ENSC 388

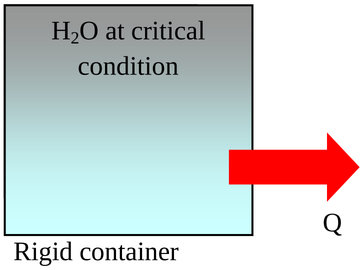
Assignment #2 (Properties of Pure Substances)

Assignment date: Wed Sept. 23, 2009

Due date: Wed Sep. 30, 2009

Problem 1

Water at the critical condition is contained in a rigid container. The system is cooled down until its temperature reaches $T = 25°\text{C}$? Calculate specific volume, internal energy, and quality for initial and final states. What would be the pressure of the system at the final state?

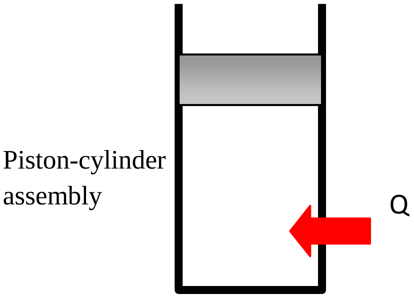


Problem 2

R-134a is contained in a piston-cylinder device at $T_1 = -12°\text{C}$ and $P_1 = 0.2MPa$. The fluid is heated isobarically such that $V_2 = 20V_1$. Heat is then added at constant volume until the temperature reaches $T_3 = 80°\text{C}$.

a) Show the process on a *T*-*v* diagram.

b) Calculate h_1, h_2 and h_3.



Problem 1:

Known:

$$T_1 = T_{cr} = 373.95℃ \qquad v_1 = v_{cr} = 0.003106 \left[\frac{m^3}{kg}\right]$$

$$T_2 = 25℃ \qquad v_2 = v_1$$

Find:

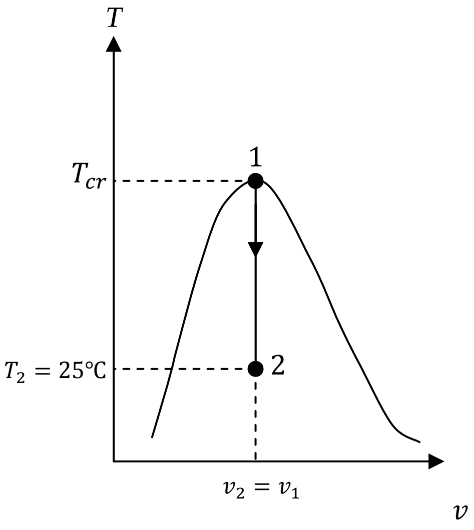
- Specific volume, internal energy and quality at states 1 and 2.
- Pressure at state 2

Assumptions:

- Rigid tank
- Negligible kinetic and potential energies

Analysis:

Since the volume of the tank and the mass of water in the container remains constant, the process is a constant specific volume process $v_2 = v_1$. This process can be shown as a vertical line in a *T*-*v* diagram.



T-*v* Diagram

We should first check to see if state 2 is a saturated mixture. Using Table A-4 at $T_{sat} = 25℃$:

$$v_{f@25℃} = 0.001003\left[\frac{m^3}{kg}\right] < v_2 = 0.003106\left[\frac{m^3}{kg}\right] < v_{g@25℃} = 43.340\left[\frac{m^3}{kg}\right]$$

Since the specific volume of state 2 falls between that of the saturated liquid and the saturated vapour at 25℃, one can conclude the state 2 is a saturated mixture.

$$v_2 = v_f + x_2 v_{fg}$$

From Table A-4:

$$x_2 = \frac{v_2 - v_{f,2}}{v_{g,2} - v_{f,2}} = \frac{0.003106\left[\frac{m^3}{kg}\right] - 0.001003\left[\frac{m^3}{kg}\right]}{43.340\left[\frac{m^3}{kg}\right] - 0.001003\left[\frac{m^3}{kg}\right]} \rightarrow\ x_2 = 0.0000485$$

$$u_2 = u_{f,2} + x_2 u_{fg,2} = 104.83\left[\frac{kJ}{kg}\right] + 0.0000485 \times 2304.3\left[\frac{kJ}{kg}\right] = 104.94\left[\frac{kJ}{kg}\right]$$

$$u_1 = u_{cr} = 2015.7\left[\frac{kJ}{kg}\right]$$

Since state 2 is a saturated mixture from Table A-4:

$$\boxed{P_2 = P_{sat@25℃} = 3.1698[kPa]}$$

Problem 2:

Known:

$$P_1 = 0.2MPa \qquad T_1 = -12℃$$

$$P_2 = P_1 \qquad V_2 = 20V_1$$

$$V_3 = V_2 \qquad T_3 = 80℃$$

Find:

$$h_1,\ h_2\ \&\ h_3$$

Assumptions:

- Saturated liquid properties for compressed liquid state considered, except for property *h*.

Analysis:

Let's starting with properties of the system at states 1, 2 and 3:

State	$T[℃]$	$P[kPa]$	$v\left[{m^3}/{kg}\right]$	$h\left[{kJ}/{kg}\right]$
1	-12	200	v_1	h_1
2	T_2	$P_2 = P_1$	$v_2 = 20v_1$	h_2
3	80	P_3	$v_3 = v_2$	h_3

State 1:

With $T_1 = -12℃ \;\rightarrow\; P_{sat} = 185.37[kPa] = 0.185[MPa]$ (Table A-11)

Since $(P_1 > P_{sat,1})$, state 1 is compressed liquid. Because we do not have a compressed liquid Table for R-134a, it is assumed:

$$v_1 \approx v_{f,1@-12℃} = 0.0007499 \left[\frac{m^3}{kg}\right]$$

$$h_1 = h_{f,1} + v_{f,1}(P_1 - P_{sat@-12℃})$$

Using Table A-11:

$$h_1 = 35.92 \left[\frac{kJ}{kg}\right] + 0.0007499 \left[\frac{m^3}{kg}\right](200 - 185.37)\left[\frac{kN}{m^2}\right] = 35.93 \left[\frac{kJ}{kg}\right]$$

State	$T[℃]$	$P[kPa]$	$v\left[{}^{m^3}/_{kg}\right]$	$h\left[{}^{kJ}/_{kg}\right]$
1	-12	200	0.0007499	35.93
2	T_2	200	0.015	h_2
3	80	P_3	0.015	h_3

State 2:

$$v_2 = 20v_1 = 0.015 \left[\frac{m^3}{kg}\right]$$

From Table A-12 at $P_2 = 200kPa$:

$$v_{f,2} = 0.0007533 \left[\frac{m^3}{kg}\right] < v_2 = 0.015 \left[\frac{m^3}{kg}\right] < v_{g,2} = 0.099867 \left[\frac{m^3}{kg}\right]$$

The quality of the mixture can be found:

$$x_2 = \frac{v_2 - v_{f,2}}{v_{g,2} - v_{f,2}} = \frac{0.015 \left[\frac{m^3}{kg}\right] - 0.0007533 \left[\frac{m^3}{kg}\right]}{0.099867 \left[\frac{m^3}{kg}\right] - 0.0007533 \left[\frac{m^3}{kg}\right]} = 0.144$$

Therefore:

$$h_2 = (1 - x)h_f + xh_g$$

$$h_2 = (1 - 0.144) \times 38.43 \left[\frac{kJ}{kg}\right] + 0.144 \times 244.46 \left[\frac{kJ}{kg}\right] = 68.1 \left[\frac{kJ}{kg}\right]$$

State	$T[°\mathrm{C}]$	$P[kPa]$	$v\left[{m^3}/{kg}\right]$	$h\left[{kJ}/{kg}\right]$
1	-12	200	0.0007499	35.93
2	10.09	200	0.015	68.1
3	80	P_3	0.015	h_3

State 3:

During process 2-3, the volume and the mass of the system remain constant; therefore it is a constant specific volume process, i.e.:

$$v_3 = v_2 = 0.015 \left[\frac{m^3}{kg}\right]$$

A constant specific volume process can be shown as a vertical line in a *T*-*v* diagram. Using saturated Table A-11 at $T_3 = 80°\mathrm{C}$:

$$v_g = 0.006436 \left[\frac{m^3}{kg}\right] < v_3 = 0.015 \left[\frac{m^3}{kg}\right]$$

Thus state 3 is superheated vapour and Table A-13 should be used. Looking for the range of v_3 in Table A-13 one can find:

$$1.4[MPa] < P_3 < 1.6[MPa]$$

where

$$v_{@1.4MPa} = 0.017023 \left[\frac{m^3}{kg}\right]$$

$$\vartheta_{@1.6MPa} = 0.014362 \left[\frac{m^3}{kg}\right]$$

Interpolating for h_3:

$$\frac{h_3 - h_{@1.4MPa}}{h_{@1.6MPa} - h_{@1.4MPa}} = \frac{v_3 - v_{@1.4MPa}}{v_{@1.6MPa} - v_{@1.4MPa}}$$

$$\frac{h_3 - 308.34 \left[\frac{kJ}{kg}\right]}{305.07 \left[\frac{kJ}{kg}\right] - 308.34 \left[\frac{kJ}{kg}\right]} = \frac{0.015 \left[\frac{m^3}{kg}\right] - 0.017023 \left[\frac{m^3}{kg}\right]}{0.014362 \left[\frac{m^3}{kg}\right] - 0.017023 \left[\frac{m^3}{kg}\right]}$$

$$\rightarrow h_3 = 305.85 \left[\frac{kJ}{kg}\right]$$

State	$T[°\mathrm{C}]$	$P[kPa]$	$v\left[{m^3}/{kg}\right]$	$h\left[{kJ}/{kg}\right]$
1	-12	200	0.0007499	35.93
2	10.09	200	0.015	68.1
3	80	1447.8	0.015	305.85

The properties of the three states are: state 1, compressed liquid, state 2, saturated mixture and state 3, superheated vapour. Thus the *T*-*v* diagram can be plotted as Fig. 1.

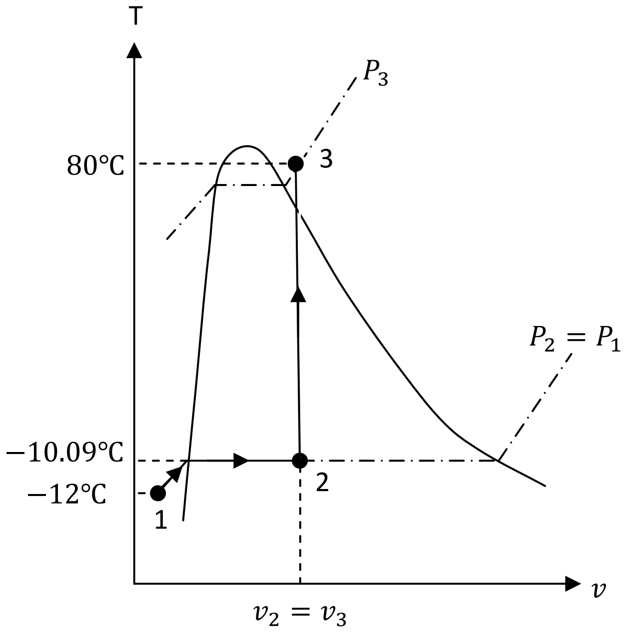


Fig.1. *T*-*v* diagram of the process

Note: In the absence of compressed liquid data, a general approximation is to treat compressed liquid as saturated liquid at the given temperature. However, the error in *h* at low to moderate pressures and temperatures can be reduced significantly by evaluating it from:

$$h \cong h_{f@T} + v_{f@T}(P - P_{sat@T})$$