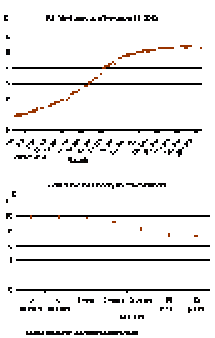
Term Structure Estimation With Missing Data

Krisztina Nagy
Department of Economics, University of Washington, Box. 353330, Seattle, WA 98195, e-mail: knagy@u.washington.edu

Research question: How can we estimate the term structure of interest rates (the yield curve) in the presence of missing (sparse) bond price data?

Yield Curves



Note that the time intervals between maturities plotted are not equal.

1. The Term Structure of Interest Rates

- relationship between term to maturity and the corresponding discount/forward rate;
- contains info about market expectations of future nominal interest rates and inflation;
- used for:
  - monetary policy
  - pricing fixed income securities
  - risk management and hedging
  - predicting future economic activity;

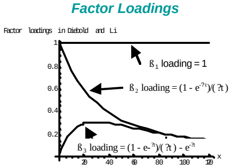
2. Challenges

- ***the term structure is not observed***
  - ➔ need to estimate three unobserved components: level, slope and curvature
- ***sparse bond price data in developing bond markets***
  - ➔ need to identify/develop an estimation technique to deal with missing data
  - ➔ need to use the non-linear price curve instead of the linear yield curve

3. Literature/ Modeling Framework

- *Literature:*
  - Many papers estimating the terms structure
  - Few papers using sparse bond price data
- *Framework:*
  - Nelson-Siegel yield curve

Factor Loadings



Nelson-Siegel Yield curve

$$y_t(t) = b_{1t} + b_{2t}\left(\frac{1-e^{-lt}}{lt}\right) + b_{3t}\left(\frac{1-e^{-lt}}{lt} - e^{-lt}\right)$$

τ – maturity (months) ;
$y_t(\tau)$ – yield at maturity τ on zero-coupon bonds (%);
β_{1t} – long-term component (level);
β_{2t} – short-term component (slope);
β_{3t} – medium-term component (curvature);
λ – time constant associated with the equation; determines the rate of exponential decay;

5. Methodology

A. Estimate the Nelson-Siegel Yield Curve Using State Space Framework

General form of the measurement equation:

$$\begin{pmatrix} y_t(t_1) \\ y_t(t_2) \\ \vdots \\ y_t(t_N) \end{pmatrix} = \begin{pmatrix} 1 & L_{2t}(t_1) & L_{3t}(t_1) \\ \vdots & \ddots & \vdots \\ 1 & L_{2t}(t_N) & L_{3t}(t_N) \end{pmatrix} \begin{pmatrix} b_{1t} \\ b_{2t} \\ b_{3t} \end{pmatrix} + \begin{pmatrix} e_t(t_1) \\ e_t(t_2) \\ \vdots \\ e_t(t_N) \end{pmatrix}$$

General form of the transition equation:

$$\begin{pmatrix} b_{1t} - m_1 \\ b_{2t} - m_2 \\ b_{3t} - m_3 \end{pmatrix} = \begin{pmatrix} f_{11} & 0 & 0 \\ 0 & f_{22} & 0 \\ 0 & 0 & f_{33} \end{pmatrix} \begin{pmatrix} b_{1t-1} - m_1 \\ b_{2t-1} - m_2 \\ b_{3t-1} - m_3 \end{pmatrix} + \begin{pmatrix} v_{1t} \\ v_{2t} \\ v_{3t} \end{pmatrix}$$

Assumptions:

$e_t \sim iid N(0, I_N)$ $v_t \sim iid N(0, I_3)$

$E[e_t v'_t] = 0$

The initial state vector is uncorrelated with any realization of et and vt

B. Estimate the Non-linear Bond Price Equation

$$P_{t^*} = C^* \sum_{m=1}^{t^*} e^{-\left[b_1 + b_2\left(\frac{1-e^{-l^* m}}{l^* m}\right) + b_3\left(\frac{1-e^{-l^* m}}{l^* m} - e^{-l^* m}\right)\right]\frac{m}{100}} + FV^* e^{-\left[b_1 + b_2\left(\frac{1-e^{-l^* t^*}}{l^* t^*}\right) + b_3\left(\frac{1-e^{-l^* t^*}}{l^* t^*} - e^{-l^* t^*}\right)\right]\frac{t^*}{100}}$$

τ^* – maturity (years) ;
$y_t(\tau^*)$ – yield at maturity τ on zero-coupon bonds (%);
?* -- ? * 12;

C. Estimate the Non-linear Bond Price Equation Using State Space Framework (work in progress)

Bond price equation in state space form

Measurement Equation:

$$\begin{pmatrix} B_t(t_1) \\ B_t(t_2) \\ \vdots \\ B_t(t_N) \end{pmatrix} = \begin{pmatrix} C^* \sum_{m=1}^{t_1} e^{-[\bar{b}_1 + \bar{b}_2 L_2 + \bar{b}_3 L_3] m} + FV^* e^{-[\bar{b}_1 + \bar{b}_2 L_2 + \bar{b}_3 L_3] t_1} \\ \vdots \\ C^* \sum_{m=1}^{t_N} e^{-[\bar{b}_1 + \bar{b}_2 L_2 + \bar{b}_3 L_3] m} + FV^* e^{-[\bar{b}_1 + \bar{b}_2 L_2 + \bar{b}_3 L_3] t_N} \end{pmatrix} + \begin{pmatrix} GT & GT^* L_2(t_1) & GT^* L_3(t_1) \\ \vdots & \ddots & \vdots \\ GT & GT^* L_2(t_N) & GT^* L_3(t_N) \end{pmatrix} \begin{pmatrix} b_{1t} \\ b_{2t} \\ b_{3t} \end{pmatrix} + \begin{pmatrix} e_t(t_1) \\ e_t(t_2) \\ \vdots \\ e_t(t_N) \end{pmatrix}$$

Transition Equation:

$$\begin{pmatrix} b_{1t} \\ b_{2t} \\ b_{3t} \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} + \begin{pmatrix} f_{11} & 0 & 0 \\ 0 & f_{22} & 0 \\ 0 & 0 & f_{33} \end{pmatrix} \begin{pmatrix} b_{1t-1} \\ b_{2t-1} \\ b_{3t-1} \end{pmatrix} + \begin{pmatrix} v_{1t} \\ v_{2t} \\ v_{3t} \end{pmatrix}$$

Where $\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = \begin{pmatrix} \bar{b}_1(1-f_{11}) \\ \bar{b}_2(1-f_{22}) \\ \bar{b}_3(1-f_{33}) \end{pmatrix}$

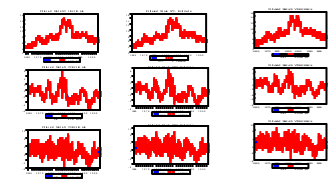
Where,

$$GT = \left[C^* \sum_{m=1}^{t_i} (-m)^* e^{-[\bar{b}_1 + \bar{b}_2 L_2 + \bar{b}_3 L_3] m} + FV^*(-t)^* e^{-[\bar{b}_1 + \bar{b}_2 L_2 + \bar{b}_3 L_3] t_i}\right]$$

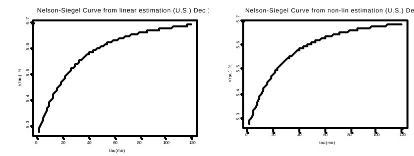
6. Results

State Space Estimation Results – with artificial missing data

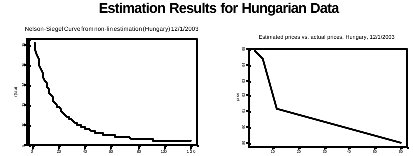
3 and 6 month rate | 3, 6, and 12 month rate | 3, 6, 1 2, and 60 month rate



Estimation Results for US data



Estimation Results for Hungarian Data



7. Conclusions and next steps

- State Space Framework handles missing data well
- Yield curve and price curve estimation produce comparable results
- Finalize the Estimation of the Non-linear Bond Price Equation Using State Space Framework

Thank You for visiting!