**Thermoregulation in Honeybee Swarms**

Jeremy Chiu, MSc Student Applied Mathematics  |  Supervisors: JF Williams, John Stockie

---

**Problem Motivation**

Hiveless honeybee swarms are capable of surviving in temperatures as low as 0°C. We derive and simulate a model describing how honeybees thermoregulate, based on the model presented by Watmough and Camazine (1).

**Mathematical Formulation**

Introduce two independent variables: time $t$ and space $r$. Also introduce temperature $T(t, r)$, bee density $\rho(t, r)$, and radius of spherical swarm $R(t)$.

**For $T(t, r)$, use the heat equation:**

$$\frac{\partial T}{\partial t} = \nabla \cdot (\lambda(\rho) \nabla T) + \rho f(T),$$

where $\lambda$ is the heat capacity of the swarm, $\lambda$ the (non-constant) thermal conductivity, and $f$ a source term that describes the metabolic heat output per bee.

**For $\rho(t, r)$, use a thermotaxis equation:**

$$\frac{\partial \rho}{\partial t} = \nabla \cdot (\mu(\rho) \nabla \rho) - \nabla \cdot (\chi(T) \nabla T),$$

where $\mu$ is the bee motility (a barrier function) and $\chi$ a thermotactic velocity.

**Modelling in 2D**

Extend (1)’s model to non-spherical swarms. Requires new numerics, and raises new modelling questions:

- what is exterior shape $R(\phi)$?
- what are the roof boundary conditions?

Contour plots from experimental data in (2).

---

**Assumptions**

1. The swarm contains a large enough population of bees that it can be approximated as a continuum.
2. Bees are self-organized and have no centralized controller.
3. A bee will move in the direction of decreasing temperature when too warm. Similarly, a bee will move in the direction of increasing temperature when too cold.
4. Bees generate heat both passively and actively, especially when cold.
5. The swarm is spherically symmetric.
6. Heat transfer is through conduction.
7. Bees are conserved and hence no bees leave or enter the swarm.

---

**Numerics**

**Problem**: Numerics in (1) did not conserve bees!  
**Solution**: Use integral equation for $R$ rather than differential equation. We created three numerical schemes:

- WC, a replication of scheme in (1);
- FC, the same but with a new equation for $R$;
- MATLAB’s fully implicit solver ode15s.

The latter two methods use the integral equation to compute $R(t)$.

---

**Physical Results**

- For simplicity, set boundary $R(\phi)$ as a semicircle.
- Impose no-flux boundary conditions at roof; hence 2D problem is spherically symmetric and thus comparable to 1D results.

---

**References**

