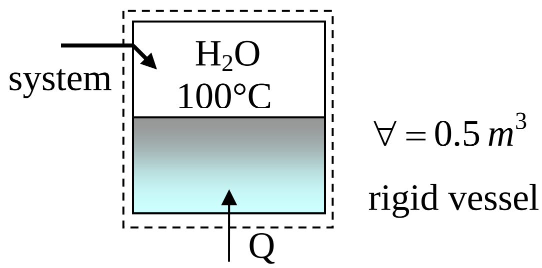
ENSC 388 Week # 3, Tutorial # 2– Properties of Pure Substances

Problem 1: A *0.5* m^3 rigid vessel initially contains saturated liquid-vapor mixture of water at *100°C*. The water is now heated until it reaches the critical state. Determine the mass of the liquid water and the volume occupied by the liquid at the initial state.

<u>Solution</u>

Step 1: Draw a diagram to represent the system showing control mass/volume of interest.



Step 2: Write out what you are required to solve for (this is so you don't forget to answer everything the question is asking for)

Find:

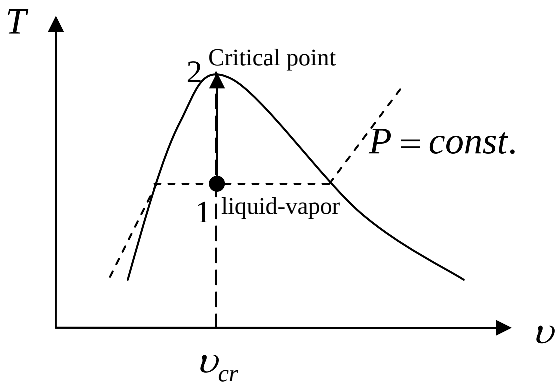
a) m - mass of the liquid water
b) $\forall_f$ - volume occupied by the liquid at the initial state

Step 3: Prepare a data table

Data	Value	Unit
$\forall$	0.5	$[m^3]$
T_1	100	$[^\circ C]$

Step 4: Calculations

This is a constant volume process ($\upsilon = \forall / m = const.$) to the critical state, and thus the initial specific volume will be equal to the final specific volume, which is equal to specific critical volume. The critical specific volume for water is $\upsilon_{cr} = 0.003106\ m^3/kg$, using Fig. **3-16** or Table **A-1** and **A-4**.



The mass of liquid-vapor mixture can be evaluated by:

$$m = \frac{\forall}{\upsilon} = \frac{0.5\,[m^3]}{0.003106\left[\frac{m^3}{kg}\right]} = 160.98\ kg \qquad \text{(Eq1)}$$

From Table **A-4** @ $T_{sat} = T_1 = 100°C : \begin{cases} \upsilon_{f1} = 0.001043\left[m^3/kg\right] \\ \upsilon_{g1} = 1.6720\left[m^3/kg\right] \end{cases}$

The quality of vapor at initial state is:

$$x_1 = \frac{\upsilon_1 - \upsilon_{f1}}{\upsilon_{fg1}} = \frac{0.003106\left[m^3/kg\right] - 0.001043\left[m^3/kg\right]}{1.6720\left[m^3/kg\right] - 0.001043\left[m^3/kg\right]} = 0.001235 \qquad \text{(Eq2)}$$

Then, the mass of the liquid and its volume at the initial state are:

$$m_{f1} = (1 - x_1)\,m = (1 - 0.001235) \times 160.98 = 160.78\ kg \qquad \text{(Eq3)}$$

$$\forall_{f1} = m_{f1}\, \upsilon_{f1} = 160.78\,[kg] \times 0.001043\, \left[\frac{m^3}{kg}\right] = 0.1677\; m^3 \qquad \text{(Eq4)}$$

Step 5: Concluding Statement

The mass of the liquid water was found to be $160.98\,kg$ and the volume occupied by the liquid at the initial state to be $0.1677\; m^3$.

Problem 2: Water contained in a piston-cylinder assembly undergoes two processes in series from an initial state where the pressure is 1 MPa and the temperature is 400 *°C.*

Process 1-2: The water is cooled as it is compressed at a constant pressure of 1 MPa to the saturated vapour state.

Process 2-3: The water is cooled at constant volume to 150 *°C.*

a) Sketch both processes on *T-v* and *p-v* diagrams.
b) Determine the specific volume, enthalpy and internal energy at states 1, 2 and 3.
c) Find the quality for states 2 and 3.

<u>**Solution**</u>

Step 1: Draw a diagram to represent the system showing control mass/volume of interest.

Step 2: Write out what you are required to solve for (this is so you don't forget to answer everything the question is asking for)

Indicate the initial and final states on a $T-\upsilon$ diagram;
Find:

a) *T-v* and *p-v* diagrams for both processes
b) v_1, v_2 *and* v_3 - The specific volume at states 1, 2 and 3, in $\frac{m^3}{kg}$.

c) h_1, h_2 *and* h_3 - The enthalpy at states 1, 2 and 3, in $\frac{kJ}{kg}$.

d) u_1, u_2 *and* u_3 - The internal energy at states 1, 2 and 3, in $\frac{kJ}{kg}$.

e) x_3, x_3 - The quality for states 2 and 3.

Step 3: Prepare a data table

Data	Value	Unit
T_1	400	$°C$
P_1	1	MPa
P_2	1	MPa
T_3	150	$°C$

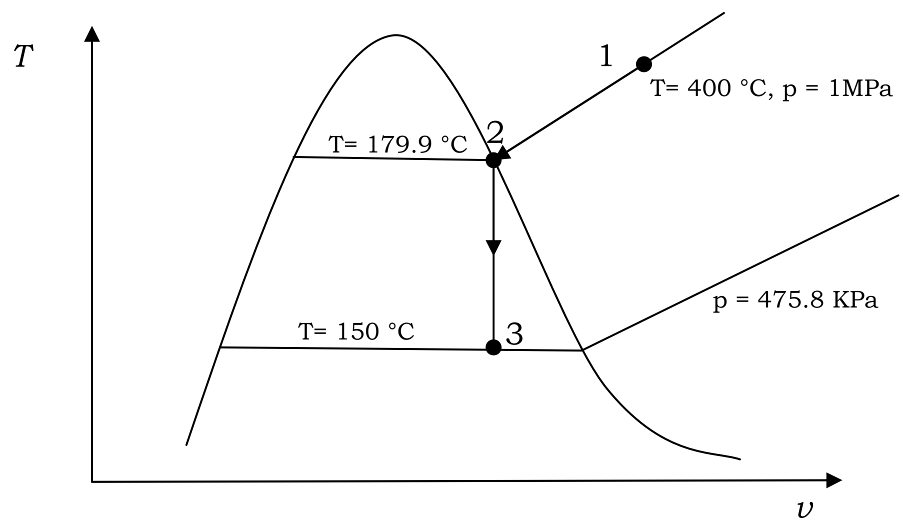
Step 4: State your assumptions (you may have to add to your list of assumptions as you proceed in the problem)

Assumptions:

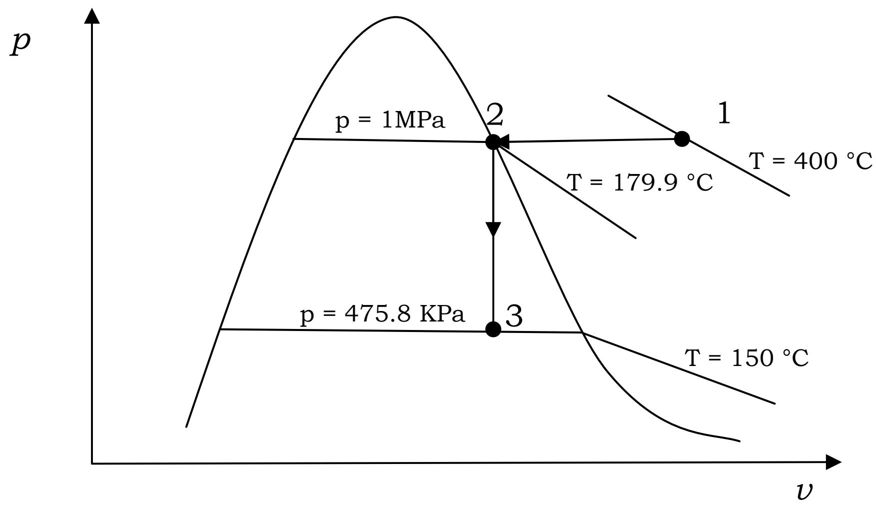
1) The water in the cylinder is a closed system.

Step 5: Calculations

The accompanying *T-v* and *p-v* diagrams show the two processes. The $T-\upsilon$ diagram for the process can be shown as:



The $p-\upsilon$ diagram for the process can be shown as:



Since the temperature at state 1, $T_1 = 400^\circ C$, is greater than the saturation temperature corresponding to $P_1 = 1\ MPa$: 179.9 °*C*, state 1 is located in the superheated region.

The specific volume at state 1 is found from Table A-6 using $P_1 = 1\ MPa$ and $T_1 = 400^\circ C : v_1 = 0.3066 \frac{m^3}{kg}$. Also $u_1 = 2957.3\ \frac{kJ}{kg}$ and $h_1 = 3264.5\ \frac{kJ}{kg}$. The specific volume at state 2 is the saturated vapour value at 1 MPa (Table A-5): $v_2 = 0.1944 \frac{m^3}{kg}$. Also $u_2 = 2582.8\ \frac{kJ}{kg}$ and $h_2 = 2777.1\ \frac{kJ}{kg}$ and $x_2 = 1$.

Since T_3 is given and $v_2 = v_3 = 0.1944 \frac{m^3}{kg}$, two independent intensive properties are known that together fix state 3. To find u_3, first solve for the quality:

$$x_3 = \frac{\upsilon_3 - \upsilon_{f3}}{\upsilon_{g3} - \upsilon_{f3}} = \frac{0.1944 \left[m^3/kg \right] - 0.10905 \times 10^{-3} \left[m^3/kg \right]}{0.3928 \left[m^3/kg \right] - 0.10905 \times 10^{-3} \left[m^3/kg \right]} = 0.494 \qquad \text{(Eq1)}$$

Where v_{f3} and v_{g3} are from Table A-4 at 150 °*C*. Then

$$u_3 = u_{f3} + x_3(u_{g3} - u_{f3}) = 631.68 + 0.494 \times (2559.5 - 631.68) = 1583.9\ [kJ/kg] \qquad \text{(Eq2)}$$

Where u_{f3} and u_{g3} are from Table A-4 at 150 °*C*. And

$$h_3 = h_{f3} + x_3(h_{g3} - h_{f3}) = 632.18 + 0.494 \times (2745.9 - 632.18) = 1676.4\ [kJ/kg] \qquad \text{(Eq2)}$$

Where h_{f3} and h_{g3} are from Table A-4 at 150 °*C*.

Step 6: Concluding Statement

The requested properties are:

a) The specific volume at states 1, 2 and 3 are 0.3066 and $v_2 = v_3 = 0.1944 \frac{m^3}{kg}$, respectively.

b) The enthalpy at states 1, 2 and 3 are 3264.5, 27771.1 and 1676.4 $\frac{kJ}{kg}$, respectively.

c) The internal energy at states 1, 2 and 3 are 2957.3, 2582.8 and 1583.9 $\frac{kJ}{kg}$, respectively.

d) Also $x_2 = 1$ and $x_3 = 0.494$