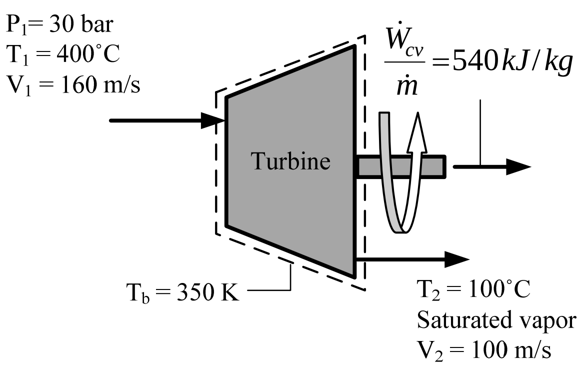
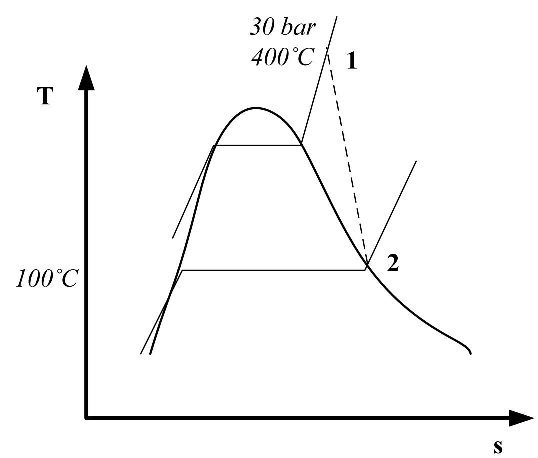
ENSC 388 Week # 7, Tutorial #6 – Entropy

Problem 1: Steam enters a turbine with a pressure of *30 bar*, a temperature of *400°C*, and a velocity of *160 m/s*. Saturated vapor at *100°C* exits with a velocity of *100 m/s*. At steady state, the turbine develops work equal to *540 kJ* per *kg* of steam flowing through the turbine. Heat transfer between the turbine and its surroundings occurs at an average outer surface temperature of *350 K*. Determine the rate at which entropy is produced within the turbine per kg of steam flowing, in *kJ/kg.K*. Neglect the change in potential energy between inlet and exit.

<u>Solution</u>

Step 1: Draw a diagram to represent the system showing control mass/volume of interest.





Step 2: Write out what you are required to solve for (this is so you don't forget to answer everything the question is asking for)

Find:

$s_{gen,cv}$ - the rate of entropy production within the turbine per kg of steam

Step 2: Prepare a data table

Data	Value	Unit
T_1	400	$[^\circ C]$
P_1	30	$[bar]$
V_1	160	$[m/s]$
T_2	100	$[^\circ C]$
V_2	100	$[m/s]$
$\dot{W}_{cv}/\dot{m}$	540	$[kJ/kg]$

Step 3: State your assumptions (you may have to add to your list of assumptions as you proceed in the problem)

Assumptions:

1) The control volume shown on the accompanying sketch is at steady state.
2) Heat transfer from the turbine to the surrounding occurs at a specified average outer surface temperature.
3) The change in potential energy between inlet and exit can be neglected.

Step 4: Calculations

To determine the entropy production per unit mass flowing through the turbine, begin with mass and entropy rate balances for the one-inlet, one-exit control volume at steady state:

$$0 = \dot{m}_1 - \dot{m}_2 \qquad \text{(Eq1)}$$

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}_1 s_1 - \dot{m}_2 s_2 + \dot{S}_{gen,cv} \qquad \text{(Eq2)}$$

Since heat transfer occurs only at T_b = *350 K*, the first term on the right side of the entropy rate balance reduces to $\dot{Q}_{cv}/T_b$. Combining the mass and entropy rate balances

$$0=\frac{\dot{Q}_{cv}}{T_b}+\dot{m}(s_1-s_2)+\dot{S}_{gen,cv} \tag{Eq3}$$

where $\dot{m}$ is the mass flow rate. Solving for $\dot{S}_{gen,cv}/\dot{m}$

$$s_{gen,cv}=\frac{\dot{S}_{gen,cv}}{\dot{m}}=-\frac{\dot{Q}_{cv}/\dot{m}}{T_b}+(s_2-s_1) \tag{Eq4}$$

The heat transfer rate, $\dot{Q}_{cv}/\dot{m}$, required by this expression is evaluated next. Reduction of the mass and energy rate balances results in

$$\frac{\dot{Q}_{cv}}{\dot{m}}=\frac{\dot{W}_{cv}}{\dot{m}}+(h_2-h_1)+\left(\frac{V_2^2-V_1^2}{2}\right) \tag{Eq5}$$

where the potential energy change from inlet to exit is dropped by assumption 3. From Table **A-6** at 30 bar, *400°C,* $h_1=3231.7\ kJ/kg$, and from Table **A-4**, $h_2=h_g(100^\circ C)=2675.6\ kJ/kg$. Thus

$$\frac{\dot{Q}_{cv}}{\dot{m}}=540\left[\frac{kJ}{kg}\right]+(2675.6-3231.7)\left[\frac{kJ}{kg}\right]+\left(\frac{(100)^2-(160)^2}{2}\right)\left[\frac{m}{s}\right]^2 \times\left[\frac{1\ N}{1\ kg.m/s^2}\right]\left[\frac{1\ kJ}{1000\ N.m}\right]=540-556.1-7.8=-23.9\ kJ/kg \tag{Eq6}$$

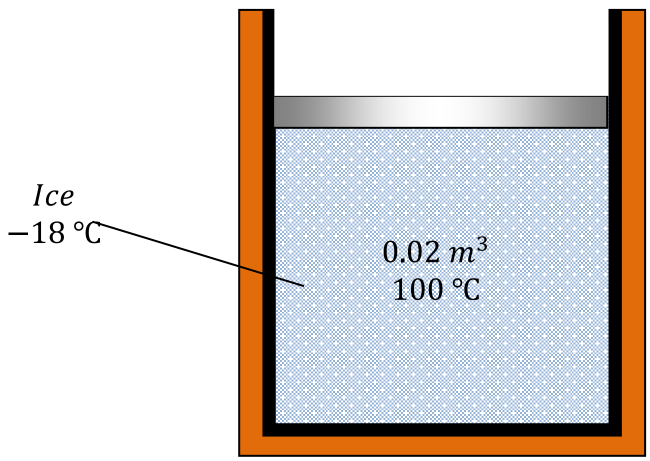
From Table **A-4**, $s_2=s_g(100^\circ C)=7.3542\ kJ/kg.K$, and from Table **A-6**, $s_1=6.9235\ kJ/kg.K$. Inserting values into the expression for entropy production, Eq4,

$$s_{gen,cv}=-\frac{(-23.9\ kJ/kg.K)}{350\ K}+(7.3542-6.9235)\left[\frac{kJ}{kg.K}\right] =0.0683+0.4307=0.499\ \text{kJ/kg.K} \tag{Eq7}$$

If the boundary were located to include a portion of the immediate surroundings so heat transfer would take place at the temperature of the surroundings, T_f = *293 K*, the entropy production for the enlarged control volume would be *0.511 kJ/kg.K*.

Problem 2

An insulated piston-cylinder device initially contains $0.02\ m^3$ of saturated liquid-vapour mixture of water with a quality of 0.1 at $100\ ℃$. Now some ice at $-18\ ℃$ is dropped into the cylinder. If the cylinder contains saturated liquid at $100\ ℃$ when thermal equilibrium is established, determine (a) the amount of ice added and (b) the entropy generation during this process. The melting temperature and the heat of fusion of ice at atmospheric pressure are $0℃$ and $333.7\ kJ/kg$.



<u>Solution</u>

Known:

$$Initial\ and\ final\ states\ of\ the\ process$$
$$Ice\ temperature$$

Find:

- The amount of ice added to the water.
- The entropy generation.

<u>Assumptions:</u>

- Thermal properties of the ice are constant.
- The cylinder is well-insulated and thus heat transfer is negligible.
- There is no stirring by hand or a mechanical device (it will add energy).

Analysis:

(a) We take the contents of the cylinder (ice and saturated water) as our system, which is a closed system. Noting that the temperature and thus the pressure remains constant during this phase change process and thus $W_b + \Delta U = \Delta H$, the energy balance for this system can be written as:

$$\underbrace{E_{in} - E_{out}}_{\substack{Net\ energy\ transfer \\ by\ heat,\ work\ and\ mass}} = \underbrace{\Delta E_{system}}_{\substack{Change\ in\ internal\ energy \\ potential,\ etc.\ energies}}$$

$$\rightarrow W_b = -\Delta U$$

$$\rightarrow \Delta H = 0$$

$$\Delta H_{ice} + \Delta H_{water} = 0$$

or

$$\left[mc_p(0\ ℃ - T_1)_{solid} + mh_{if} + mc_p(T_2 - 0\ ℃)_{liquid}\right]_{ice} + [m(h_2 - h_1)]_{water} = 0$$

The specific heat of ice at about 0℃ is $c_p = 2.11\ kJ/kg \cdot °C$ (Table A-3). The melting temperature and the heat of fusion of ice at 1 atm are 0℃ and 333.7 kJ/kg. The properties of water at 100℃ are (Table A-4):

$$v_f = 0.001043\ \left[\frac{m^3}{kg}\right], \quad v_g = 1.6720\ \left[\frac{m^3}{kg}\right]$$

$$h_f = 419.17\ \left[\frac{kJ}{kg}\right], \quad h_{fg} = 2256.4\ \left[\frac{kJ}{kg}\right]$$

$$s_f = 1.3072\ \left[\frac{kJ}{kg.K}\right], \quad s_{fg} = 6.0490\ \left[\frac{kJ}{kg.K}\right]$$

$$v_1 = v_f + x_1 v_{fg}$$

$$= 0.001043\ \left[\frac{m^3}{kg}\right] + 0.1\left(1.6720\ \left[\frac{m^3}{kg}\right] - 0.001043\ \left[\frac{m^3}{kg}\right]\right)$$

$$= 0.16814\left[\frac{m^3}{kg}\right]$$

$$h_1 = h_f + x_1 h_{fg} = 419.17\ \left[\frac{kJ}{kg}\right] + 0.1 \times 2256.4\ \left[\frac{kJ}{kg}\right] = 644.81\left[\frac{kJ}{kg}\right]$$

$$s_1 = s_f + x_1 s_{fg} = 1.3072\ \left[\frac{kJ}{kg.K}\right] + 0.1 \times 6.0470\ \left[\frac{kJ}{kg.K}\right]$$

$$= 1.9119\left[\frac{kJ}{kg.K}\right]$$

$$h_2 = h_{f@100\,℃} = 419.17 \left[\frac{kJ}{kg}\right]$$

$$s_2 = s_{f@100\,℃} = 1.3072 \left[\frac{kJ}{kg.K}\right]$$

$$m_{steam} = \frac{V_1}{v_1} = \frac{0.2\ [m^3]}{0.16814 \left[\frac{m^3}{kg}\right]} = 0.119\ [kg]$$

Noting that $T_{1,ice} = -18\ ℃$ and $T_2 = 100\ ℃$ and substituting gives:

$$m\left\{\left(2.11\left[\frac{kJ}{kg.℃}\right]\left(0\ ℃ + \overbrace{18℃}^{-(-18℃)}\right) + 333.7 \left[\frac{kJ}{kg}\right] + 4.18\left[\frac{kJ}{kg.℃}\right](100℃ - 0℃)\right)\right\}$$

$$+ 0.119[kg]\left(419.17\left[\frac{kJ}{kg}\right] - 644.81\left[\frac{kJ}{kg}\right]\right) = 0$$

$$\rightarrow \text{m} = 0.034\ [\text{kg}] = 34\ [\text{g}]\ \text{ice}$$

(b) We take the ice and the steam as our system, which is a closed system. Considering that the tank is well-insulated and thus there is no heat transfer, the entropy balance for this closed system can be expressed as:

$$\underbrace{S_{in} - S_{out}}_{\substack{Net\ entropy\ transfer \\ by\ heat\ and\ mass}} + \underbrace{S_{gen}}_{\substack{Entropy \\ generation}} = \underbrace{\Delta S_{system}}_{\substack{Change\ in \\ entropy}}$$

$$0 + S_{gen} = \Delta S_{ice} + \Delta S_{steam}$$

$$\Delta S_{steam} = m(s_2 - s_1) = 0.119[kg]\left(1.3072\left[\frac{kJ}{kg.K}\right] - 1.9119\left[\frac{kJ}{kg.K}\right]\right)$$

$$= -0.0719 \left[\frac{kJ}{K}\right]$$

$$\Delta S_{ice} = \left(\Delta S_{solid} + \Delta S_{melting} + \Delta S_{liquid}\right)_{ice}$$

$$\Delta S_{solid} = \left(mc_p ln\frac{T_{melting}}{T_1}\right)_{solid}$$

$$= 0.034[kg] \times 2.11\left[\frac{kJ}{kg.K}\right] \times ln\frac{273.15[K]}{255.15[K]} = 0.0049\left[\frac{kJ}{K}\right]$$

$$\Delta S_{melting} = \frac{mh_{if}}{T_{melting}} = \frac{0.034[kg] \times 333.7 \left[\frac{kJ}{kg}\right]}{273.15[K]} = 0.0415\left[\frac{kJ}{K}\right]$$

$$\Delta S_{liquid} = \left(mc_p ln\frac{T_2}{T_1}\right)_{liquid} = 0.034[kg] \times 4.18\left[\frac{kJ}{kg.K}\right] \times ln\frac{373.15[K]}{273.15[K]}$$
$$= 0.0443\left[\frac{kJ}{K}\right]$$

$$\rightarrow \Delta S_{ice} = 0.0049\left[\frac{kJ}{K}\right] + 0.0415\left[\frac{kJ}{K}\right] + 0.0443\left[\frac{kJ}{K}\right] = 0.0907\left[\frac{kJ}{K}\right]$$

Then,

$$\rightarrow S_{gen} = 0.0907\left[\frac{kJ}{K}\right] - 0.0719\ \left[\frac{kJ}{K}\right] = 0.0188\left[\frac{kJ}{K}\right]$$