

14-67 Two glasses of a double pane window are maintained at specified temperatures. The fraction of heat transferred through the enclosure by radiation is to be determined.

Assumptions 1 Steady operating conditions exist. 2 Air is an ideal gas with constant properties. 3 The air pressure in the enclosure is 1 atm.

Properties The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (280+336)/2 = 308 \text{ K} = 35^\circ\text{C}$ are (Table A-22E)

$$k = 0.02625 \text{ W/m}\cdot^\circ\text{C}$$

$$\nu = 1.655 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7268$$

$$\beta = \frac{1}{T_f} = \frac{1}{308 \text{ K}} = 0.003247 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the distance between the two glasses, $L_c = L = 0.4 \text{ m}$. Then,

$$Ra_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2} \text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003247 \text{ K}^{-1})(336 - 280 \text{ K})(0.4 \text{ m})^3}{(1.655 \times 10^{-5} \text{ m}^2/\text{s})^2} (0.7268) = 3.029 \times 10^8$$

The aspect ratio of the geometry is $H/L = 1.5/0.4 = 3.75$. For this value of H/L the Nusselt number can be determined from

$$Nu = 0.22 \left(\frac{\text{Pr}}{0.2 + \text{Pr}} Ra \right)^{0.28} \left(\frac{H}{L} \right)^{-1/4} = 0.22 \left(\frac{0.7268}{0.2 + 0.7268} (3.029 \times 10^8) \right)^{0.28} \left(\frac{1.5}{0.4} \right)^{-1/4} = 35.00$$

Then,

$$A_s = H \times W = (1.5 \text{ m})(3 \text{ m}) = 4.5 \text{ m}^2$$

$$\dot{Q}_{\text{conv}} = kNuA_s \frac{T_1 - T_2}{L} = (0.02625 \text{ W/m}\cdot^\circ\text{C})(35.00)(4.5 \text{ m}^2) \frac{(336 - 280)\text{K}}{0.4 \text{ m}} = 578.9 \text{ W}$$

The effective emissivity is

$$\frac{1}{\varepsilon_{\text{eff}}} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 = \frac{1}{0.15} + \frac{1}{0.90} - 1 = 6.778 \longrightarrow \varepsilon_{\text{eff}} = 0.1475$$

The rate of heat transfer by radiation is

$$\begin{aligned} \dot{Q}_{\text{rad}} &= \varepsilon_{\text{eff}} A_s \sigma (T_1^4 - T_2^4) \\ &= (0.1475)(4.5 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(336 \text{ K})^4 - (280 \text{ K})^4] = 248.4 \text{ W} \end{aligned}$$

Then the fraction of heat transferred through the enclosure by radiation becomes

$$f_{\text{rad}} = \frac{\dot{Q}_{\text{rad}}}{\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}} = \frac{248.4}{578.9 + 248.4} = \mathbf{0.30}$$

