

ENSC 388

Assignment #3

Assignment date: Wednesday Sept. 30, 2009

Due date: Wednesday Oct. 7, 2009

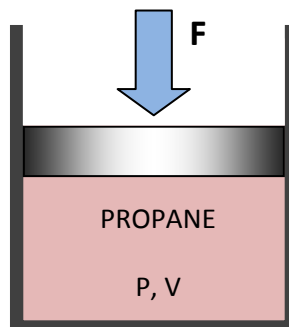
Problem 1

A cylinder fitted with a piston contains propane ($T_{cr} = 370\text{ K}$, $P_{cr} = 4.26\text{ MPa}$, $R = 0.1885\text{ kJ/kg}\cdot\text{K}$) gas at 100 kPa and 300 K , where the total volume of the system is 200 L . The gas is now slowly compressed according to the relation:

$$PV^{1.1} = \text{constant}$$

until the final temperature reaches 340 K .

- What is the final pressure?
- Elaborate on why the ideal gas assumption can be used?
- How much work is done during the process?



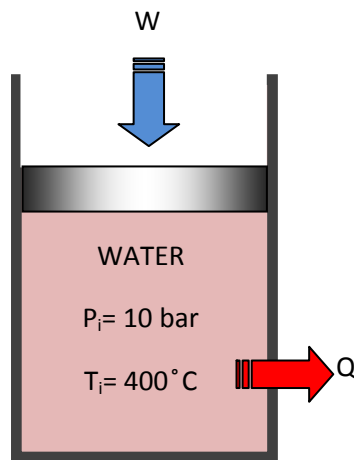
Problem 2

Water contained in a piston-cylinder assembly undergoes two processes in series from an initial state where the pressure is 10 *bar* and the temperature is 400°C.

Process 1-2: The water is cooled as it is compressed at constant pressure of 10 *bars* to the saturated vapour state.

Process 2-3: The water is cooled at constant volume to 150°C.

- Sketch both processes on T - v and P - v diagrams.
- For the overall process determine the work, in kJ/kg .
- For the overall process determine the heat transfer.



Problem 1:

Known:

Critical properties of propane:

$$T_{cr} = 370 \text{ K}, P_{cr} = 4.26 \text{ MPa}, R = 0.1885 \text{ kJ/kg}\cdot\text{K}$$

Initial condition:

$$P_1 = 100 \text{ kPa}, T_1 = 300 \text{ K}, V_1 = 200 \text{ L}$$

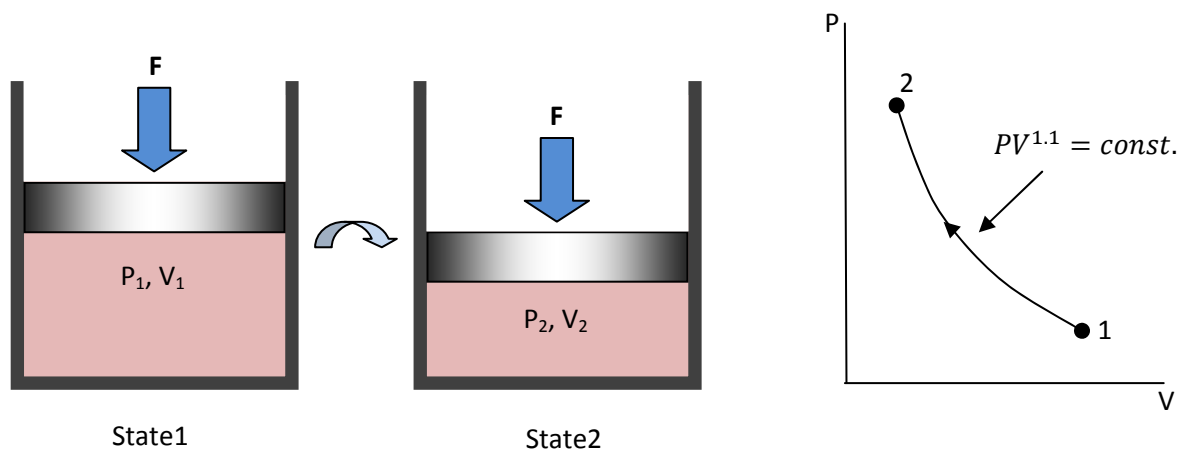
Find:

- Final pressure of the process.
- The work is done during the process.

Assumptions:

- Weight of the pistons is negligible.
- Equilibrium condition.
- Propane is ideal gas

Analysis:



From ideal gas assumption:

$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}$$

$$\frac{P_2 V_2}{340 [K]} = \frac{100 [kPa] \times 0.2 [m^3]}{300 [K]} \rightarrow P_2 V_2 = 22.667 [kPa \cdot m^3]$$

Also, from compression process:

$$P_2 V_2^{1.1} = P_1 V_1^{1.1} = 100 [kPa] \times 0.2^{1.1} [m^{3.3}] = 17.027 [kPa \cdot m^{3.3}]$$

Thus:

$$\frac{P_2 V_2^{1.1}}{P_2 V_2} = V_2^{0.1} = \frac{17.027 [kPa \cdot m^{3.3}]}{22.667 [kPa \cdot m^3]}$$

$$\rightarrow V_2 = 0.0572 [m^3]$$

From ideal gas equation of state:

$$P_2 = \frac{mRT_2}{V_2}$$

Applying ideal gas equation of state for state 1:

$$m = \frac{P_1 V_1}{RT_1} = \frac{100 [kPa] \times 0.2 [m^3]}{0.1885 \left[\frac{kJ}{kg \cdot K} \right] \times 300 [K]} = 0.345 [kg]$$

Thus:

$$P_2 = \frac{0.345 [kg] \times 0.1885 \left[\frac{kJ}{kg \cdot K} \right] \times 340 [K]}{0.0572 [m^3]}$$

$$\rightarrow P_2 = 396.6 [kPa]$$

(b) Since pressures and temperatures are far from the critical state, the ideal gas equation of state can be used in this problem with no error.

(c) Work in a close system is calculated from:

$$W = \int_1^2 P dV$$

P can be related to V with $P = C/V^{1.1}$ (C is constant):

$$W = \int_1^2 \frac{C}{V^{1.1}} dV = \frac{C}{1-1.1} V^{1-1.1} \Big|_1^2 = -\frac{C}{0.1} V^{-0.1} \Big|_1^2$$

$$\rightarrow W = -\frac{C}{0.1} (V_2^{-0.1} - V_1^{-0.1})$$

Substituting $C = P_1 V_1^{1.1} = P_2 V_2^{1.1}$:

$$W = -\frac{P_2 V_2^{1.1} V_2^{-0.1} - P_1 V_1^{1.1} V_1^{-0.1}}{0.1} = -\frac{P_2 V_2 - P_1 V_1}{0.1}$$

Thus:

$$W = -\frac{396.6[kPa] \times 0.0572 [m^3] - 100[kPa] \times 0.2 [m^3]}{0.1}$$

$$\rightarrow W = -26.5 [kJ]$$

Problem 2:

Known:

The two processes

Initial condition: $P_1 = 10 \text{ bar}$, $T_1 = 400^\circ\text{C}$

Find:

- Processes on T - v and P - v diagrams
- Overall work and heat transfer

Assumptions:

- Weight of the pistons is negligible.
- Equilibrium condition.

Analysis:

(a) Let us determine the three states of the process:

	State	$T[^\circ\text{C}]$	$P[\text{kPa}]$	$v \left[\frac{\text{m}^3}{\text{kg}} \right]$	$u \left[\frac{\text{kJ}}{\text{kg}} \right]$
1	?	400	1000	v_1	u_1
2	Saturated Vapour	T_2	$P_2 = P_1$	v_2	u_2
3	?	150	P_3	$v_3 = v_2$	u_3

State 1:

$$P_1 = 10 [\text{bar}] = 1 [\text{Mpa}] \quad T_1 = 400[^\circ\text{C}]$$

From Table A-5, $T_{\text{sat}@1000 \text{ kPa}} = 179.88 \text{ }^\circ\text{C} < T_1 = 400 \text{ }^\circ\text{C}$. Thus state 1 is superheated vapour. Reading Table A-6 at $P_1 = 1 \text{ MPa}$ and $T_1 = 400^\circ\text{C}$:

$$v_1 = 0.30661 \left[\frac{\text{m}^3}{\text{kg}} \right] \quad u_1 = 2957.9 \left[\frac{\text{kJ}}{\text{kg}} \right]$$

State		$T[^\circ\text{C}]$	$P[\text{kPa}]$	$v \left[\frac{\text{m}^3}{\text{kg}} \right]$	$u \left[\frac{\text{kJ}}{\text{kg}} \right]$
1	Super heated vapour	400	1000	0.30661	2957.9
2	Saturated Vapour	T_2	1000	v_2	u_2
3	?	150	P_3	$v_3 = v_2$	u_3

State 2:

Using Table A-5 for saturated vapour at $P_2 = P_1 = 1 \text{ Mpa}$:

$$v_2 = v_g = 0.19436 \left[\frac{\text{m}^3}{\text{kg}} \right] \quad u_2 = u_g = 2582.8 \left[\frac{\text{kJ}}{\text{kg}} \right]$$

State		$T[^\circ\text{C}]$	$P[\text{kPa}]$	$v \left[\frac{\text{m}^3}{\text{kg}} \right]$	$u \left[\frac{\text{kJ}}{\text{kg}} \right]$
1	Super heated vapour	400	1000	0.30661	2957.9
2	Saturated Vapour	179.88	1000	0.19436	2582.8
3	?	150	P_3	0.19436	u_3

State 3:

Process 2-3 is constant volume, hence $v_3 = v_2 = 0.19436 \text{ m}^3/\text{kg}$.
Reading Table A-4:

$$v_{f@150^\circ\text{C}} = 0.001091 \left[\frac{\text{m}^3}{\text{kg}} \right] < v_3 = 0.19436 \left[\frac{\text{m}^3}{\text{kg}} \right] < v_{g@150^\circ\text{C}} = 0.39248 \left[\frac{\text{m}^3}{\text{kg}} \right]$$

As a result, state 3 is saturated mixture and:

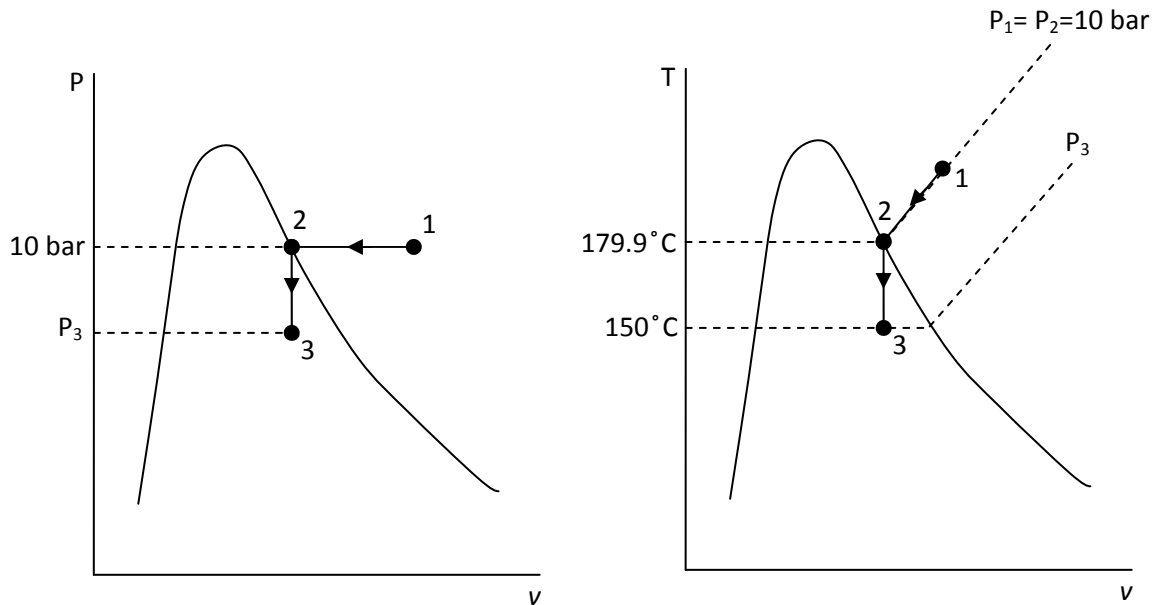
$$x_3 = \frac{v_3 - v_f}{v_g - v_f} = \frac{0.19436 \left[\frac{\text{m}^3}{\text{kg}} \right] - 0.001091 \left[\frac{\text{m}^3}{\text{kg}} \right]}{0.39248 \left[\frac{\text{m}^3}{\text{kg}} \right] - 0.001091 \left[\frac{\text{m}^3}{\text{kg}} \right]} = 0.494$$

$$u_3 = u_{f@150^\circ\text{C}} + x_3 u_{fg@150^\circ\text{C}} = 631.66 \left[\frac{\text{kJ}}{\text{kg}} \right] + 0.494 \times 1927.4 \left[\frac{\text{kJ}}{\text{kg}} \right]$$

$$\rightarrow u_3 = 1584.1 \left[\frac{\text{kJ}}{\text{kg}} \right]$$

	State	$T[^\circ\text{C}]$	$P[\text{kPa}]$	$v \left[\frac{\text{m}^3}{\text{kg}} \right]$	$u \left[\frac{\text{kJ}}{\text{kg}} \right]$
1	Super heated vapour	400	1000	0.30661	2957.9
2	Saturated Vapour	179.88	1000	0.19436	2582.8
3	Saturated Mixture	150	476.16	0.19436	1584.1

T - v and P - v diagrams are:



(b) The overall process work is:

$$W = \int_1^3 P dV = \int_1^2 P dV + \int_2^3 P dV$$

Since process 1-2 is constant pressure and process 2-3 is constant volume ($dV = 0$):

$$W = \int_1^2 P dV = P(V_2 - V_1)$$

or per unit mass:

$$w = P(v_2 - v_1) = 1000 \text{ [kPa]} \times \left(0.19436 \left[\frac{\text{m}^3}{\text{kg}} \right] - 0.30661 \left[\frac{\text{m}^3}{\text{kg}} \right] \right)$$

$$\rightarrow w = -112.2 \left[\frac{\text{kJ}}{\text{kg}} \right]$$

(c) Applying the first law of thermodynamics:

$$q_{13} - w_{13} = \Delta u_{13} \rightarrow q_{13} = \Delta u_{13} + w_{13}$$

$$q_{13} = \left(1584.1 \left[\frac{kJ}{kg} \right] - 2957.9 \left[\frac{kJ}{kg} \right] \right) + \left(-112.2 \left[\frac{kJ}{kg} \right] \right)$$

$$\rightarrow q = q_{13} = -1485.4 \left[\frac{kJ}{kg} \right]$$

Note (1): Since the deviation of gas from ideal gas behaviour is greatest in the vicinity of the critical point, when pressure and temperature are far from this point, ideal gas assumption can be used without any essential error.

Note (2): At very low pressures gases behave as an ideal gas regardless of temperature

Note (3): At high temperatures, ideal gas behaviour can be assumed with good accuracy regardless of pressure (except for $P \gg P_{cr}$).