobservable properties/attributes (causal sources) that populate the realm of psychological phenomena are *out there*. They exist regardless of whether they are perceived to exist, and whether, in the end, they are, in fact, detected.

(ii) A given unobservable property/attribute (causal source) has an identity independent of human conceptualization. This identity is the concept whose name would, if known, be employed in place of the generic *latent variate to* ***X***.

---

3 Slogans such as "latent variates (factors) are causes" or "latent variates (factors) are properties/attributes" are unsophisticated versions of the claims that the concept *latent variate to* ***X*** *designates* a cause (property/attribute)." They are unsophisticated versions because they inadvertently conflate the concept *latent variate to* ***X***, the symbol $\boldsymbol{\theta}$ in the equations of a latent variable model, the scores that comprise the distribution of the random variate $\boldsymbol{\theta}$, and the real-world property/attribute (causal source) believed to be signified by the concept *latent variate to* ***X***, and implied in the discussions of latent variable modellers. Concepts themselves, they being elements of a language, do not have causal effects. Nor do random variates, nor the scores that comprise the distribution of a random variate. A particular entity or force may have causal effects, and the researcher might be able to measure a particular entity or force in respect a particular property. Likewise, it is not the concept *height* that is measured, but the heights of trees, people, etc. A measurement is *taken* of the height of something, and then, it might be said, this number, just because it was produced *as such*, is signified by the concept of *height*. The slogan "the common factor of **X** is a cause of **X**" only has coherence if it is unpacked as, "The concept *latent variate to* ***X*** signifies a constituent of natural reality that is a cause of the phenomena represented by the manifest variates, and measurements (of the individuals under study) taken with respect this constituent comprise the distribution of the latent variate to **X**." Similarly, "the latent variate is a property or ability" only makes sense if it is taken as short-hand for "the concept *latent variate to* ***X*** signifies a property/attribute of the phenomena represented by the manifest variates, and measurements (of the individuals under study) taken with respect this property/attribute comprise the distribution of $\boldsymbol{\theta}$."

(iii) A given unobservable property/attribute (causal source) may be characterized by a list of empirical properties, including its relationships with other unobservable properties/attributes (causal sources), and, in particular, its place in causal networks.

CA2: Unobservability

(i) If $\underline{\Omega_T}\subset M_{lvm}$, i.e., $\underline{\mathbf{X}}$ is described by lvm, the attribute/property (causal source) signified by *latent variate to* $\underline{\boldsymbol{X}}$ is *unobservable*. It *underlies* the phenomena represented by the manifest variates (in senses described by CAC and CAM), but is not perceptually available.

(ii) The unobservability of the property/attribute (causal source) detected when $\underline{\mathbf{X}}$ is described by lvm is problematic. It means that, even following its detection: a) the identity of this property/attribute (causal source) (i.e., the concept-name by which it should rightly be called, and which should properly replace the generic *latent variate to* $\underline{\boldsymbol{X}}$) cannot be known to the researcher; b) The empirical nature of this property/attribute (causal source) cannot be known to the researcher.

comment: It is as if the referent were hidden behind a barrier. The latent variable modeler has, in a sense, located the *position* of the referent behind the barrier (for, when $\underline{\mathbf{X}}$ is described by lvm, *latent variate to* $\underline{\boldsymbol{X}}$ does signify *some* property/attribute (causal source)), but cannot remove the barrier to observe it. The referent, thus, cannot be identified. In a given instance, the identity of the referent might be e.g., *anxiety* or *self-esteem*, but, given unobservability, the modeller cannot know.

(iii) Latent variable models are needed precisely because the "entities" to be detected or discovered are unobservable (else, no special methodology would be required, and latent variable modeling would have no reason for being).

(iv) Because unobservability is an insoluble problem, neither the identity, nor empirical nature, of a detected property/attribute (causal source) is *knowable*. Partial knowledge, gained by way of model-based inferences is all that is achievable.

CA3: Model and Testability

(i) There can exist a property/attribute common to (causal source of) the phenomena that are represented by the manifest variates $\underline{\mathbf{X}}$. If such a property/attribute (causal source) does, in fact, exist, then it is unobservable. It follows then that standard methodologies cannot be used to investigate its possible existence, nor, if it exists, its identity and empirical nature. Special tools, latent variable models, are required to detect and describe these unobservable entities.

(ii) A test of whether $\underline{\Omega_T}\subset M_{lvm}$, i.e., whether the distribution of $\underline{\mathbf{X}}$ is described by lvm, is a test of an hypothesis of the existence of an unobservable property/attribute (causal source) of the phenomena represented by $\underline{\mathbf{X}}$. A latent variable model is a tool of detection, and one may express this fact by stating that the manifest variates *pick-up*, *detect*, and *tap into* these unobservables.

(iii) It follows from (ii) that the sense in which a latent variable model is testable is distinct from mere statistical testability. To test a latent variable model is not simply to test the truth of a claim about the value of a parameter that governs the distribution of $\underline{\mathbf{X}}$, but, rather, to test an existential hypothesis about an unobservable property/attribute (causal source) of the phenomena represented by the $\mathbf{X}_j$.

(iv) The special sense of testability associated with the employment of a latent variable model is *not* a feature of component and other statistical models. With a component model the researcher cannot test for the existence of unobservable properties/attributes (causal sources). Component models do not "go beyond the data."

(v) A latent variable model is a "model" in the classical sense of the term. That is, if $\underline{\Omega}_T \subset M_{lvm}$, lvm describes a particular state of natural reality. The $\mathbf{X}_j$ terms each represent a phenomenon of interest, the $\boldsymbol{\theta}$ term represents the detected unobservable property/attribute (causal source) of these phenomena, scores on which are signified by a concept whose name must be inferred by the researcher. Moreover, the model equations and distributional specifications that link the $\mathbf{X}_j$ terms to the $\boldsymbol{\theta}$ terms represent the natures of the relationships that exist between the phenomena represented by the $\mathbf{X}_j$, and the detected property/attribute (causal source) represented by $\boldsymbol{\theta}$.

CA4: Inference and estimation

(i) If $\underline{\Omega}_T \subset M_{lvm}$, i.e., $\underline{\mathbf{X}}$ is described by lvm, the latent variable modeller can rightly only make the claim that an unobservable property/attribute (causal source) has been detected. Even if he is correct in regard this claim, he still does not know the identity of this unobservable entity, nor anything about its empirical nature. Moreover, as is clear from CA3, he cannot *know* these facts. He must, instead, rely on inferential procedures. In particular, he must: a) Make an inference regarding the identity of the detected property/attribute (causal source). This inference is made by employing the parameter estimates, $\hat{\underline{\Pi}}$, (notably, the estimates of the correlations between the manifest variates and the latent variate) obtained in an application of lvm to a sample from P_T, to "interpret the latent variate (factor)." By this, it is meant that the researcher makes an informed guess as to the concept that signifies the detected property/attribute (causal source). In doing so, he replaces the generic *latent variate to* $\underline{\boldsymbol{X}}$ with a concept-name, e.g., *general intelligence*, *ability* _, *trait* _, etc., whose choice seems most appropriate in light of $\hat{\underline{\Pi}}$; b) Make inferences about the empirical nature of the detected property/attribute (causal source). These inferences are also made on the basis of $\hat{\underline{\Pi}}$, and, in particular, the conditional moments of $\boldsymbol{\theta}$ given $\underline{\mathbf{X}}$.

(ii) Inferences regarding the identity and empirical nature of the referent of *latent variate to* $\underline{\boldsymbol{X}}$, i.e., the detected property/attribute (causal source), are made by the researcher with the understanding that he may be incorrect, and that other researchers might make, based on the same data, conflicting inferences.

(iv) If $\underline{\Omega}_T \subset M_{lvm}$, an unobservable property/attribute (causal source) has been detected, and each individual in P_T has a score with respect this property/attribute (causal source). These scores comprise the distribution of the random variate $\boldsymbol{\theta}$ to $\underline{\mathbf{X}}$. Because the detected property/attribute (causal source) is unobservable, it is not possible to know an individual's score with respect it. The researcher must employ manifest information to predict/estimate this latent measurement.

CA5: Epistemological specialness

(i) Theses one to five lead unavoidably to the conclusion that the latent variable model is special on epistemological grounds. It may be used to detect unobservable properties/attributes (causal sources) of the phenomena represented by the manifest variates. It is a tool that can be used to make inferences as to the identities and empirical natures of these unobservable entities. Simply put, no other brand of statistical model makes this particular contribution to the scientific endeavour.

(ii) Latent variable models are especially useful when employed within the behavioural and social sciences, because the phenomena of interest within these domains of investigation are very often unobservable.

(iii) Component models, involving mere transformations of the manifest variates, are, in contrast to latent variable models, not epistemologically priviledged.

As well as informing technical work undertaken on latent variable models, the Central Account informs empirical work involving latent variable models, and, hence, shapes work in many branches of the social and behavioural sciences. The CAM, for example, licenses the researcher to interpret the residual terms of latent variable models as measurement error variances, and the latent variable model as a detector of properties (thus encouraging him to bypass the definitional work that is a hallmark of sound scientific practice).

3. *The CA in the work of experts and applied researchers*

Commitment to either of the causality or measurement pictures is as old as latent variable modeling itself. Blinkhorn (1997, p.180) puts it well: "The words they used, for example 'primary mental abilities' (Thurstone) or 'source traits' (R.B. Cattell), are witness to the faith and trust placed in factor analysis as revealing the psychological analogues of the periodic table of elements, or the list of subatomic particles." However, the measurement and causality pictures have different histories, even though Spearman invented classical true-score theory and linear factor analysis at roughly the same time, and, as Bentler (1986, p.40) notes, the true score of the classical true-score model is "..a very limited latent variable, that is, a measured variable purified of random error."[4] Whereas the causality picture has been a feature of latent variable modeling since the early history of these models, the measurement picture did not come to full prominence until later. Its ascendancy coincided with the evolution of item response theory (a tool that has been used extensively in the attempt to evaluate measurement claims), the 1952 publication of Frederic Lord's book "A theory of test scores", the 1968 publication of Lord and Novick's book "Statistical theories of mental test scores", and the publication of Cronbach and Meehl's 1955 paper on construct validation[5].

3a. A selection of primary sources

C. Spearman

---

4 McDonald (1981) has reminded the latent variable modeler that *true score random variate* and *common factor* are not synonymous concepts, "defined" as they are on the basis of different principles. Exactly how great is the cleavage between the two is a topic for later.

5 While this paper is clearly founded on a primitive empirical realism, and, hence, is committed to the idea of the detection of latent *causes* (as is all of Meehl's later work on taxometrics), it effectively wed latent variable thinking to measurement concerns in the mind of the practicing social scientist.

In the writings of Charles Spearman are found a mixture of CAC and CAM, almost outrageously hopeful claims regarding the powers of factor analysis, and subtle cautionary notes regarding its use. It would be incorrect to portray Spearman as strongly in favour of any one portrayal of *g*. His writings are inconsistent in their portrayal of *g*. Some would undoubtedly have it that this was an indication of the fact that his thinking on the matter was undergoing change. But in truth, Spearman's writing often give the impression that he was more interested in appearing the victor in the battles he was waging over his two-factor theory, than in achieving a consistent, coherent statement of this theory. This is perhaps understandle given that he was breaking new ground in psychological science. On the other hand, his responses to critics of his approach often involved a subtle spin-doctoring of key facts.

In Chapter VI of his great book, "The abilitities of man, their nature and measurement", Spearman discusses the criterion of tetrad difference. He comments that

> ...whenever the tetrad equation holds throughout any table of correlations, and *only* when it does so, then every individual measurement of every ability (or of any other variable that enters into the table) can be divided into two independent parts which possess the following momentous properties. The one part has been called the "general factor" and denoted by the letter *g*; it is so named because, although varying freely from individual to individual, it remains the same for any one individual in respect of all the correlated abilities. The second part has been called the "specific factor" and denoted by the letter *s*. It not only varies from individual to individual, but even for any one individual from each ability to another (p.75).

He claims to have provided, in the Appendix, a "proof" of this claim. This proof was *not* seen by Spearman as a proof of a mathematical uniqueness, but of a uniqueness of the entities he believed his approach to have detected.[6] That is, he believed that if the two-factor theory was correct, then it proved the existence of a set of properties (general and specific intelligences) with respect to which each individual had true values. Later, he reiterates that the demonstration of the tetrad condition is equivalent to a demonstration that "...*g* and *s* certainly exist" (p.87), and claims that it enters "into all abilities whatsoever", this being the "justification for attributing so much importance to *g*, despite its purely formal character" (p.76). Spearman's 1904 paper, "The proof and measurement of the association between two things", contains an early version of the causality picture: "...another-theoretically far more valuable-property may conceivably attach to one among the possible systems of values expressing the correlation; this is, that a measure might be afforded of the *hidden underlying cause of the variations*" (p.74). This view is very similar to that of another of the fathers of correlational methodology, Karl Pearson. In discussing the work of Galton, Pearson commented that "It is easy to see that co-relation must be the consequence of the variations of the two organs being partly due to common causes. If they were wholly due to common causes, the co-relation would be perfect..." (Pearson, 1920, p.39). In summary, Spearman's writing shows commitment to both CAM and CAC.

With regard to what type of causal entity or property was *g*, Spearman entertained a number of possibilities, each of which was clear in its portrayal of the referent of *common factor*

---

6 If it was, indeed, a mathematical uniqueness that Spearman had in mind, he would have had to take the work of E.B. Wilson on the indeterminacy of his theory (see chapter 5) as a refutation of his "uniqueness" claim. Instead, he saw indeterminacy as the unpredictability inherent to the prediction of a single, "unique", property *g*.

*to **X*** as something with a material existence, whose nature was open to later discovery by means independent of factor analysis. In the end, he seemed to prefer "mental energy" as the referent:

> The next explanation of *g* to be mentioned takes the adventurous step of deserting all actually observable phenomena of the mind, and proceeding instead to invent an underlying something which- by analogy with physics- has been called mental energy. To this view the present writer attaches such great importance, that it will be reserved for exposition in two chapters specifically devoted to it (p.89)[7]

Earlier, Hart and Spearman (1912) had interpreted 'g' as "intellective energy", and claimed that it had much in common with "clear awareness" and "attention." They were also quick to outline the social policy implications of Spearman's theory: "Indeed, so many possibilities suggest themselves that it is difficult to speak freely without seeming extravagant...It seems even possible to anticipate the day when there will be yearly official registration of the "intellective index", as we will call it, of every child throughout the kingdom...The present difficulties of picking out the abler children for more advanced education, and the "mentally defective" children for less advanced, would vanish in the solution of the more general problem of adapting education to all...Citizens, instead of choosing their career at almost blind hazard, will undertake just the professions really suited to their capacities. One can even conceive the establishment of a minimum index to qualify for parliamentary vote, and above all for the right to have offspring" (pp.78-79). Certainly, Spearman's contemporaries were in no doubt as to his position with regard factor analysis. Thorndike states that "I may add that other studies of correlation made by my students and myself are unanimous in contradicting Spearman's ingenious hypothesis of one sole common element as the cause of all positive correlations" (Thorndike, Lay & Dean, 1909, p.368), while Eysenck (1952, p.109) describes both Thurstone and Spearman as acting "as if" factors were causal agencies.

G. Thomson

Godfrey Thomson was far more cautious than Spearman about the claims that factor analysis could support. However, as is clear from his comments on factor indeterminacy (see Chapter V), he nevertheless saw factor analysis as, potentially, yielding discoveries about single, imperfectly measured properties. That is, he was a proponent of the CAM. Moreover, in his 1939 book *The Factorial Analysis of Human Ability*, he too could not resist taking the referent of *common factor to **X*** to be some sort of material entity, specifically, "neural bonds, some of which are more closely linked together into 'pools'."

L.L. Thurstone

In his 1947 book "Multiple Factor Analysis", Thurstone explains factor analysis both in terms of the CAC and CAM. References to the causality picture include: Within the behavioural sciences, abilities may be "...postulated as primary causes of individual differences in overt accomplishment..." (p.52), there being support for such claims if widely different achievments are "...demonstrable functions of a limited number of reference abilities" (p.52); Factor analysis was developed "…for the study of individual differences among people, but the individual

7 In a later reply to Piaggio (1931), he downplays the role of mental energy in his theory.

differences may be regarded as an avenue of approach to the study of the processes which *underlie* [italics added] these differences" (P.55). Individual of Thurstone's statements are occasionally a mixture of the causality and measurement pictures: "one of the simplest ways in which a class of phenomena can be comprehended in terms of a limited number of concepts is probably that in which a linear attribute of an event is expressed as a linear function of primary causes" (p.1975); The principal purpose of factor analysis is to "...discover the parameters or factors and something about the nature of the individual differences that they *produce* [italics added]. The individual subjects are examined not for the purpose of learning something about them individually, but rather for the purpose of discovering the *underlying factors* [italics added]" (1947, P.325).

The general tone of Thurstone's writing has factor analysis playing the role of detector of fundamental or principal dimensions of empirical domains of inquiry, a stance consistent with CAM. This is evident in the following quotes: "The factorial methods were developed primarily for the purpose of identifying the principal dimensions or categories of mentality..." (P.55); "A factor problem starts with the hope or conviction that a certain domain is not so chaotic as it looks. The object of factor analysis is to discover the principal dimensions or categories of mentality and to indicate the directions along which they may be studied by experimental laboratory methods" (1940, p.189); In employing factor analytic techniques, the psychologist is dealing with an "...underlying trait" (P.322); The problem of factor analysis "...is to identify, if possible, the psychological trait which is common to these tests..." (1938, p.80).

According to Thurstone, one way to investigate a particular empirical domain is to "...invent a hypothesis regarding the processes that underlie the individual differences, and one can then set up a factorial experiment, or a more direct laboratory experiment, to test the hypothesis" (P.55). The researcher is hoping to "...discover in the factorial analysis the nature of the underlying order" (P.56). The notion of "underlying order" (later, "latent domain") has distinctly platonic undertones, as if there existed a realm containing inaccessible properties whose identities are *as such*, independent of human perception and conceptualization. These properties are simply in need of detection and identification by the scientist. Thurstone describes the kind of property the researcher might detect:

> Factor analysis is not restricted by assumptions regarding the nature of the factors, whether they be physiological or social, elemental or complex, correlated or uncorrelated. For example, some of the factors may turn out to be defined by endocrinological effects. Others may be defined in biochemical or biophysical parameters of the body fluids or of the central nervous system. Other factors may be defined by neurological or vascular relations in some anatomical locus; still others may be defined in terms of yxperience and schooling. Factor analysis assumes that a variety of phenomena within a domain are related and that they are determined, at least in part, by a relatively small number of functional unities or factors. The factors may be called by different names, such as "causes," "faculties," "parameters," "functional unities," "abilities," or "independent measurements." The name of the factor depends on the context, on one's philosophical preference and manner of speech, and on how much one already knows about the domain to be investigated (Thurstone, 1947, P. 56)

The "factors" discovered in a factor analysis are (note the peculiar "defined by") of possibly a biochemical, biophysical, or neurological nature. In an example involving the consideration of the correlations amongst a set of gymnastic stunt variables, Thurstone equates the factors of linear factor analysis with "functional unities": "We might then find that the correlations can be comprehended in terms of a small number of functional unities, such as sense of balance, arm strength, or speed of bodily movement. Each of the gymnastic tests might require one or several of these functional unities; but it is not likely that every test will require every one of the functional unities that are represented by the whole set of gymnastic tests. A factorial analysis would reveal these functional unities, and we would say that each of them is a primary factor in the battery of tests" (1947, P.57). Once again, the view appears to be that *balance*, *arm strength*, and *speed of bodily moverment* may be out there, elements of the underlying order. If they truly are, the employment of factor analysis may well detect them as such.

C. Burt

Burt's contributions reflect an interesting blend of apparent unease with the CAC, and reiteration of this very picture. In part 1 of his book, "The Factors of the Mind: An Introduction to Factor-analysis in Psychology" (1940), he attempts an explanation of the concept *common factor*. Mill's method of concommitant variations is reviewed: "Whatever phenomenon varies in any manner, whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or else is connected with it *through some fact of causation*" (Mill, 1843, p.441). What follows is a clear accounting of many of the ingredients of CAC: i) The processes of interest to psychology "..are too involved for us to isolate, as a directly observable "phenomenon," either as a simple "cause" or a simple "effect": at most, we can only surmise that some underlying "fact of causation" connects the visible changes we can usually observe. Hence, statistical analysis has to be used to supplement or take the place of experimental analysis, in order that we may allow for the complex mass of irrelevant influences which, in the simpler sciences like physics or chemistry, we should usually be able to remove or control" (pp.3-4); ii) The most important device for the detection of such "facts of causation" is the correlation coefficient: "The degree of such partial dependence is measured by a coefficient of correlation or its equivalent; and, as Spearman puts it, "the system of correlation proposed by Galton and elaborated by Pearson...may be conceived as expressing the *hidden underlying cause* of the variations investigated"" (p.4); iii) "To designate the supposed "underlying cause"- Mill's "common connecting fact of causation"- the name "factor" is now regularly used in psychology" (p.4); iv) "On testing a group of childern for the chief school subjects, it was found that, "as a rule, those who are bad at reading are bad at spelling as well; their arithmetic is also below the average for their age, but by no means as bad as their reading or spelling." Now, we cannot suppose that weakness in spelling is (to borrow Mill's language) "either a cause or an effect" of weakness in arithmetic. Consequently, we infer that both are "connected" through some more fundamental cause, which we term a "common factor"" (1940, p.5); v) By applying factor analysis to scores generated in the employment of mental tests,

> ...psychologists have hoped to reach an inventory of what, in Spearman's phrase, have been described as "the abilities of man." Verbal ability, arithmetical ability, mechanical ability, retentivity, quickness, perseveration, oscillation of attention, and above all a general factor of intelligence that enters into all we say or do or

> think- these, or qualities somewhat like them (for, to avoid misconception, the factorists prefer to designate their factors by letters rather than by concrete names), are supposed to be the "primary abilities" that make up the human mind (p.6).

Thus, Burt takes the cause of phenomena to be common factors. With respect to the issue of the number of such abilities, Burt cites Spearman disciple W. Stephenson (1936, p.208): "It seems to be a fact that there is only a limited number of such fundamental tendencies in the human being: Spearman has found five or so; Thurstone specifies seven; the Thorndike unitary traits committee expects to find anything between one and twenty...the implication is that a few fundamental factors account for, or are the cause of, all human conduct." The view here is that there are in existence, regardless of whether or not they are discovered, from one to twenty mental traits "out there". The job of the factor analyst is to reveal and identify these traits. Burt acknowledges that

> It is frequently implied that these 'factors of the mind' are innate factors- fundamental elements in the individual's mental endowment handed on to him at birth. Thus, in one of the earliest investigations on intelligence tests, an attempt was made to show that the factor which they tested was not only general but also inborn...And some of the earlier investigations into type-factors, particularly those that appeared to be associated with temperament, race, or sex, suggested the possibility that the most fundamental of all would be those attributable to genetic elements, obeying Mendelian laws and producing traits either linked or segregating freely" (p.8).

He takes Thomson as equating the factors of factor analysis with "..fundamental capacities or traits" (1940, p.10), while Spearman's aim is characterized as being to "discover the causal mechanisms of the mind and the general laws which they obey" (1940, p.12).

R.B. Cattell

Cattell's commitment to the CAC is somewhat legendary. Messick (1981, p.578), for example, offers Cattell as a protoype of the believer in the causality picture. In Cattell's 1952 book "Factor Analysis: An Introduction and Manual for the Psychologist and Social Scientist", he claims that "It would seem that in general the variables highly loaded in a factor are likely to be the causes of those which are less loaded, or, at least that the most highly loaded measure-the factor itself-is causal to the variables that are loaded in it" (P.362).

P. Lazarsfeld

It would be a mistake to underestimate the influence that the work of sociologist Paul Lazarsfeld has had on applied latent variable modeling within the social and behavioural sciences. His numerous accounts of his latent structure analysis methodology contains a wealth of ideas and terminological innovations. Perhaps more significant than any of the technical innovations introduced by Lazarsfeld's work was the full-bodied conceptualization of the role of latent variable analysis in science that he offered to applied researchers. Whereas the contributions of many experts in psychometrics are solutions to small, well circumscribed problems, Lazarsfeld attempted to place latent variable modeling within a broader context of scientific investigation. He explained to the applied researcher how latent variates and latent variable models were to be conceptualized, and the conceptualization he offered was strongly empirical realist in its orientation. Lazarsfeld was, undoubtedly, a major force in making the CAM the dominant account of latent variable modeling.

In his 1959 treatise, *Latent Structure Analysis*, Lazarsfeld opens with the claim that "All the social sciences deal with concepts which seem somewhat vague. Who can, in practice, recognize an extrovert personality? Who has not read many discussions as to the real meaning of public opinion..." (p.477). There are, in other words, conceptual problems in the social sciences. Why do such problems arise? Lazarsfeld explains: "There are various reasons why the social scientists' language has so many of these terms, which at first sight seem to be ill defined and even at their best are "fuzzy at the fringe." In some cases we can, by the nature of the concept, only observe symptoms, behind which we assume a more permanent reality" (p.477). Here, then, is the root of CA2. The idea[8], prevalent in the social sciences, is that conceptual difficulties abound in the social sciences because humans can only perceive the messy realm of observables (symptoms), and cannot access the realm of "truths" within which the pure and unproblematic essences of concepts reside. All the scientist can hope for is to make inferences about this "permanent reality." Latent structure analysis is "...one special procedure by which it is possible to make what one might call inferential classifications..." (p.477).

How can latent structure analysis be employed to make inferences about the elements of the "more permanent reality" underlying the observable symptoms? In the first place, the trait concepts of interest to the psychologist are to be equated with "inferential concepts" possessing "indicators" (pp. 479-480). Trait concepts develop as follows: "We experience, say, anxiety, and its role in our own course of action (R). We observe how other people act in situations (S) which would, we know, bring on our anxieties; we notice that their reaction R is similar to ours. As a result, we file away in our minds that as a rule such stimuli S are likely to be followed by responses R. We "explain" such S-R sequences with the help of an intervening variable: anxiety. The value of this construct becomes particularly apparent if many S-R situations are observed where the S and R vary, but where the same intervening variable (anxiety) seems appropriate." (p.481). Lazarsfeld was evidently not sensitive to the standard rebuttal that, if such an S-R paradigm is to be taken as defining *anxiety*, as seems his intention, such a definition would require the perceiver to be able to *identify* anxious experiences as such ("We experience, say, anxiety, and its role in our own course of action), and that the capacity to identify anxious experiences as such presupposes the capacity to correctly employ the concept *anxiety*. But the CA suggests something else: That the meaning of *anxiety* is inherent to the unobservable essence that underlies mere observable anxious experiences. Such meanings or essences exist independent of human conceptualization, and are linked to the observable realm by indicators.

However, "...because indicators have only a probability relation to traits, the crucial problem arises as to how they can be combined if they do not all go in the same direction, for example, if in a specific case R_1 makes high anxiety and R_2 low anxiety probable" (p.482). Lazarsfeld expresses his disapproval of Tolman's view that one can get away with just one

8 Later in this book, we will say confusion.

indicator for each intervening variable: "...to an intervening variable there will correspond a variety of indicators and that they will have to be reconciled in some way" (p.483). He cites Koch (1954) with approval: "all alternate experimental variables to which a given independent variable is reducible...must be brought to converge by appropriate scaling techniques" (p.65). Hence, "We find that a tradition has grown in psychology whereby the intended classification required by concepts like trait, attitude, intervening variable, etc., is performed by using indicators directly accessible to the investigator. These indicators are presumed to have a probability relation to the "underlying" (intended) variable; and because of it, if we use- as we invariably do- a number of indicators simultaneously, we will always get into "contradictions" which have to be reconciled" (p.484). Thus, Lazarsfeld suggests the "truth" (intended variate) and indicator (truth plus error) notion.

The logic of this position is, according to Lazarsfeld, well explained by empirical realist philosophy. He states that "In recent writings of logicians, one can find frequent discussions of "disposition terms" which refer not to a directly observable characteristic, but rather to a disposition on the part of some physical objects to display specific reactions under specifiable circumstances" (p.484). Carnap's 1937 paper *Testability and Meaning*, which is often taken as an explication of disposition terms, as well as Hempel's (1952) simplified presentation, are cited. Hempel, of course, discussed how the indeterminacy inherent to the meaning of open concepts, of which dispositional terms were taken to be examples, could, in theory, be eliminated, in the limit, by adding more and more reduction sentences. This idea would later be a cornerstone of the construct validity program of test validation introduced to psychology by Cronbach and Meehl (1955). By taking the trait concepts of interest to the psychologist to be akin to the dispositional concepts discussed by empirical realist philosophers, and, hence, to be intervening variables, and, hence, open concepts, Lazarsfeld arrives at an important conclusion: "...we find to our pleasant surprise that the modern logician is disclosing a practice of the natural sciences, which was considered to be embarrasing by many social scientists. That is, they define important concepts as "intervening variables" or underlying constructs by reference to a series of test situations, which all have to be used together" (p.485).

Here, then, is an explicit equating of the concepts that denote psychological phenomena, these very often ordinary language concepts, with the open concepts described in the empirical realist program of philosophy. According to Lazarsfeld, the problems that the psychologist faces in dealing with these messy, problematic concepts, are explained by recognizing that the meanings of these concepts cannot be accessed directly, but must be inferred on the basis of observable indicators. Lazarsfeld deduces that "This has immediate bearing on the enterprise in which we are engaged here. Indeed we shall concentrate on certain measurement procedures to clarify how we create "underlying" concepts like traits, attitudes, group characteristics, etc.: their role is to summarize a variety of empirical observations and to store them, one might say, for systematic use in a "theory" which we hope will one day develop...the Carnap explication of disposition concepts is fully transferable to our problem area" (p.485). Furthermore, "... something which seems to be an embarassing shortcoming of social science concepts, such as IQ, introversion, or cohesion, becomes the common property of a large group of concept formations in all sciences. In all such cases we must decide what items should be included in the base of observations from which intervening variables of any kind are inferred. The explication of disposition concepts thus certainly covers all the elements we are concerned with: the use of indicators to place people correctly into an "underlying" order required by a more abstract conceptualization, the somewhat fluid choice of these indicators, their probability relation to the intended "ordering", the consequent fact that they will not *all* point in the same direction and that, therefore, they have to be combined into a kind of "index" or "measurement" which represents the best inference which can be made from the manifold of our empirical observations. But the formulation of the logicians is so general that it does not lead directly to concrete research operations. If they are our goal, one more translation has to be attempted. The notion of "property space" seems to serve this purpose best" (p.487). Concrete reseach activities include, among other things, the use of latent variable models, including latent structure analysis. Once again, in using such techniques "We are dealing with *latent characteristics*, in the sense that their parameters must somehow be *derived* from *manifest observations*" (p.490). That is, "Empirical observations locate our objects in a manifest property space. But this is not what we are really interested in. We want to know their location in a latent property space. *Our problem is to infer this latent space from the manifest data*" (p.490).

F. Lord and M. Novick

Lord and Novick's 1968 book "Statistical theories of mental test scores" has been credited with placing test theory on firm statistical footing. It also offered an explicit account of the role of latent variable models as tools for the investigation of measurement claims. Lord and Novick explain that the aim of test theory is to "measure the "ability" of an examinee" (p.13). There exists "a classical theory of error appropriate to measurement in the physical sciences" (p.13), but this theory is not appropriate for work in the social and behavioural sciences. A theory of mental testing founded on a theory of error tailored to the social and behavioural sciences is needed because "...mental test scores contain sizable errors of measurement" and, importantly, because "...the abilities or traits that psychologists wish to study are usually not directly measurable; rather they must be studied indirectly, through measurements of other quantities. We cannot directly measure a person's mathematical ability; we can only measure his performance on a number of mathematical test items. Indeed these abilities are in general given mathematical definition only in terms of specified measurements" (P.13). Here again is the notion that what is of interest to the researcher is not directly accessible, but is, rather, an essence lying somewhere behind observed phenomena.

Lord and Novick elaborate on this key idea. The need for such a theory of errors does arise in other areas, for example, in the analysis of voting behaviour, "...in which fluctuations from the "normal vote" prevent the direct measurement of that construct" (P.14). Similarly, the "..."genetic composition" of a herd of cattle cannot be measured directly, but the effects of this composition can be studied through controlled breeding." These problems are similar to those that arise in social and behavioural research in that "Each postulates constructs that are not directly measurable and each observes phenomena that manifest these latent constructs but that also exhibit fluctuation due to other factors..." (P.14). Latent constructs, not directly measurable, are also called "hypothetical" (p.14) or "theoretical" (p.15). Hence, the problem that must be addressed is that what is to be measured in psychology are properties existing independ of humand perception and conceptualization, that are, additionally, not "directly measurable." They are "unobservable."

The researcher must, then, hypothesize/postulate the existence of a particular "latent" or unobservable property, and find a way of detecting its presence. In this regard, it is helpful to know that theoretical constructs (hypothetical constructs/latent constructs) "...are often related to the behavioral domain through observable variables by considering the latter as *measures* or *indicants* of the former. And conversely, "theoretical constructs are often abstracted from given observable variables" (P.19). But still, how is the researcher to use this fact to detect such hypothetical or theoretical constructs? The answer is the latent variable model. The "classical test theory model deals with two kinds of random variables, *manifest* or observable variables and *latent* or unobservable variables" (P.530). Latent variable models "...have in part been offered as a means of linking the more precise but limited mathematical models of behavior with the more broadly conceived psychological theories" (P.19). For example, "The factor analytic model is one of a number of models that give concrete form to the concepts used in theoretical explanations of human behavior in terms of *latent traits*. In any theory of latent traits, one supposes that human behavior can be accounted for, to a substantial degree, by isolating certain consistent and stable human characteristics, or *traits*, and by using a person's values on those traits to predict or explain his performance in relevant situations" (P.537).

To reiterate, on Lord and Novick's account the trait concepts of psychology are hypothetical or latent constructs, and, hence, are real, but unobservable. Their existence must be detected through their relations to observables. They exist independently of human conceptualization, but must be "isolated." The latent variates of latent variable models stand in correspondence to these unobservables. Latent variable models are the appropriate tools to isolate these "consistent and stable human characteristics." What types of traits might be isolated in a latent variable analysis? Lord and Novick (1968, P.537) explain that these might include "...quantitative aptitude, verbal aptitude, mathematical ability, subject-matter knowledge in psychology..perseverence and creativity." The problem of "...identifying and defining such traits in terms of observable variables and determining what traits are important in a given behavioral context is the major problem in any theory of psychology" (P.537). That is, the referent of the concept *latent variate to* ***X*** may turn out to be, e.g., denoted by the ordinary language concept *creativity*. Human characteristics, or traits, are, is it were, "out there", in need of detection and identification. Through the employment of latent variable models the scientist can detect and identify these traits.

R.P. McDonald

Within the context of linear factor analysis, McDonald has previously pointed to what are, herein, called the causality and measurement pictures, as organizers of work within the domain of latent variable modeling. He states that "Attempts to discuss the scientific status of common factors treat them as either some kind of hypothetical cause of behaviour or as abstractive concepts describing behaviour" (McDonald, 1981, p.107). Moreover, he claims that the measurement picture has now achieved dominance over the causal picture:

> By reading several hundred published applications of factor analysis (if they have not already done so), readers will easily confirm that very few studies yield a theoretical account of a factor as a determinant of a set of behaviours, and almost all of them treat the factor as an abstractive concept that serves as a summary description of those behaviours... In using the common factor model, then, with rare exceptions the researcher makes the simple heuristic assumption that two tests are correlated because they in part measure the same trait, rather than because they are determined by a common cause, or linked together in a causal sequence, or related by some other theoretical mechanism. It is a consequence of

> this heuristic assumption that we interpret a common factor as a characteristic of the examinees that the tests measure in common, and, correlatively, regard the residual (the unique factor) as a characteristic of the examinees that each test measures uniquely (McDonald, 1981, p.107)

His own own opinion is now that a latent variable model is a detector or discoverer of properties/attributes common to the phenomena represented by the manifest variates analyzed. Such properties/attributes are not directly accessible, and inferences must be made in regard both the identities of these detected/discovered properties/attributes, and measurements of individuals with respect them. The 1979 paper of McDonald and Mulaik, for example, is fully immersed in this way of thinking, and contains a discussion of the use of further variates to measure a "discovered attribute" more precisely. The CAM is reiterated in many of McDonald's more recent (post 1974) publications. For example, he claims that "...I am describing the rule of correspondence which I both recommend as the normative rule of correspondence, and conjecture to be the rule as a matter of fact followed in most applications, namely: In an application, the common factor of a set of tests/items corresponds to their common property" (1996b, p.670); "..the interpretation of a common factor in terms of the common attribute of the tests that have high loadings on it..."; "what attribute of the individuals the factor variable represents" (McDonald & Mulaik, 1979, p.298). The properties/attributes that he believes are detected in the application of a latent variable model he calls "abstractive properties", and suggests (e.g., McDonald, 1996a, p.598) that this interpretation implies an infinite variate domain foundation for latent variable modeling. His ideas in regard this topic will be reviewed in Chapter XV. However, the reader will come to see in McDonald's many publications on factor analysis numerous and varied faces of the Central Account. His early work cleaved to the causality picture, while his more recent work favours the abstractive property notion. Throughout, however, it insists upon the idea that *latent variate to* ***X*** signifies a single, true, thing, whose identity and properties, existing independently of human perception and conceptualization, cannot be directly accessed, but only inferred.

3b. Secondary sources

CAC: Causality picture

While the CAM has now become the favoured interpretation of latent variable modeling, there nevertheless can still be detected in modern work signs of commitment to the CAC. First, attempts have been made to give the causal picture a technical grounding. Rozeboom, for example, states that

> When one has gone through the labor of decomposing a set of data variables into factorial components, it is hard not to feel, consciously or otherwise, that one has thereby earned the right to construe these factors as *causal explanations* of the observed correlations...But even if the **n** data variables have been found to have a factor basis whose dimensionality, **m**, is appreciably smaller than **n**, and this is, in fact, due to the existence of **m** causal determinants shared by these variables, it

> would still be an extraordinary coincidence if the **m** linear combinations of the data variables by means of which this dimensionality was discovered also happened to coincide with those linear functions of the data variables which best estimate these underlying determinants. (Rozeboom, 1966, p.236)

In fact, Rozeboom has, elsewhere, argued convincingly (1984, p.220) that the detection of causal sources has been a chief aim of the applied personality researcher who employs factor analysis. In his 1970 book *Applied Factor Analysis*, Rummel offers advice to the practising researcher as to how to view and carry out factor analytic research. He notes that there has been "Much controversy in the literature..on whether a factor is a cause of the pattern it defines" (P.25), and concludes that "If we define "cause" as uniform relationships, then, does factor analysis uncover factors that are causes of the relationships they delineate? The answer must be "yes." Each of the variables analyzed is mathematically related to the factors. Through this relationship the factors describe the regularities in the data and it is these regularities that define a causal *nexus*" (P.26). In discussing MAXCOV, a member of his suite of taxometric models, and, additionally, a unidimensional monotone latent variable model, Meehl (1992) states that "If a causal conjecture substantively entails the existence of a taxon specifying (on theoretical grounds) its observable indicators, a clear-cut nontaxonic result of taxometric data analysis discorroborates the causal conjecture"(p. 152). Meehl reiterates his view that latent variable models are detectors of causes throughout his writings: "In order to give "empirical meaning to the variates in those questions...a clear distinction [must be] made between manifest behavior indicators and the inferred (latent, causal) factors" (1986, p.222). In his review of factor analytic techniques, Vincent (1953, p.107) explains that they are "Techniques, developed mainly by psychologists for dealing with their problems, which, by analyzing the inter-correlations between sets of measurements, attempt to identify the causes that are operating to produce the variance within each set, and to evaluate the contribution due to each cause." Finally, Borsboom, Mellenbergh, and van Heerden (2003) have recently re-stated, and endorsed, the causality picture of latent variable modelling.

The portrayal of latent variable models as tools for the detection of causes has been taken by some as a natural consequence of the way in which latent variable models are constructed. In particular, the fact that the conditional statistical independence of the manifest variates given the latent variate is a key ingredient of these models has very often been taken as a technical definition of "causal relationship." As Sobel (1997, p.20) has observed "The principle of common cause (Reichenbach, 1956) is sometimes used to justify invoking the axiom of conditional independence. Roughly, the principle states that if X and Y are two associated random variables, then either (say X is prior to Y) X causes Y or there is a prior variable Z that causes both X and Y. In the case where X does not cause Y, the X-Y association is supposed to vanish, conditioning on Z....In this vein, a number of authors (for example, Anderson (1984) and Bartholomew (1987)) have applied this principle to latent variable models, arguing that latent variables (Z or a vector $\underline{Z}$) cause the indicators and the association bectween the indicators is due to these common causes."

Second, commitment to the CAC is evident in the applied literature. In discussing an employment of Meehl's MAXCOV procedure, Gangestad and Snyder (1985), for example, state that "From the perspective of construct validation (e.g., Cronbach & Meehl, 1955), then, there appears to be strong evidence for the existence of a latent causal factor- as yet not directly observed- that partially accounts for a specifiable, yet extensive and extensible, domain of observable behavior..." McDonald (1981, p.107) has noted that Eysenck's theory of cortical inhibition, cortical inhibition a "factor" according to Eysenck, portrays cortical inhibition as a determinant of behaviours signified by the concept of extraversion: "...we might say that a given quantity of cortical inhibition causes the collection of behaviours characteristic of extraverts."

Third, experts in latent variable modeling and applied researchers alike, employ pseudo-causal terminology which implies the CAC. Lawley and Maxwell, for example, suggest that "On the theoretical side there is the belief that the *determinants* [italics added] of human behaviour, whatever their basic nature, may prove capable of description in terms of a relatively small number of mental factors or attributes, or may be shown to function *as if* they were organized in some such manner" (1963, p.4). Bartholomew (1995, p.213) states that "By 1914, Spearman recognized that the clearest way of providing a theoretical foundation for factor analysis was by reference to the theory of partial correlation. If the factor G *existed* [italics added] and if it was the only thing common to two observable variables 1 and 2 then the partial correlation of 1 and 2 *given* G should be zero. For if we hold constant the variable which *gives rise to* [italics added] the correlation then the correlation must vanish." Now, it may be the case that Bartholomew has in mind mathematical senses for "existence" and "gives rise to", but, if so, he does not make clear what these senses are. His talk sounds strikingly like Core thesis 1: i.e., when the linear factor model describes $\underline{\mathbf{X}}$, there has been detected a cause which brings about (gives rise to) the phenomena represented by the manifest variates. The same may be said of many other of Bartholomew's descriptions of linear factor analysis: "If the *y*s are common factors they *influence* [italics added] the values of all *x*s and thus *induce* [italics added] the correlations between them" (1995, p.218); "The first approach is along the lines of traditional factor analysis in terms of the α_{ij} which in this context are known as the *factor loadings*. A large value of α_{ij} means that the *j*th factor exerts a big *influence* [italics added] on the *i*th manifest variable. By identifying a set of manifest variables for which *y*j is an important *determinant* [italics added] we might hope to find an interpretation of *y*j" (1999, p.28).

Allen and Yen (1979, p.111) attempt to define the term "factor" in one sentence: "A *factor* is a hypothetical variable that *influences* [italics added] scores on one or more observed variables." Steinberg and Thissen (1995, p.162) assert that "Unidimensional IRT is based on the assumption that the item responses *depend on* [italics added] a single, continuous latent variable." Now, it is possible that Steinberg and Thissen mean to use the concept *dependency* purely in the technical sense of statistical dependency. But if this were the case, one would expect the claim to be that the form of the *distribution* of the manifest variates, rather than the item responses themselves, depends on the latent variate. For it is one thing to demonstrate statistical dependency, and possibly quite another to demonstrate that natural phenomenon A depends, in some sense, upon phenomenon B. Edwards (1957, p.101-106) takes the latent variates of latent variable models to be those sources that "*...produce* [italics added] certain behavior or responses" (p.101). In an article on the application of item response theory to personality variates, Damarin (1970, p.26) states that "The existence of personality traits that *govern* [italics added] actual behavior is a plausible explanation for correlations among personality items when these are answered accurately by a substantial majority of subjects."

Fourth, latent variable models, and, in particular, covariance structure models, have commonly been called "causal models." In their review of structural equation modeling, Bielby and Hauser (1977, p.137) state that "More than a decade ago, methods for modeling the structure of relationships among variables with systems of equations began to diffuse among sociologists. Expositions and applications have typically referred to *causal models* or *path* *analysis*...Sociologists speak of "causal models" because the term provides a convenient description of what a structural equation system does." Blalock called his book on path analysis, *Causal inference in nonexperimental research*. The statistical models tested by programs such as LISREL are often themselves called by applied researchers, causal models: "...many of us seek to use confirmatory [latent variable models] to evaluate causal models" (Mulaik, 1986, p.30).

Fifth, and perhaps most strikingly, are the comments made by certain of the contributors to the debate on the indeterminacy property of linear factor analysis[9]. Mulaik's (1976, p.252) comments on the issue are rife with CAC inspired imagery: "To use Guttman's measure of indeterminacy in factor analysis, we need not assume that the factors *generating the data* [italics added] on the observed variables..."; "which set of variables actually are the factors that *generated the data* [italics added]..."; "He then establishes a set of empirical operations that will measure that *causal factor* [italics added]..." Later in the same paper, in considering the idea of factors as constructed variates, he states that "such artificial variables in **x** could not serve as causal explanations of the variables in **n**" (Mulaik, 1976, p.254). Rozeboom (e.g., 1966, 1988), commenting on the issue of isolating, from amongst the factor constructions, those which are most important, states that "Moreover, <**A**,$\mathbf{M}_{FF}$,F> is the particular solution of (1) that most closely aligns F with an m-tuple of Z's causal sources" (1988, p.211). He claims that this way of thinking "is already an implicit presumption in most applied multivariate research" (Rozeboom, 1988, p.211), and that "Although our understanding of causality is still primordial,...there can be little doubt that any tuple Z of data variables does in fact have causal sources which, moreover, comprise just a vanishingly small subset of the variables with which Z is jointly distributed" (Rozeboom, 1988, p.211). The closing sentence of his 1998 paper on indeterminacy is telling:

> In short, the feeling of unease occasioned by classic factor-indeterminacy is legitimate and indeed important. But its proper target of concern is not Variety-**AM** indeterminacy of factor scores but our flaccid conceptual grip on the logic of causality and the ontology of scientific variables (Rozeboom, 1988, p.225).

In other words, indeterminacy is really just a problem of failing to provide a sufficiently technical grounding of what is called here the causal picture of the Central Account.

There are, of course, those who, in the same sentence, are able to endorse *both* the causality and measurement pictures: "One way to think of exploratory factor analysis is a process of discovering and defining latent variables and a measurement model that can provide the basis for a causal analysis of relations among latent variables" (Loehlin, 1998, p.143).

CAM: Measurement picture

There exist, in the literature on latent variable modeling, several species of the CAM.

a) The manifest variates are fallible measures of the latent variate, the latter a "trait" or "ability":

Examples of this version of the CAM are found in both Lazarsfeld and Lord and Novick. The referent of *latent variate to **X*** is "real", existing independently of human perception, and possessing an identity independent of human conceptualization. The identity of the referent is

9 A topic that will be addressed in the next chapter.

given by the ordinary language concept that signifies the referent. However, due to the referents unobservability, this conceptual linkage cannot be known. In characterizations of latent variable modeling which commit to this version of CAM, there is the tell-tale equating of the referent of *latent variate to* ***X*** with the referent of an ordinary language ability or trait concept. For example, in his review of test theory, Lewis (1986) endorses Gulliksen's view of the central problem of test theory as being the study of the relationship between an individual's ability and his observed score on the test. Lawley (1943, p.273), in justifying restricting his attentions to *unidimensional* item response models, states that "we shall assume that all items composing a given test are measuring the same ability." In their introduction to a discussion of latent trait models, Allen and Yen (1979, p.240) explain that: "In *latent-trait theories*, it is assumed that the most important aspects of test performance can be described by an examinee's standing on one *latent trait*- a hypothetical and unobserved characteristic or trait (for example, verbal ability, knowledge of history, or extroversion). Latent-trait theories propose models that describe how the latent trait influences performance on each test item."

b) The latent variate is a signal, and the manifest variates, noise corrupted "receivers" of the signal:

The signal/noise dichotomy is commonplace within engineering and statistics, and these areas may have been the source of this version of the CAM. Dobson (1990), for example, includes the following description: "The transmission and reception of information involves a message, or signal, which is distorted by noise. It is sometimes useful to think of scientific data as measurements composed of signal and noise and to construct mathematical models incorporating both of these components. Often the signal is regarded as deterministic (i.e., non-random) and the noise random. Therefore, a mathematical model of the data combining both signal and noise is probabilistic and it is called a statistical model" (p.10). Steiger (1996, pp.539-540), while not endorsing the conception, provides a linear factor analytic example which is very clearly organized in terms of signal and noise:

> Suppose it is *believed* that a signal may be emitted ten times (*and only ten times*) at one hour intervals from point X, starting at 1:00 P.M. Point X can never be observed directly. Receivers are constructed to measure the signal at points Y_1, Y_2, Y_3, Y_4. However, the signal is "jammed" by a noise countersignal at each receiving point. An additional signal, at point W, is received directly without any noise degradation. Fortunately, it is known from intelligence sources that (a) all signal and noise distributions have zero means, (b) the signal and noise components are additive, and (c) they are precisely uncorrelated over the ten observations taken, and (d) that the ten signals have a variance of exactly 1....Clearly, the signal believed to exist can be considered a random variable X that satisfies the definition of a common factor for the 4 random variables in vector $\mathbf{Y}$.

c) The latent variate is "truth", and the manifest variates, truth+error:

Holland and Rosenbaum provide a clear statement of this version of the CAM: "...a scalar U easily lends itself to the interpretation as an underlying "true" quantity that is fallibly measured by the observable responses in **X**- e.g., the true "ability" that is fallibly measured by the exam responses **X**" (1986, p.1526). The truth plus error conception is also invoked in slogans such as "Common factor analysis should be routinely applied as the standard analysis because it recognizes we have error in our variables... " (Gorsuch, 1990, p.39). In expressing his preference for latent variable, over component, models, Loehlin (1990, p.30) provides the advice that "If one lives in the real world in which measures contain error and/or specific variance, why not fit models that reflect this?" If there is *error* in the variates of the social scientist, what then is *truth*? Bollen (1989, p.11) provides the standard reply: "The observed variables or indicators of a latent variable contain random or systematic measurement errors, but the latent variable is free of these"; "Latent random variables represent unidimensional concepts in their purest form" (1989, p.11). As Steiger (2001, p.332) acknowledges, "These factors, generally based on two or more observed variables (called indicators) are thought to represent "purified" versions of the concepts under study." Recall that Lazarsfeld, in his classic work on latent structure analysis, speaks straightforwardly of his aim in using his technique: To reveal the positions of empirical observations within a *latent* (purified) property space, on the basis of the fallible manifest variates.

d) The latent variate is a fundamental or principal variate or dimension which underlies the empirical realm of interest:

As was seen, this idea was prominent in the writings of Thurstone ("The factorial methods were developed primarily for the purpose of identifying the principal dimensions or categories of mentality...", 1947, P.55) and, not surprisingly, given his influence, has been echoed in the writings of many others. For example, in his book *Psychometric Methods*, Guilford explains that "The task of isolating the independent aspects of experience has been a difficult one. Armchair methods dominated by deductive logic rather than by observation led to the faculty psychologies, traditionally unacceptable to modern psychology. Direct observation has likewise failed to arrive at any set of unitary traits which even approach a universal acceptance. Factor analysis or some similar objective process had to be brought into the *search for the unitary traits of personality* [italics added]" (1954, p.470). Furthermore, "In spite of the great social and scientific usefulness of psychological tests it must be acknowledged that for the most part we have had very inadequate ideas as to what it is they actually measure. The plea is frequently offered in defense of tests that, by analogy, we do not know the whole truth about electricity and yet we do not question the right of the physicist or the engineer to measure it. Let us be ready to recognize that, although the full nature of electricity is not known, some of the real variables of electricity, such as potential, resistance, and inductance, have been isolated and laws of their interrelatioships have been stated. The fundamental variables or dimensions of human ability and of human personality in general are still well within the unexplored territory reserved for psychologists. To meet this situation, a statistical approach such as factor analysis is necessary" (1954, p.470). Once again, this is the key Central Account picture of abilities and traits existing as unobservable constituents of natural reality. Latent variable models are necessary to bring about their detection and identification.

Fruchter, in his *Introduction to Factor Analysis*, speaks of the aim of factor analysis as being to discover "basic categories" that account for a set of observations. He (1954, p.2) comments that "It has been pointed out by Royce...that a proper order for research programs might be, forst, to use a set of *a priori* measures in a field of investigation and factor analyze them to determine the basic traits or other sources of variance operating..." In the words of Joseph Royce, "Why does he carry out a factor analysis? Essentially, he is looking for the best way to describe things. He wants to identify the fundamental behavioral variables" (1958, p.139). Shortly thereafter, he invokes a number of facets of the Central Account: "...a limited number of abilities or factors are isolated as determiners of behavior in the domain in question" (p.139).

Version (iv) of the CAM describes a scenario in which the basic properties/attributes relevant to the characterization of humans exist "out there" in nature. They must be detected, identified, and catologued. Factor analysis is a tool specially designed to meet the demands inherent to this task. Lists of properties/attributes believed to have been detected in the use of factor analysis have, in fact, been published. Such list-keeping was especially popular in the early history of applied factor analysis. In Cureton and D'Agostino's (1983, p.3) text on factor analysis, they provide the reader with some idea as to the kinds of "entities" that can be detected in a factor analysis:

> If the variables are psychological and/or educational tests, then common factor will be interpreted as the principal underlying abilities. Examples are verbal comprehension, numerical ability, abstract reasoning...
>
> If the variables are athletic and gymnastic measures, the common factors will be such things as static strength...explosive strength...speed of movement...
>
> If the variable are the items of an opinion questionnaire, the common factors will be main underlying attitudes.

e) The latent variate is an "abstractive property."

McDonald's abstractive property notion is a more subtle version of the CAM. He claims that this position implies an infinite behaviour domain basis to latent variable modeling, and this claim is discussed in Chapter XV.

CA2: Unobservability

While Thurstone (1947), with its references to "underlying traits" and its portrayal of linear factor analysis as a tool of detection of such traits, certainly contains elements of "unobservability talk", this kind of talk is not a prominent feature of early discussions of latent variable modeling[10]. It *is* a prominent feature of contemporary discussions, and its development coincided with the mid-century rise of the empirical realist philosophy of Feigl (1950, 1953, 1956), Hempel (1965), and Sellars(1972), and its popularization in the work of Cronbach and Meehl (1955) and Lazarsfeld (1950). Empirical realism can be outlined as follows[11]: i) Natural reality is comprised of observable phenomena and the unobservable causes of these phenomena; ii) These phenomena and causes exist independently of whether they are perceived by humans; iii) Scientific concepts can, not uncontroversially, be categorized as either theoretical or

---

10 Thurstone (1940, p.216) contains: "...if we want to know whether these measures are related by some underlying order which will simplify our comprehension of the whole set of measures, then a factor analysis is called for."
11 It must be acknowledged that empirical realists are a diverse collection of philosophers (see Slaney, 2003, for a detailed discussion of this point). Tuomela's 1973 and Rozeboom's 1984 accounts will, herein, be taken as prototypes.

observational terms. Observational terms are semantically unproblematic, as they designate observable phenomena. They are defined in data language terms (i.e., in terms of observables). On the other hand, a theoretical term, e.g., *electron*, *phlogiston*, *neutrino*, is a term that is introduced by a scientific theory and that designates an unobservable cause of a set of observable phenomena. As a result, a theoretical term cannot be defined on the basis of observational terms. This makes the theoretical term both essential to what the theory claims about natural reality and semantically problematic[12]; iv) A theoretical term is an *open concept* in that it is not explicitly definable in terms of observables. Instead, it is implicitly, and incompletely, defined by the system of laws and core presumptive hypotheses which comprise the theory in which it is embedded; v) While the theoretical term is implicitly defined by its embedding theory, if the theory turns out to be correct, the theoretical term does, in fact, *name* the causal entity it denotes. In the words of Rozeboom (1984, pp.211-212),

> Logical positivism courageously sought to face down this doubt in the only way that classical epistemology could envision, namely, by proposing that theoretical sentences which seem to make claims about unobserved entities do not really do so...The thesis that theoretical expressions are always equivalent to data-language constructions if meaningful at all was a large promissory note whose lack of cash backing eventuated in positivism's banruptcy during the 1950s...logical positivism gave way in the 1950s to the empirical realism long championed by Feigl (cf. 1950, 1956) under the title "logical empiricism." This is the thesis that although theoretical terms get their meanings from the data-language contexts in which they are used, what they semantically *designate* are causal features of natural reality generally concealed from perception but knowable through their data consequences.

Empirical realist philosophy seems to have been taken by many psychometricians and behavioural scientists, Lazarsfeld a notable case in point, as explaining why scientific work on phenomena denoted by psychological concepts seems to present so many difficulties. Operationism was manifestly a failure, and a certain, perhaps incautious, reading of empirical realism could see it as claiming that: i) The rueful complexity of the psychological concepts employed in the social and behavioural sciences is attributable to *their* unobservability; and ii) the multifariousness and diffuseness of the criteria of psychological concepts, a feature often erroneously identified by social and behavioural scientists as being equivalent to the "abstractness" of these concepts, is the result of these criteria being, in fact, multiple "indicators" of an unobservable nexus in need of detection. Primitive versions of (i) can be traced within psychometrics at least as far back as Spearman (1927), he viewing rampant equivocation with respect the concept *intelligence* as a prime motivator for the development of linear factor analysis. Burt (1940) similarly took the fact that phenomena of interest were "too involved" as implying that these phenomena were unobservable. The reasoning inherent to (ii) is evident in Krantz's (1991) discussion of the differences between the fundamental (axiomatic) and psychometric traditions of measurement. He suggests that the convergence of multiple distinct operational definitions of a *construct* may indicate an underlying nexus.

---

12 To the logical positivist, a theoretical term is, at best, a "useful fiction" which may be used to structure the theory (Slaney, 2001; Worrall, 1982). If it has any meaning, a statement of its meaning is reducible to statements made in terms of observables.

With the help of empirical realist philosophy, unobservability came to be seen as a characteristic feature of the conceptual issues that arise in the social and behavioural sciences. As Lazarsfeld explained, it was then fortuitous that there existed a set of statistical tools, latent variable models, indigenous to the social and behavioural sciences, that were tailor-made for dealing with unobservables. Influential works such as Lord and Novick (1968) and Cronbach and Meehl (1955), the latter, a dissertation on construct validity, popularized these ideas. Based on these accounts, the justification believed to be provided by empirical realist philosophy of the unobservability theses of the Central Account can be described as follows:

i) A model is "...an attempt to *describe* structures and mechanisms which are often unavailable to observation, even with the use of scientific instruments" (Keat & Urry, 1975, p.34);
ii) Psychological concepts are often theoretical or open concepts that denote unobservable property/attribute (cause) of observable behavioural phenomena;

iii) The symbol $\boldsymbol{\theta}$ in the equations of a latent variable model stands for an open concept which, if the model is correct, designates an unobservable property/attribute (cause) of the phenomena represented by the manifest variates;

iv) Hence, the latent variable model is the proper tool for the detection and study of the unobservables that underlie behavioural phenomena.

Paul Meehl, in his many accounts of his taxometric latent variable methodologies, and William Rozeboom (e.g., 1988), in various publications, have made explicit this interpretation of latent variable models. Royce's 1963 paper *Factors as theoretical constructs* decribes in detail factor analysis from the perspective of the empirical realist. He states that "...by a factor we shall mean a true variable, a process or determinant which accounts for covariation in a specified domain of observation" (p.523), and that "...factors are O variables intermediate between S and R variables. We infer functional unities which are determinants of the covarying response pattern. These mediating variables may be either intervening variables or hypothetical constructs, depending on their depth of penetration into the nomological net" (p.525).

Now, the claim here is most certainly *not* that latent variable modellers are all card-carrying empirical realists, but, instead, that empirical realist philosophy has contributed greatly to the social and behavioural scientist's ease with respect the notions of unobservability and latency, and, hence, with the Central Account. It seems to have provided license for the social and behavioural scientist to view concepts indigenous to his area of inquiry as theoretical concepts, open concepts, hypothetical variates, and the like. Any signs of equivocation or uncertainty with regard a psychological concept are immediately attributed to unobservability. The perceived solution is to forego the enormous effort other sciences have invested in clarifying the conceptual bases of their empirical investigations, in favour of taking concepts to be "represented" by latent variates. For example, in discussing causal modeling Bentler (1980, p.433) jumps immediately to the conclusion that *cognitive dissonance* is a "hypothetical variate", and, hence, a latent variate, without providing any discussion of the correct employments of this concept.

The unobservability talk inherent to the practice of latent variable modeling is, nowadays, comprised of a dense network of figurative and technical terms, organized in terms of a number of adjective clusters. Prominent examples are reviewed below.

a) Cluster 1: *Unobservable*

In published discussions of latent variable modeling, the adjective by which manifest and latent variates are principally contrasted is that of *unobservability*. In many treatments, *observability* is taken as a synonym for *manifestness*, and *unobservability* for *latency*. Many of the major texts on factor analysis introduce the factor analysis model as one in which the "relationships amongst a set of observed (manifest) variates are explained by their regressions on a smaller set of unobserved (latent) variates." Cureton and D'Agostino (1983, p.3), for example, explain that "Factors are random variables that cannot be observed or counted or measured directly, but which are presumed to exist in the population. Because they are not observed or counted or measured directly, they are sometimes termed *latent variables*." Throughout his comprehensive text on linear factor analysis, Mulaik (1976) describes factors as unobservable. Crocker and Algina (1986, p.288) state that "A factor is an unobservable or latent variable, just as a true score is unobservable", while Holland & Rosenbaum (1986, p.1524) explain that "The manifest variables, which are real or integer valued, can be observed directly while the latent variable is unobservable." On the use of the term "causal modeling" to describe covariance structure analysis, Bentler (1980, p.420) explains that "...it is not necessary to take a stand on the meaning of "cause" to see why the modeling process is colloquially called causal modeling (with latent variables). The word "cause" is meant to provide no philosophical meaning beyond a shorthand designation for a hypothesized unobserved process...In such a definitional context, one need not worry about the criticism that "causal analysis does not analyze causes" (Guttman, 1977, p.103)." One cannot help wondering whether Guttman would have found the unclarified "hypothesized unobserved process" of any greater comfort than "cause."

In contrast to the vast majority of treatments, McDonald's 1974 paper on factor indeterminacy contains an attempt to define the concept *unobservable*. He states that

> It is traditional in the course of a definition of the common factor model to remark that the components of $\boldsymbol{\eta}$ are "observable", whereas the components of ξ, (hence of ζ) are "unobservable" or "latent." In a general way, we feel we know what these words mean. Yet they seem to require an explicit definition in applications to random variables (1974, p.216). The following definition fits usage:- Let $\boldsymbol{\tau}$ be a vector defined on a given outcome space(sample space). Then $\boldsymbol{\tau}$ is *observable* if (a) we can know the value $\boldsymbol{\tau}=\mathbf{t}$ of any observation drawn from the outcome space, or, (b) we have a rule for drawing an observation at random from a subspace of the outcome space in which $\boldsymbol{\tau}=\mathbf{t}$, where t is any fixed value, known or not, in the domain of $\boldsymbol{\tau}$. It is unobservable only if it not observable, as defined (p.216).

This definition was the subject of some controversy (see, e.g., Guttman, 1975), and will be analyzed in detail in Chapter V. Lohmoller's (1989, p.81) discussion of different *degrees* of latency rests on a distinction between different degrees of unobservability:

> Latent variables can be more or less latent.

(i) Firstly, they can be completely latent, unknown, hidden, invisible, undercover, unmanifested and then their scientific purpose is as obscure as the LVs themselves.
(ii) Secondly, due to some assumptions on the nature of the LVs their deductive aspects may become patent, estimable, palpable, evident, and thus their scientific usefulness emerges. In a common factor model the factors are considered as "known" when the factor loadings, structures and correlations are known.
(iii) Thirdly, due to some more stringent assumptions the scores for the LVs become estimable, the total LV thus becoming known. In a principal Component model the weights, scores and loadings are considered and estimated from the beginning, whereas the estimation of weights and scores in FA requires additional assumptions beyond the factor model...

According to Lohmoller, then, unobservability can, in certain cases, be a perceptual matter (note his use of "hidden", "invisible", "undercover", these akin to Rozeboom's use of "concealed"), and in certain cases something entirely different ("unknown", "unmanifested", and only resolvable by the making of further "assumptions"). He does not explain what these different terms mean, nor how these apparently different brands of unobservability (is something "unknown" also "hidden" and "invisible") can all be dealt with through the use of latent variable models.

b) Cluster 2: *Underlying*

Another commonly employed descriptor is *underlying*. As was previously mentioned, this descriptor, present in the writing of both Spearman and Thurstone, appears to have been one of the first elements of the unobservability component of the Central Account. This is not surprising, given that *underlying* seems to be used as a synonym for "causal, but unobservable", and that the CAC predated the CAM. Edwards (1957, p.101-106), for example, states that a latent variable is "any variable that might be considered to underlie or produce certain behavior or responses." The term *underlying* is often employed in conjunction with other terms. Harman, for example, claims that "It is the object of factor analysis to represent a variable z_j in terms of several underlying *factors*, or hypothetical constructs" (1960, p.14), while Bartholomew and Knott (1999, p.2) explain that "We have already spoken of these measures as indices or hypothetical variables. The usual terminology is *latent variables* or *factors*. We prefer to speak of latent variables since this accurately conveys the idea of something underlying what is observed."

In an early discussion of latent structure analysis, Rozeboom invokes roughly the same imagery: "Generically, latent structure theory postulates that *behind* [italics added] data variables $\mathbf{X}_1,..., \mathbf{X}_n$ there dwell sources $\boldsymbol{\tau}_1,...,\boldsymbol{\tau}_m$ such that in the contingent joint distributions of $\mathbf{X}_1,..., \mathbf{X}_n$ given any constellation of values on the $\boldsymbol{\tau}_j$, the $\mathbf{X}_i$ are fully independent of one another" (Rozeboom, 1966, p.530). The term *underlying* is, of course, also employed to describe the context of application of biserial and tetrachoric correlations: "Both coefficients are obtained by hypothesizing the existence of a continuous "latent" variable underlying the "right"-"wrong" dichotomy imposed in scoring a dichotomous item" (Lord and Novick, 1968, p.337).

Hartley (1952) discusses the propriety of describing common factors with terms such as "underlying unity", and concludes that it is, indeed, legitimate to do so. He claims that when properties such as "underlyingness" are used to describe a factor, the factor may then be either "right" or "wrong." The criteria for judging whether a factor is "right" or "wrong" are, according to Hartley, provided by the empirical realist philosophy of Carnap (1950). In fact, Hartley believes that when a factor is interpreted and the attempt is made to give it an "empirical basis", it has been given surplus meaning, and, hence, converted into a hypothetical construct.

c) Cluster 3: *Construct/concept/theoretical variate/hypothetical variate*

In the Central Account, latent variates are often equated with constructs, hypothetical or theoretical variates, or concepts, a practice which underscores the close ties of modern latent variable thinking to empirical realist philosophy in general, and the construct validation theory of Cronbach and Meehl in particular. In discussing two senses of the concept *latent variate*, Bartholomew (1980, p.295) considers the case in which "The latent variable is not "real" meaning that it could not be measured directly, even in principle. It is a mental construct used to facilitate economy of thought. Attitudes and abilities largely come into this category." In his review of causal modeling, Bentler (1980, p.420) states that "...a latent variable is a variable that an investigator has not measured and, in fact, typically cannot measure. Latent variables are hypothetical constructs invented by a scientist for the purpose of understanding a research area; generally there exists no operational method for directly measuring these constructs." Bentler does not explain how the researcher should go about inventing a hypothetical construct, nor what he means, in the first place, by the term, nor how it could be verified that a particular invented hypothetical construct is the same thing as whatever is signified by the concept *latent variate to* ***X***.

Gorsuch (1983, p.259) likens latent variates to constructs, and, at the same time, invokes the unobservability property: "The present approach is to consider factors as constructs that will, hopefully, aid in theory development. Constructs are always abstractions from data and never observed." He also likens them to "undefined theoretical scores" (Gorsuch, 1983, p.258). Earlier in his book, he paints a slightly different picture, calling factors "representatives" of corresponding constructs: "The smaller set of variables [factors] can be used as operational representatives of the constructs underlying the complete set of variables" (Gorsuch, 1983, p.4). In their text on factor analysis, Cureton and D'Agostino provide a similar account: "We use the term "factors" to designate latent variables; the term "variable" (or "test") will *always* designate a manifest variable. The factors are actually hypothetical or explanatory *constructs*" (1983, p.3). Green, in *The Handbook of Social Psychology*, states that "...The responses are said to covary because they are all mediated by the same hypothetical variable" (1954, p.335), Maxwell (1977, p.47) refers to factors as "hypothetical dimensions of variability", and Lawley and Maxwell (1963, p.88) explain that "...it is sometimes desirable to go a step further and find equations by which the scores on the hypothetical factors may be estimated from a set of observations of the variates."

Rummel's 1970 text contains a chapter titled "Factors as Concepts", the ideas within owing much, in his estimation, to the work of empirical realist philosopher Carl Hempel. Rummel distinguishes between three types of concept, the formal or analytic concept, which denotes "a class of logical or mathematical relationships among symbols" (p.19), the theoretical concept or construct, whose "validity" is established by its place "...within the structure of a theory and not its factual content" (p.19), and the empirical concept, which defines existential classes of things, events, or processes. The important feature of an empirical concept is its "...truth value-the degree to which the phenomena defined by the concept have operational meaning" (p.19). According to Rummel, the concept *factor* can be, depending on the purpose of investigation, any of these three types of concept. As constructs,

> ...factors measure the inner working of the "black box" through which observed inputs are transferred to observed outputs. Imagine a box with a set of buttons at one end and a set of colored lights at the other. Let the input observations be the order and timing of the buttons pressed; let the output be the colors and patterns of flashing lights. We can observe input and output, but not the intervening mechanism by which pressed buttons are associated, with flashing lights. This may be some highly complex function (e.g., $y=axwe^{-z}$) or some simple function ($y=a+bx$), but, whatever it is, the mechanism is hidden in the box (1970, p.20).

If one were to factor analyze the input and output observations, "...the resulting dimensions will define a model of the box's inner mechanism" (p.20). That factors are constructs "...involves the belief that they define the causal nexus underlying the observed patterns" (p.20). As Rummel notes, Royce (1963) also "...views factors as variables that mediate between S (stimulus) inputs and R (response) outputs in psychological data" (1970, p.20). Although the mechanism through which these causes operate may be unknown, "the factors will delineate the phenomena that are involved" (p.20).

In his text on covariance structure analysis, Bollen treats the concepts *concept*, *construct*, *hypothetical variate*, and *latent variate* as interchangeable:

> Since all latent variables correspond to concepts, they are hypothetical variables. Concepts and latent variables, however, vary in their degree of abstractness. Intelligence, social class, power, and expectations are highly abstract latent variables that are central to many social science theories. Also important, but less abstract, are variables such as income, education, population size, and age. The latter type of latent variables are directly measurable, whereas the former are capable of being only indirectly measured (1989, p.11)

Following traditional psychometric practice, Bollen does not bother to explain what he means by the claim that a latent variate, a function defined on a sample space, "corresponds to a concept." Nor does he clarify what he intends in the employment of "Concepts and latent variables". Is he, for example, equating the two? The reasoning seems to be that a concept is a latent variate, which implies, in turn, that it is a hypothetical variate. If it is an "abstract" hypothetical variate, it follows, then, that it is not directly measureable. Hayduck, another covariance structure modeller, presents a like-minded account, in which concepts possess the capacity to cause each other: "If a concept is directly caused or influenced by any of the other concepts, it is classified as endogenous..." (1987, p.88).

Green offers up the standard depiction of psychological concepts as being "imprecise" or "messy", they possessing of many different "symptoms", this implying that they are hypothetical or unobservable: "To obtain a more precise definition of attitude, we need a mathematical model that relates the responses, or observed variables, to the latent variable." Thus, to Green, psychological concepts are "hypothetical variables" that have "...been called *traits*, *intervening variables*.., *latent variables*...,*genotypes*..., and *factors*..." (1954, p.725). As a result, "...an attitude is a latent variable" (1954, p.727).

CA3: Model and testability

The Central Account claims that, for population P_T, particular set of manifest variates $\underline{\mathbf{X}}$, and latent variable model lvm, a test of $\underline{\Omega}_T \subset M_{lvm}$, i.e., whether $\underline{\mathbf{X}}$ is described by lvm, is a test of an existential hypothesis about a putative unobservable property/attribute (causal source) of the phenomena represented by the manifest variates. Upon inspection, this belief is evident in the oft-heard refrain that latent variable models are preferable to component models on the grounds that they are "testable". Testability, according to Bentler and Kano (1990, p.68), is a key basis for distinguishing between the linear factor and component models: "This is one of its virtues, as compared to components analysis, because the ability to reject a model is a fundamental aspect of data analysis using structural models (Bentler, 1989; Joreskog & Sorbom, 1988). In fact, the component *model*, or class of models as described by Velicer and Jackson, is not a model at all. As noted by Dunteman (1989, p.56), "Principal components analysis is a procedure to decompose the correlation matrix without regard to an underlying model. There is no hypothesis tested with the model. It is a nonfalsifiable procedure for analyzing data that can always be applied with any data set. Of course, there is a sampling theory for eigenvalues of a sample covariance matrix that can be used to test hypotheses about population roots of a covariance matrix (e.g., Anderson, 1963), but such hypotheses are typically hard to frame and not very informative about the structure of a correlation matrix..." Joreskog states that "Principal component analysis is not a model in the usual sense. It is merely a descriptive method of analysis that can be used to analyze all kinds of quantitative variables. Factor analysis, on the other hand, postulates a certain model...which is to be tested against empirical data. The equations... are not capable of direct verification, since the p variables x_i are expressed in terms of $p+k$ other variables which are not observable" (1979, p.17).

However, there exist different senses of *testable*, senses between which latent variable modellers have not always been careful to distinguish. In the first place, there is statistical testability. The correctness of *any* restriction whatsoever placed on the joint distribution of a set of variates is, in principle, testable via the employment of statistical methodology. For example, for a given set of rules of score production, $\{r_1, r_2, ..., r_p\}$, and population P of objects under study, let $\mathbf{X}_j$, j=1..p, be a set of random variates, the distribution of the jth containing scores produced by application of r_j to each member of P. Then the $\mathbf{X}_j$, j=1..p, have a joint distribution in P. This distribution might be idealized as being reasonably well approximated by the multivariate normal distribution with mean vector $\underline{\mu}$ and covariance matrix Σ. It is standard statistical inference to then test, e.g., the hypothesis that $\Sigma = \Sigma_o$ against the alternative that $\Sigma \neq \Sigma_o$. Such hypotheses are restrictions on the distribution of $\underline{\mathbf{X}}$, and, based on a sample drawn from P, a decision must be made as to whether or not the restriction is correct. A given unidimensional latent variable model places restrictions on the joint distribution of the manifest variates, and hence, is "testable" in this sense of the term. However, it is not at all clear that *statistical* testability can be the basis for a distinction between between component and latent variable models. For example, the Principal Component hypothesis that a set of p variates is unidimensional is to claim that, in population P, Σ is of rank unity. This, and other like hypotheses, is statistically testable.

The sense of *testable* cited as a special property of latent variable models depends for its meaning, not on statistics, but on ontological commitment. As Henrysson (1957, p.89) candidly attests:

> ...Spearman, Holzinger, Burt, and Thomson, mainly limit their use of factor analysis to the testing of hypotheses already formulated. They employ a procedure where one or more factor analyses, together with other information, will show whether hypotheses, previously advanced, as to the nature of the underlying processes can be confirmed or not.

This is precisely the idea at the root of the CA: To test the conformity of **X** to a given latent variable model is to a test an hypothesis of existence in regard the unobservable referent of *latent variate to* ***X***. In Bentler's assessment, "Spearman's mathematical achievment was originally accompanied by an application that emphasized the theory testing nature of the method: The theory of a general ability plus varieties of uncorrelated specific abilities in test performance amounted to a hypothesis that could, in principle, be refuted by data" (1986, p.40). That is, Spearman was testing an existential hypothesis about "general ability." Schönemann and Haagen (1987, p.836) further clarify: "The purpose of this theory was not, as some statisticians have surmised (e.g., Lawley and Maxwell, 1971, p.2; Press, 1972, p.303), to merely account for the correlations among the observed variables...Rather, as Spearman had made clear in the title of his epochal paper (1904a), he wanted to "objectively determine and measure" *g*, the general intelligence factor." By *determine*, Spearman meant "prove the existence of", a point made by both Thomson (1935) and Wolfle (1940, p.7).

In the words of Eysenck:

> ...for Spearman, Thurstone, and those who follow their methodology there quite clearly is such a reference. This causal implication characterizes not only the interpretation of factors as suggestive of a hypothesis, but also the next level of factors as proving a hypothesis, and since from the psychological point of view this causal implication is precisely what lends interest and value to factor analysis, it may be opportune here to give a definition of factor which brings out this element...A factor is a hypothetical causal influence underlying and determining the observed relationships between a set of variables (1953, p.108)

Burt (1966, pp. 34-35) gives a clear statement of the belief that linear factor analysis provides a test of the various hypothesized roles concerning intelligence: "It will be noted that the methodology thus proposed treats factor analysis as a method for falsifying or verifying alternate hypotheses already tentatively suggested by antecedent evidence or arguments..." (1966, pp.34-35). Holland and Rosenbaum, on the other hand, state that "The use of latent variable models is about testing "for the existence of a one-dimensional latent variable *U* that underlies a set of data" (1986, p.1524). If this were merely a statistical issue, and the issue was one of *mathematical* existence and statistical testability, then there would be no reason for the concept *underlying* to make an appearance. Certainly, the pages of Feller, seemingly every psychometrician's convenient citation on statistical theory, contain no mention of *underlying*. In fact, Holland and Rosenbaum are invoking senses of *existence* and testability which go well beyond mathematics.

The belief that latent variable models allow for tests of existential hypotheses is also expressed in a number of less direct ways, including the portrayal of manifest variates as indicators or detectors of unobservables: "As revealed by their correlations with the self-monitoring class variable, these factors thus serve as fallible indicators of the class variable" (Gangestad & Snyder, 1985, p.333). If manifest variates are *indicators*, they must, of course, be indicators of something. To know that *c* is an indicator of φ presupposes that one has a means of antecedently identifying φ. The Central Account explains that the thing to be detected, the referent of *latent variate to* ***X***, is a property/attribute (causal source) whose identity can be inferred after its detection. The manifest variates are said to *tap*, *detect*, or *pick up on* the pure signal sent out by the referent of *latent variate to* ***X***. This language is, evidently, drawn from the domain of resource extraction, and portrays the latent variable modeller as "tunneling" beneath the superficial, surface features of reality (the phenomena represented by the manifest variates) to get at the valuable material (the pure signal) that is of interest, but is obscured from view. Such language is commonplace in the applied literature: "...the subjective feeling tone is only one aspect of the entire depressive syndrome, and there is evidence that the failure to *tap* [italics added] the other dimensions of depression impairs the usefulness of this kind of test" (Beck, 1967, p.187); "The scale was originally designed to *tap* [italics added] into a single source of variance.." (Hoyle and Lennox, 1991, p.511); "The inclusion of more affect markers that *reflected* [italics added] states of high positive arousal (e.g., elated) and high negative arousal (e.g., angry) would enable a more adequate test for an arousal dimension of affect expression" (Hamid and Cheng, 1996, p.1011). Benet and Waller (1995, p.702) exploit features of the resource extraction metaphor when they state that "...in the Big Seven, Extraversion and Neuroticism are called Positive Emotionality and Negative Emotionality, respectively, in recognition of the emotional *core* [italics added] of these higher order factors."

The special sense of testability believed to attend the use of latent variable models goes hand- in-hand with the employment of a special sense of *model*. Whereas statisticians call many things models, latent variable models are seen to be models in a very special sense: They represent the relations between the phenomena represented by the manifest variates, and an unobservable property/attribute (causal source) of these phenomena. McArdle (1990, p.81) states, for example, that:

> The key algebraic difference between the CFA and the PCA is the explicit estimation of the unique factor variance in CFA. In a CFA model for *p* observed variables the *p* unique test factors are assumed to (a) be decomposable into *p* random error variables and *p* specific test variables, (b) are statistically independent from the *k* common factors, and (c) are statistically independent of each other. The classical CFA separation of common and unique factors is precisely the form of the first structural equation models described by Spearman (1904). Are these assumptions correct? Who knows. The one strength of the CFA model comes from the fact that the assumption (c) of uncorrelated specific factors is a falsifiable hypothesis.
>
> (p.86) My main point here is that PCA is not a model for psychological behaviours- PCA is simply a calculation device for the CFA model.

McArdle does not explain why he believes that latent variable models are models for "psychological behaviours" while PCA is not. McDonald (1996b, p.668) argued explicitly that a latent variable model is a model in the classical sense, i.e., that it is a representer of a particular state of affairs: "...we need a few remarks about (mathematical or nonmathematical) propositions that are offered as a *model*. Such propositions are necessarily offered as a model for...-they do not reflexively model themselves, or remain self-contained. There will be rules of correspondence between (some) features of the model, and (some) features of the modeled."

It was suggested earlier that empirical realist philosophy has been taken as justification of the unobservability components of the Central Account. But it is clear that this brand of philosophy also provides a suggestive interpretation of the manner in which latent variable models are uniquely testable. For, if the latent variable model, lvm, can be seen as a "theory" that introduces theoretical term *latent variate to* $\underline{\mathbf{X}}$, this term denoting an unobservable causal source, φ, of the phenomena represented by the $\mathbf{X}_j$, should such a causal source exist, then it is natural to see a test of $\underline{\Omega}_T \subset M_{lvm}$ as a test of the hypothesis that such a φ does, in fact, exist. Moreover, the ubiquitous claim that "the manifest variates are multiple indicators of a common factor or latent variate", is understood as "the manifest variates are multiple indicators of an unobservable nexus (either a cause or common property)". Rozeboom (1984) discusses the causal reading of this claim.

CA4: Inference and estimation

Commitment to CA4 is evident in several tasks standardly undertaken following model testing and parameter estimation. The latent variable modeler:

i) "interprets the factor (latent variate)";

ii) estimates latent variate (factor) scores;

iii) makes inferences as to the empirical properties of the interpreted latent variate.

In most texts on latent variable modeling there are chapters devoted to explaining to the researcher the steps he should take to interpret the latent variate. The product of latent variate interpretation is the assignment of a concept label to the property/attribute (cause) believed to have been detected in the employment of a model. Mulaik (1986, p.24) describes the chief task undertaken by Thurstone and his followers as being to determine "…what are the invariant properties revealed by a factor analysis?" Hence, the view is that there are in existence properties, these properties are in need of detection, factor analysis is a tool by which such properties can, in fact, be detected, and, once such a property has been detected, the task is to identify *which* property it is. As was seen, Lazarsfeld (1950), and many others, have offered up the received view that, in carrying out these inferential tasks, one is making inferences about unobservable objects and properties existing in latent domains.

CA4 is implied by oft-repeated slogans of the following sort: "...factor scores cannot be determined precisely, but only "estimated" (Pawlik, 1968, p.163); "Since Spearman (1922) it has been known that we cannot determine the common factor score of each of a set of subjects, on the basis of a sample of tests of finite size" (McDonald, 1977, p.165); "In a common-factor analysis we have n measured variables, from which we derive m+n factors: m common factors and n unique factors. It is evident then, because we have more unknowns (factors) than measured variables, that scores on the m common factors can only be estimated, for example, by regression" (Cureton & D'Agostino,1983, p.339); "The theoretical scores are not available: consequently, several systems for estimating the scores have been proposed (Tucker, 1971, p.427); "IRT models are used to make formal connections between latent and manifest variables and to estimate the values of a person on the latent variables as well as the values of the item parameters from the observable manifest variables (e.g., item responses). Such estimations are necessary, because latent variables are principally unobservable" (Strauss, 1999, p.19). It is also implied by the close ties of latent variable modeling to the the empirical realist account of unobservable entities.

Finally, in distinguishing between what he calls the *descriptive* and *inferential* aims of factor analysis, Hartley (1954) gives a clear account of the essence of CA4:

> It may be possible, then, to characterize adequately inferential factor analysis as that form of factor analysis in which the factor matrix has empirical meaning in addition to that expressed by the correlation matrix. Two different factorial solutions for the same correlation matrix could, then, express different and even contradictory meanings. The meaning of the factor matrix which is in addition to that expressed by the correlation matrix is the reference to the unobserved but hypothesized "factors". Since these factors are not referred to by the correlation matrix, the correlation matrix would seem to be serving as the evidence from which the factors are inferred by some process or other (p.197)

CA5: Epistemological specialness

Methodologists and applied researchers alike assert in a number of different ways the belief that latent variable modeling is epistemologically special, relative to other brands of statistical modeling. First, belief in CAM or CAC implies that latent variable models are special, for these pictures describe powers possessed by no other statistical model: When $\underline{\mathbf{X}}$ is described by a latent variable model, the researcher has detected an unobservable property/attribute (causal source) of the phenomena represented by the $\mathbf{X}_j$. The latent variable model is a tool that yields information about the unobservable constituents of latent domains. Second, there can be found in the literature claims that imply this belief. Rummel, for example, calls linear factor analysis the "calculus of the social sciences" (1970, p.4) and claims that it represents a formalism analogous to "that of quantum theory" (1970, p.28). Royce (1958, p.158) states that "Factor analysis is one of the most powerful methodological tools and theoretical models in the hands of the research psychologist. Viewed broadly, it represents a profound and monumental contribution to psychological science as well as a unique contribution to general scientific methodology."

Third, the epistemological specialness of latent variable models is expressed in comparisons of latent variable models to other similar statistical models. Excellent examples are found in the responses to the target article of Velicer and Jackson, which appeared in the 1990 issue of *Multivariate Behavioral Research*. The Velicer and Jackson article offered a theoretical and empirical comparison of the common factor and principal component models. Among other conclusions, they put forth that the results yielded by the two models were often very similar, that certain of the advantages supposed to hold in the employment of the common factor model were illusory, and that the principal component model was preferable for a number of reasons, including that component variates are determinate. Evidently, few discussants felt at ease with Velicer and Jackson's claim that the two models "often yield similar results". For, indeed, it is pointless to compare parameter estimates generated under each of the two models, when these estimates are deprived of the meaning afforded them by their place within an analytic practice. What is the point of reporting that a set of factor loadings are roughly identical to a set of variate/component correlations, when, as implied in rebuttals by Mulaik (1990) and others, the former are conceptualized in an entirely different fashion than the latter. In particular, factor analytic output is understood in terms of the Central Account.

Mulaik's (1990) rebuttal put it well: "Potentially a common factor variable can have theoretical applications independent of any particular set of observed variables by being, say, the common cause of variation in and the basis for covariance between variables other than those observed variables originally giving rise to the concept of the common factor in question. The indeterminacy of latent common factors (and unique factors also) in the common factor model is an unavoidable aspect of what makes the common factor model (where applicable) appropriate for formulating objective general conceptions of the basis for relationships between observed variables in nature" (p.53). In contrast "...the essential feature of a component variable in a component analysis model is that a component variable is defined as some specific linear composite of a set of observed variables chosen in some particular way to optimize some quantitative criterion. Component variables are thus tied inextricably to a particular set of variables as linear combinations of them." (p.53). Hence, according to Mulaik, latent variable models somehow "go beyond" the information inherent to the manifest variates, something that cannot be achieved in the use of component models. The implication is that "determinate models like component analysis may not have as many scientific virtues as do indeterminate models like common factor analysis..." (Mulaik, 1990, p.54). Mulaik concludes that, "One might wish to do component analysis because computationally it is simpler. But using component analysis in this way need not mean one has abandoned thinking of the data in terms of the common factor model...one would make a grievous error in thinking that a component variable could represent the common cause of and reason for the covariation among a set of manifest variables in the same way that a latent variable can in common factor analysis and structural equation modeling. That effect variables are asymmetric functional relations of causal variables precludes causal variables' being in turn functional relations of effect variables and thereby uniquely determined by them also..." (p.54). In an earlier summary of the development of factor analysis that appeared in *Psychometrika*, Mulaik (1986, p.29) states that "...it is difficult to regard component factor models as causal models, for ordinarily causes are not uniquely determined by a sample of their effects, which is what is implied by the component factors' being uniquely determined by the observed variables." In favoring factor analysis over principal component analysis, Heermann presents a line of reasoning very similar to that of Mulaik (1990): Components "are always contained in the test space, and cannot be expected to represent anything which goes beyond the original measures" (1964, p.380).

Bentler and Kano's (1990, p.67) rebuttal contains many interesting insights, but is firmly rooted to a belief in CA5. They ask, "Does one believe, in a particular application, that each variable to be analyzed is *generated* [italics added] in part by a random error variate, and that these error variates are mutually uncorrelated? If so, the dimensionality of any meaningful model is greater than the dimensionality of the measured variables, and hence by definition a latent variable model, here, factor analysis, is called for." But what needs anwering is why one's *beliefs* should matter when it comes to application of a model that is supposed to provide *adjudgment* as to whether the variates were *generated* (whatever that means) in the manner described. Are Bentler and Kano truly wishing to claim that *belief* in random error justifies the use of high dimensional models? Moreover, they later state that "With a small number of variables...the factor model is typically more appropriate to understanding the structure of the covariance or correlation matrix... (1990, p.73). What exactly does this mean? By "structure", Bentler and Kano evidently mean the particular set of values assumed, in a given population, by the elements of a particular covariance or correlation matrix. Now, a particular matrix of values, say Σ_o, can be decomposed in a great variety of ways, e.g., as per the Cholesky factorization (if Σ_o is positive definite), in terms of eigenvectors and eigenvalues, or, if Σ_o satisfies certain restrictions, as per the linear factor model. Let's say that Σ_o happens to be m-dimensional linear factor representable, say, $\Sigma_o=\Lambda_m\Lambda_m'+\Psi$, Ψ, diagonal and positive definite. How will this particular decomposition help the researcher to achieve a better "understanding" of Σ_o than, say, the Cholesky decomposition of Σ_o? On its own, the answer is that it won't. What Bentler and Kano (1990) fail to mention is their belief in the Central Account. For it is the Central Account, with its description of underlying generative sources, that they tacitly take to constitute an *explanation* as to why Σ_o is the way that it is, and, hence, why the factor representation is "appropriate". Rozeboom's (1990, p.63) stance regarding the powers of factor analysis is well known. His rebuttal to Velicer and Jackson includes the claim that "..for those of us who regard common factoring as our best way to search out the causal origins of manifest covariances still have serious work ahead."

Finally, the perceived epistemological specialness of latent variable models is shown in the fact that, through the years, researchers and experts have felt the need to debunk this perception. Cyril Burt, for example, commented that:

> When we analyze table salt into sodium, chlorine and a residuum of impurities, we effect an actual physical separation; and we consequently infer that the component atoms or elements are as concrete as the particles of salt. With some such analogy in mind, the student of factor-analysis in psychology is tempted to reify the factors named, and to visualize a logical analysis as a physical separation, tacitly assuming that, if distinct abilities are ever to be discovered they will be concrete and separate "organs", like the heart or the lungs, and that the "mental mechanisms" which form them will be localized in separate brain centres or cortical areas...Our factors, therefore, are to be thought of in the first instance as lines or terms of reference only, not as concrete psychological entities (Burt, 1940, p.17).

In a 1964 *Psychological Bulletin* paper, Overall (1964, p.270) attempted a similar corrective, insisting, and attempting to demonstrate, that "The results need not and do not have inherent in them any necessary relationship to "real" or "primary" characteristics of the objects or persons being measured." In the language of the current work, Overall seems to be claiming that there is no reason to believe that, when a particular $\underline{\mathbf{X}}$ is described by the linear factor model, the referent of *latent variate to* $\underline{\boldsymbol{X}}$ is a property/attribute (self-esteem, dominance, agreeableness) of the phenomena represented by the manifest variates. That is, he is taking exception to key elements of the CAM.