Non–Atomic Distributivity and Typicality
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1. Introduction

Non–atomic distributivity involves distribution over a sub–plurality of a plural expression. Consider the following sentence, from Schwarzschild (1996).

1) The vegetables are too heavy for the gray scale and too light for the black scale.

Out of the blue, (1) may have a number of interpretations, most notably a “collective” and a “distributive” interpretation. Under the collective interpretation, it means that all the vegetables together are too heavy for the gray scale and too light for the black scale. Under the distributive reading, the sentence conveys that each one of the vegetables is both too heavy for the gray scale and too light for the black scale. Now suppose that (1) is uttered in the following context (also from Schwarzschild 1996):

2) Two merchants want to price some vegetables. The vegetables are sitting before the merchants, piled up in several baskets. To determine their price, the vegetables need to be weighed. Unfortunately, the merchants do not have an appropriate scale. Their gray retail scale is too fine, meant to weigh only a few vegetables at a time, and their black wholesale is coarse, meant to weigh huge truckloads.

Sentence (1), judged to be true on the basis of these facts, cannot be either collective (true of all the vegetables together) or fully distributive (true of each one of the vegetables). (1) can only be true if one considers baskets of vegetables, otherwise the sentence would not make sense in the scenario described by (2). Theories of plurality and distributivity have referred to these readings as “non-atomic”, “partially distributive” or “intermediate”. The sentences in (3) show a similar three–way ambiguity.

3) a. Rodgers, Hammerstein and Hart wrote musicals.  
   [Gillon 1987]
   b. The shoes cost $50.  
   [Lasersohn 1995]

The three composers in example (3a) never wrote a musical together, nor did any of them ever write one all by himself: Rodgers and Hammerstein wrote the musical Oklahoma together, and Rodgers and Hart wrote the musical On your toes together. Similarly, (3b) can be true in the eventuality where some amount of shoes together cost $50, each pair of shoes costs $50, or each individual shoe costs $50. These are all non–atomic distributive interpretations of the subject.

This paper discusses a particular case of non–atomic distributivity that has not received much attention. The cases of intermediate readings that are most often discussed in the literature on plurals involve plural definite DPs. The novel observation is that only some DPs can be quantified over without losing the relevant non–atomic reading. As an illustration, consider the following examples, which form minimal pairs with (1) and (3b):

4) a. All the vegetables are too heavy for the gray scale and too light for the black scale.
   b. All the shoes cost $50.

It is difficult, if not impossible, to understand (4a) as being true in the situation described above in (2); the predicate can now only be true either of the collective body of vegetables or of each of the individual vegetables separately. The example in (4b), on the other hand, does not show such restriction: it can be truthfully uttered in situations where what is at stake is the cost of pairs of shoes, as opposed to individual shoes or bigger groupings

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of shoes. Notice that this contrast is not a particularity of the quantifier *all*: the non–atomic distributive reading of (4b) survives also with a variety of quantifiers.

\[
\begin{align*}
a & \text{. Most} \\
b & \text{. Many} \\
c & \text{. Some} \\
d & \text{. No}
\end{align*}
\]

\[\text{shoes cost } \$50.\]

In the case of (4a), changing the quantifier does not facilitate the missing intermediate interpretation. Thus, none of the examples in (6) are good follow–ups to the discourse in (2).

\[
\begin{align*}
a & \text{. Most} \\
b & \text{. Many} \\
c & \text{. Some} \\
d & \text{. No}
\end{align*}
\]

\[\text{vegetables are too heavy for the gray scale and too light for the black scale.}\]

This behavior is also not a particularity of *pairs* of objects, as the following examples show:

\[
\begin{align*}
a & \text{. Things are really expensive in this supermarket, all the eggs cost } \$3. \\
& \Rightarrow \text{All the dozens of eggs cost } \$3
\end{align*}
\]

\[
\begin{align*}
b & \text{. All the Matryoshka dolls fit well together.} \\
& \Rightarrow \text{All the dolls in a single set of dolls fit well together}
\end{align*}
\]

It is obvious that, when uttering (7a), the speaker does not mean that all the eggs in the supermarket cost $3, that would be a bargain. But she may not mean either that each individual egg costs that much. In that case, what she intends to say is that $3 is the price of some relevant amount of eggs, usually a dozen or maybe half a dozen. Similarly, example (7b) refers to the fact that all the dolls that belong to the same set of dolls fit well together in the relevant sense.

The focus of the paper is on the contrast between (4a) and (4b). The task is twofold: (i) to understand why some quantified NPs but not others allow non–atomic distributive interpretations, and (ii) to provide a principled account that determines the circumstances under which a quantifier has access to the relevant sub–groupings of a plural expression. This paper argues that, given what we know about the world, the nature of this contrast is due to the fact that all the nouns but not others, e.g., *shoes* but not *vegetables*, can form “typical groups”, that is, pluralities whose arrangement is tied to the way that the extensions of these nouns are presented to us in the world.

### 2. Non–atomic distribution

#### 2.1. The basics of distributivity

In order to model distributivity we need to quantify over parts of a plural individual; this is no different in the case of non–atomic distributivity. The additional complication of these cases amounts to determine what constitutes a relevant sub-plurality.

For concreteness, I will adopt the framework of plurality first introduced in Link (1983), whereby the domain of individuals \(E\) is closed under a sum formation operation \(\oplus\) and has the structure of an \(i\)–(individual) join semilattice.\(^1\) As a consequence, \(i\)-sums are partially ordered through a relation \(\leq\) on \(E\), which constitutes the individual part–of relation, satisfying the following equivalence:

\[
\begin{align*}
a \leq b & \Leftrightarrow a \oplus b = b
\end{align*}
\]

The pluralization operation \(\ast\) generates all the individual sums of members in the extension of any 1–place predicate \(P\). The denotation of \(\ast P\) can now be defined as a complete join–semilattice that is generated in \(E\) by operating over atoms. Atoms have to be understood as entities with no proper parts, as defined by the predicate constant \(\mathit{Ar}\) in (9).

\(^1\)The present system is adopted for convenience only, and so any claim made in this paper should easily carry over to any other system to model plurality (see for example Landman 2000, a.o.).
9) \[ At(x) = \forall y [x \leq y \rightarrow y = x] \]

In this framework, a popular method to account for the difference between collective and distributive readings is by means of a distributivity operator "D" (Link 1983 a.o.).

10) a. \[ D \overset{\text{def}}{=} \lambda P \lambda x \forall y [y \in x \rightarrow P(y)] \]
b. \[ DP \iff \forall x [P(x) \rightarrow At(x)] \]

Given this definition of the distributivity operator, it follows that for any plural individual \( X \) that satisfies \( DP \), \( P \) is true for every \( x \) such that \( x \leq X \) and \( At(x) \). For example, a sentence like Jonn and Mary carried a piano has two possible interpretations, depending on whether the operator \( D \) is applied to the predicate.

11) a. \( D^{\ast} (\text{carried--a--piano})(j \oplus m) \Rightarrow (\text{carried--a--piano})(j) \wedge (\text{carried--a--piano})(m) \)
b. \( D^{\ast} (\text{carried--a--piano})(j \oplus m) \Rightarrow (\text{carried--a--piano})(j \oplus m) \)

12) a. \( (\ast \text{carried--a--piano})(j \oplus m) \Rightarrow (\ast \text{carried--a--piano})(j \oplus m) \)
b. \( (\ast \text{carried--a--piano})(j \oplus m) \Rightarrow (\ast \text{carried--a--piano})(j) \wedge (\ast \text{carried--a--piano})(m) \)

When \( D \) applies to the predicate carry-a-piano, the sentence is only true of every atom in the extension of the plural individual \( j \oplus m \), as shown by the entailment pattern in (11), whereas this is not the case under the collective reading in (12), where the entailment pattern is reversed.

2.2. Context dependency

As it is, the system only has access to two kinds of individuals: atoms and plural individuals. However, we still need to account for cases where distribution occurs over entities other than those two. That is, as it is, the system has no means to make the \( D \) operator range over baskets of vegetables in (1). One popular solution, first advanced by Gillon (1987), is to introduce covers. Covers are sets of sets of individuals that restrict the domain of \( D \) by functioning as context-dependent domain selection variables.

13) \[ Cov(X,Y) \overset{\text{def}}{=} \left\{ \begin{array}{l} \emptyset \notin X, \\
\forall x \in X \exists y \in Y \exists y \in X \end{array} \right. \]

In words, a set \( X \) is a cover of a set \( Y \) iff \( X \) is nonempty and every member of \( Y \) belongs to some set in \( X \). The innovation of cover-based approaches is that they can relax the requirement to quantify always over atomic individuals and allow quantification over sub-pluralities in the extension of a NP.

14) \[ Cov \overset{\text{def}}{=} \lambda P \lambda x \forall y [y \in Cov \land y \leq x \rightarrow P(y)] \]

[Schwarzschild 1996]

The adoption of the variable \( Cov \) adds an extra condition for the relevant relation \( P(y) \) to hold: for every \( y \), \( y \) must be a proper subset of \( x \), and a member of the contextually determined cover \( Cov \). With this amendment to the \( D \) operator it is possible to account for the non-atomic distributive reading of (1).

15) \[ \forall y [\text{y} \in Cov \land y \leq \text{[the--vegetables]} \rightarrow \text{[heavy--to--carry]}(y)], \text{ where } \text{[the-vegetables]} = \{a,b,c,d,e,f\} \]

Depending on the contextually-relevant cover that we pick, we can get one of many possible interpretations. For instance, consider the following covers:

16) a. \( Cov^1 = \{a,b,c,d,e,f\} \)
b. \( Cov^2 = \{\{a\},\{b\},\{c\},\{d\},\{e\},\{f\}\} \)
c. \( Cov^3 = \{\{a,b\},\{c,d\},\{e,f\}\} \)

\( Cov^1 \) and \( Cov^2 \) provide the familiar collective and distributive interpretations, respectively. \( Cov^3 \), on the other hand, allows the \( D \) operator to quantify over subsets of the cover, which would correspond to baskets of
vegetables in the previous example. This is possible only because, by assumption, \( D \) requires the existence of a context through which the variable \( \text{Cov}^a \) can be resolved. Thus, any context that supports some partition of a plural noun into sub–pluralities suffices to allow the choice of a cover that corresponds to that partition.

This last point is important because, as pointed out by Lasersohn (1995), the selection of covers should not be completely free. For example, imagine that John, Mary, and Bill are the teaching assistants and each one of them was paid exactly $7,000 last year. If so, sentences (17a) and (17b) are both true, as expected under the classical definition of the \( D \) operator (i.e., by deciding whether to apply \( D \) to the predicate, as in (17a), or not, as in (17b)). Sentence (17c), however, is false, even though nothing precludes the choice of a cover that could verify the sentence, namely \{\{john \oplus mary\}, \{mary \oplus bill\}\}.

17) a. The TAs were paid exactly $7,000 last year. ✔ DISTRIBUTIVE
   b. The TAs were paid exactly $21,000 last year. ✔ COLLECTIVE
   c. The TAs were paid exactly $14,000 last year. ♠ INTERMEDIATE

The choice of these potential but illicit covers can be ruled out by the lack of a supporting context. Thus, the contrast between (1) and (17c) is predicted under the assumption that a salient context is available only in the case of (1), but not in (17c).

3. Quantifying over non–atomic individuals

Regardless the usefulness of covers for the interpretation of plural DPs, I will argue that they cannot help us in capturing the contrast in (4). In fact, one plausible way of thinking is to allow covers restrict the domain of quantifiers too (cf. Brisson 2003), but this is problematic. Were we to adopt this strategy, we still would have to restrict the kind of covers we can quantify over. For instance, we would have to stipulate that some NPs require specific covers, e.g., covers containing subsets of two elements in the case of shoes. Of course, doing so factors out the context–dependency of cover–based systems, which is precisely what makes them appealing. Moreover, this move would not help us understand what the difference between (4a) and (4b) amounts too, and we would have to further stipulate that covers of shoes can be quantified over, but not covers of vegetables.

This paper suggests an alternative route, and so it will not rely on covers. I will instead introduce and make use of the notion of “typical group”. For the moment being, assume that typical groups are simply that, pluralities of individuals that, given what we know about the world, are commonly arranged together. For example, what is typical of shoes is to be arranged in pairs, and so a pair of shoes constitutes a typical group. The general idea behind typical groups is a rather intuitive one: to know what shoe means is to know that they usually come in pairs, given human physiognomy and what shoes are usually used for. In contrast, knowing what a vegetable is does not come with any such inherent knowledge.

With respect to non–atomic distribution, the main difference between both types of NPs is that sub–groupings of vegetables can only be identified with the help of a suitable context, whereas sub–groupings of shoes in pairs are readily available without such supporting contexts. The relevant distinction can be described by appealing to two different kinds of salience: pairs of shoes are conceptually salient, whereas baskets of vegetables can only be contextually salient in (1). Thus, typical groups must be understood as pluralities whose cardinality can be established by world–knowledge alone, and not by the context. Other examples of these nouns include gloves (arranged in pairs), eggs (arranged in (half-)dozens), Matryoshka dolls (arranged in sets of variable cardinality), etc. In this sense, the composers, a pile of plates or the members of a cycling team do not count as typical groups.

The remainder of this section is devoted to show (i) that typical groups cannot be reduced to mere context–dependency and (ii) that only nouns that denote typical groups can be quantified over without losing a non–atomic distributive interpretation. The upshot of this discussion is that typical groups are not only an intuitive way to characterize a particular class of nouns, but they constitute a real linguistic object that the grammar has access to.

3.1. Context independence

I will first argue that the kind of non-atomic distributive interpretations displayed by (4b), (5) and (7) cannot be reduced to contextual saliency. The critical example has already been provided in (4a) above (see also
(6)): after quantifying over the vegetables the availability of the non-atomic distributive reading disappears. Somehow, the quantifier blocks the relevant interpretation even in the presence of a suitable supporting context. This is important: it points out that the availability of intermediate readings with quantified subjects cannot be licensed by contextual information. The conditions regulating the availability of non–atomic distribution in (1) are inaccessible for (4a), signaling that an appropriate context is insufficient to provide a sub–plurality that a quantifier can quantify over.

The corollary is that contextual information does not mediate in the intermediate interpretation of (4b) either. This speaks against a cover–based approach to non–atomic distribution for (4b), and so also for cases like (5) and (7). Covers are only capable of indicating the right partition of a plurality when this partition is provided by the context. Since the context cannot license the intermediate reading of (4a), it is reasonable to think that it fails to do so also in the case of (4b). The following examples make a similar point.

18) There is a new shop in town and Bill got in to buy a new shirt. He realized that all the shirts on the shelves are organized and displayed in piles of three shirts. He then overheard one of the shop assistants speaking to a customer:

19) a. It’s an opening sale, the shirts cost $30.
   b. It’s an opening sale, all the shirts cost $30.

In the context of (18), an interpretation where triplets of shirts cost $30 is only available for (19a), thus supporting the idea that contextual information is orthogonal to the availability of these special cases of distribution over sub–pluralities. Notice that the opposite seems to be also true: if a plurality can denote a typical group, it can do so also out of the blue, with no supporting context whatsoever.

3.2. More on group predication

Typical groups, then, do not show the same kind of context dependency effects that other sub–pluralities do. Their properties are also different from other notions of group predication proposed in the literature, such as instances of team credit attribution, as discussed in Lasersohn (1995) and Landman (1996) or substantive groupings, like the ones considered by Kratzer (2003).

It is easy to see why typical groups cannot be subsumed under the notion of team credit. What is special about team credit is that a group is attributed an action that is only performed by some –or just one– of its members. The most common case is that in which the relevant group acts as a team; hence the name. For example, (20) below does not tell us anything about the individuals that constitute the Islanders; all we know is that one of them scored a goal, although the merit is attributed to the whole team.

20) The Islanders scored a goal. \[Lasersohn 1995\]
   \[\Rightarrow\] All the Islanders scored a goal.

The distributive inference in (20) does not go through. In fact, it is a characteristic of team credit interpretations that no distributive inference can hold. Thus, the predicate score a goal can only be true of a single member of the denotation of the DP, but not all of them. This is not so in the case of non–atomic distribution: example (3b) above is true of all the relevant sub–pluralities, and so distributive inferences hold, albeit not all the way down to atomic entities. The opposite situation is also true: an individual that holds the responsibility for a group action can be ascribed team credit.

21) Al Capone killed several people in Chicago.

We can truthfully assert (21) if Al Capone was indeed responsible for the killing or if we have reasons to think that he ordered it, even if he did not physically participate. In this case too, typical groups are different from team credit in that they always require pluralities.

This is not a logical consequence, since it could be that the intermediate reading of (4a) is blocked by independent means. Whereas I cannot see why this should be the case, I can see reasons to think of shoes as being the special case, and so I will only explore the latter possibility here.
As Lasersohn (1995) points out, other cases of team credit need a richer pragmatic context. Kratzer (2003) observes, however, that pragmatic relevance alone will not be enough to characterize what counts as a group in terms of team credit. In fact, team credit predication only arises when the relevant pluralities “correspond to substantive groupings of pluralities in the actual world” (Kratzer 2003:31). In her account, team credit may be an effect of a collaborative action, but not of a mere sum of actions or individuals.

To which extent collaborative effort is needed is a matter of discussion. The sentence in (22) below makes a similar point: the journalists need not have acted collectively when asking the questions; indeed, they may very well belong to competing media. Even more, maybe some of them had no intention at all of asking any questions. What the sentence means is that the predicate asked five questions is true of a collective body of journalists, whose only common property is, precisely, to be journalists.

22) The journalists asked five questions after the press conference.

In any case, regardless of the correctness of the characterization of pluralities that can be ascribed team credit as substantive groupings, typical groups are still essentially different. In fact, typical groups may constitute substantive groupings themselves, but this notion will not be enough to provide a principled account of their behavior. This is, partly, because spatio–temporal proximity constitutes an important ingredient of what counts as a substantive grouping. This is discussed in an example by Schein (1993).

23) a. All the bunches of leaves on the lawn are allergenic.
   b. All the leaves on the lawn are allergenic.

Consider a situation with a tree whose leaves are of a particular kind: they are allergenic only when they come in bunches larger than three. If we consider a lawn where several bunches of these leaves have been stacked, sentence (23a) is true even though there might be stacks of one or two leaves that have been scattered from the original bigger bunches (by the wind, for example). In this case, the leaves that are in big bunches count as substantive groupings. But, once more, these cannot count as typical groups, for we cannot directly quantify over the leaves themselves, and so (23b) is not true in the present scenario.

4. Characterizing typical groups

A typical group cannot simply be any contextually relevant plurality nor any substantive grouping of sub–pluralities. So, what is it? As I suggested earlier, typical groups are constituted by a conceptually salient or relevant plurality in the extension of a plural NP. Given the semantic framework assumed in §2.1., a typical group will be some individual in the lattice structure that is neither an atom nor the supremum.

The claim put forth in this paper is that typical groups behave to some respects like atoms, and they do so because conceptually they are at least as relevant as atoms are. Informally, the idea is that knowing that shoes come in pairs is an essential part of knowing what a shoe is. There are practical reasons why most of the times that we encounter shoes in the world they are arranged in pairs: shoes are functional objects that would not be as useful if you only have one. The claim here is not that a single shoe is less of a shoe than a pair of shoes. What is argued is that this knowledge is at the basis of the mechanism that allows the grammar to treat groups of two shoes on a par with an atomic shoe. As a consequence, quantification over typical groups happens more easily than over groups whose cohesion is more circumstantial.

4.1. Concepts and the source of typicality

The analysis is spelled–out in terms of prototype theory, although it is by no means a defense of prototype–theory over other models of graded categorization.3 Thus, I consider the proposal put forth in this paper as a first step of an account that, ultimately, will have important ramifications for both semantic theories of gradability and theories of conceptualization.

3There are well know problems with prototype theory (see Fodor 1998) that will be ignored here. In that sense, it seems intuitive and defensible to replace the notion of typicality by some other means to allow world knowledge to break into the semantics of plural DPs without putting all the burden on contextual saliency. Exemplar theory, for instance, is a good candidate. See also the semantic account in Sassoon 2013 for extensive discussion on typicality and vagueness, in general.
In the psychological literature the nature and structure of ordinary concepts like \texttt{fruit} and \texttt{bird} has been extensively studied.\(^4\) The focus has been on the way the human mind represents objects as instances of (idealized) concepts. The so-called classical theory advocates a procedure of conceptualization consisting of spelling out the necessary and sufficient conditions for categorizing any given object as an instance of a particular concept (Margolis and Laurence 2014). For instance, the concept of \texttt{bachelor} may be defined as the intersection \texttt{unmarried} \cap \texttt{man}. One of the main problems with the classical theory is that it cannot account for “typicality preferences” in categorization. For example, it has been shown that the object \texttt{plum} is considered to be less representative of the concept \texttt{fruit} than the object \texttt{apple} but more representative than \texttt{coconut} (Rosch 1975 a.o.), and this difference cannot be recast under the reductive approach advocated by the classical theory.

A popular alternative is prototype theory, which does take into account typicality preferences. In this theory, a concept \texttt{con} does not have a definitional structure, but a probabilistic structure that measures the amount of properties that are true of some object \texttt{obj} and evaluates whether \texttt{obj} satisfies a sufficient number of properties to fall under \texttt{con}. In this model, \texttt{apple} rates “better” (in the sense of “more typical”) than \texttt{plum} with respect to the concept \texttt{fruit}.

Typicality refers to the extent to which objects, concrete or abstract, are good characterizations of concepts. According to prototype theory, a concept \texttt{con} is not understood categorically, but as cluster of attributes, only some of which may be instantiated by any one object that falls within \texttt{con}.\(^5\) A useful consequence is that we can compare two objects in order to determine which one satisfies more attributes relative to some concept. Thus, a \texttt{robin} is more typical a \texttt{bird} than a \texttt{penguin} because it satisfies more “birdy” attributes.

I suggest that applying a similar mechanism to the domain of pluralities one can account for non–atomic distributive interpretations in (4b), (5) and (7). In particular, I suggest than one of the attributes that may determine the degree of typicality of an object relative to some concept is its “typical cardinality”, that is, whether the objects that instantiate the relevant concept usually appear individually or in groups of more than one. Take the concept \texttt{bird}. Birds are found individually and in bunches of variable size. However, knowing whether particular instantiations of the concept \texttt{bird} appear individually or not has no bearing in understanding what a bird is. Thus, the notion of cardinality is not part of the cluster of attributes that determine the degree of membership relative to \texttt{bird}. In these cases, I assume that the typical cardinality is simply 1. The example of \texttt{shoe}, however, is different. For one, knowing that shoes are typically found in twos follows from the function that shoes are meant to have. In addition, most times that we encounter instantiations of \texttt{shoe}, we encounter them in groups of two. It is following this intuition that the typical cardinality of objects instantiating \texttt{shoe} is 2 instead of 1.

It is important to capitalize on the fact that typical cardinalities are determined by mechanisms that make use of world–knowledge rather than contextually available information. Typical cardinalities can be regarded as generalizations of the frequency with which objects are found arranged in particular groupings. Since typical cardinalities constitute, by assumption, one of the attributes that are relevant to establish which one of any two objects is a “better” representative of some concept, the same holds for cardinalities too. So, we have it that shoes in pairs have the potential to be as good representatives of \texttt{shoe} as individual shoes, whereas this is irrelevant in the case of a pair of birds relative to \texttt{bird}.

In what follows, I will use the notation in Kerem et al. (2009) (cf. Osherson and Smith 1997), where typicality is expressed by a function \(\Theta\) that assigns to objects values in the interval \([0, 1]\), where the higher score corresponds to a greater typicality. In this notation, then, \(\Theta_{\text{fruit}}(\text{apple}) > \Theta_{\text{fruit}}(\text{plum}) > \Theta_{\text{fruit}}(\text{coconut})\).

\section*{4.2. Accounting for typical groups}

In the framework assumed here, objects are conceptualized in a way that can be modeled by the mereological approach developed after Link (1983), by means of notions such as \texttt{atom} and \texttt{part-of} relations. Typical groups are defined in terms of a function \(\Theta_{\text{con}} : \mathbb{N} \to [0, 1]\), which maps cardinalities of objects \texttt{obj} instantiating a

\(^4\)I will use the following typographical conventions: I will use \texttt{small caps} to describe the name of concepts and \texttt{monospace} to refer to particular objects instantiating a particular concept (as in \texttt{bird} and \texttt{robin}).

\(^5\)The classical reference is Wittgenstein (1953). He raised the challenge of defining the concept \texttt{game}, an impossible task according to him. His argument proceeds by considering a number of plausible definitions followed by an obvious counterexample. For instance, he takes \texttt{games} to be some competitive activity (counterexample: a solitaire card game), and that \texttt{games} always involve winning or losing (counterexample: throwing a ball against a wall).
In many cases there may not be a significant difference between two cardinalities of the same object (consider for instance the case of \( \Theta_{\text{pen}}(3) \approx \Theta_{\text{pen}}(5) \)).

For any number \( n \) greater than 1 when applied to \( \Theta_{\text{con}}(n) \), we say that the “typical cardinality” of \( \text{con} \) is \( n \). In the case of shoes, this will only be true of the number 2, since all that we can learn from world-knowledge is that pairs of shoes are typical, but we know that there is nothing special about triplets of shoes, triplets of shoes are not conceptually salient in any way. Thus, it is the case that \( \Theta_{\text{shoe}}(2) > \Theta_{\text{shoe}}(1) \) but \( \Theta_{\text{shoe}}(3) \not\approx \Theta_{\text{shoe}}(1) \) and \( \Theta_{\text{shoe}}(1) > \Theta_{\text{shoe}}(3) \), and the typical cardinality of shoes is 2.

Typical groups are both accessible for quantification and also context independent. This suggest that they are relevant for semantic interpretation and so the grammar must have access to them in some way or other.

I recast these observations in (24), following ideas from Kerem et al. 2009 for the semantics of reciprocal expressions and from the notion of impure atomicity in Landman (2000).

24) **Typical Cardinality Hypothesis:**

A plurality denoting NP with a typical cardinality \( n \) of more than 1 can denote an impure atom by grouping \( n \) atoms together.

This hypothesis states that if a plural NP is typically arranged in sub–pluralities of cardinality \( n \) the grammar is allowed to build an impure atom out of those sub–pluralities. Impure atoms are atoms that are derived from an \( i \)–sum through the group formation operation \( \uparrow \). Once \( \uparrow \) applies to a plurality, its individual parts are no longer accessible anymore, and the newly created entity behaves like a singular atom; hence the name “impure atoms”.

25) \( I-At(x) = \exists y[y \neq x \land \uparrow(y)] \)

Creating an impure atom allows us to maintain the idea, defended extensively in Winter (2001), that distributivity can only be atomic. We have now the conditions for an NP to denote a typical group. This is implemented in the following function:

26) Typical Group Function: given a plural predicate ‘\( P \)’ and an individual–part \( x \) of ‘\( P \),

\[
\text{TypG}(x) = \forall x [x \leq P \land |x| = n \land \Theta_{\text{r}}(n) > \Theta_{\text{r}}(1) \rightarrow \exists y[y \neq x \land x = \uparrow(y)]]
\]

That is, for every \( i \)-part \( x \) of a plural predicate ‘\( P \), if \( x \) has a cardinality of \( n \), where \( n \) is more typical than a cardinality of 1 with respect to the concept \( r \) that ‘\( P \) instantiates, then an impure atom \( y \) can be build from \( x \). In the case of the plural predicate shoes, (26) says that pairs of shoes will be able to be interpreted as impure atoms.

The value attributed by \( \Theta_{\text{con}} \) to 1 is taken to be the “golden standard”: the condition is stated in such a way that any sub–pluralities \( x \) that is part of ‘\( P \) can be interpreted as an impure atom if \( x \) is more typical relative to \( r \) than the pure atoms composing ‘\( P \). For most plural NPs there will be no such sub–plurality. In these cases, the premise in (26) is falsified and no typical group is predicted to be available. If such \( x \) exists, then the prediction is that the grammar has access to three types of individuals: two kinds of atoms (pure and impure) and \( i \)-sums. Since, from a grammatical perspective, both type of atoms constitute the same type of formal object, they behave the same for all purposes. As a consequence, quantifiers can now pick either one of the two as arguments, and so the correct meanings for sentences like (4b) are derived:

6 Notice that typical groups have to be defined concept internally: a typicality function \( \Theta \) may assign different values to different cardinalities of the same object. The motivation for this is that when looking at what counts as a typical grouping of some object, say glove, one need not know which of the objects between woolen glove or workout glove is more typical with respect to the more general concept glove. In this sense, the typicality function that looks for “typical cardinalities” must be different from \( \Theta \) in Kerem et al. (2009).
In Link’s framework, plural entities denote sums of individuals, and not sets, and they are as “concrete” as the individuals that serve to define them, hence of the same logical type. The operation ↑ can be understood as a domain–shifter rather than as a regular type–shifter: divide the domain of entities E into the set of atoms AT, where atoms are defined as in (9), and the set of non–atoms SUM, where SUM = E \ AT; then, if x ∈ SUM, ↑(x) ∈ AT.

It may be that existential sentences like (28) behave differently and block the intermediate interpretation, but the matter requires more research than I am able to report on here.

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8http://dailysuitcase.blogspot.com/2014/05/more-tips-for-packing-light.html

5. Conclusions

This paper suggests that some problematic cases of non–atomic distribution can be better handled if we adopt some notion of typicality. If we allow plural NPs to be ranked on a scale of typicality according to the cardinalities that are conceptually more salient, and we adopt a principle in the vein of the Typical Cardinality Hypothesis, we understand why quantifiers allow for non–atomic distribution: the grammar allows to build impure atoms out of the most typical cardinalities for each concept (if any), and therefore, quantifiers and other kinds of operators gain access to them.

One of the advantages of precluding the operator ↑ to apply freely is that not any sub–plurality of shoes will be accessible for quantification. Letting typicality mediate in the availability of impure atoms has the further advantage of relegating the role of context to a secondary plane and make context independent non–atomic distribution available. Context–dependent intermediate interpretations of plural DPs can still be handled by covers (or your theory of choice); the important consequence is that we can dispense the theory of distributivity from having to give an account of these problematic cases, all the rest being equal.

Some aspects of the account require still further elaboration, and they will have to remain as open questions. One challenge is that relying on typical cardinalities may still not be sufficient. For instance, the typical group function in (26) predicts that any group of two shoes should qualify as a typical group. But it is not clear that this is correct. Should two left shoes qualify as a typical group? In this sense, typical groups resemble substantive groupings, in the sense of Kratzer (2003). Since, as discussed above, substantive groupings alone are not sufficient to account for the data in §1, it may be that ingredients from both approaches are needed.

An anonymous reviewer is concerned that some quantifiers may not have access to typical groups, on the basis of examples like the following:

27a. \[
[(4b)] = \forall x \cdot \text{ Shoe}(x) \rightarrow \bigwedge_{\mathbf{T}_{\mathbf{DG}}} \cdot (\text{Cost} - 50)(x)
\]

b. \[
[(4b)] = \forall x \cdot \text{ Shoe}(\text{TypG}((x)) \rightarrow \bigwedge_{\mathbf{T}_{\mathbf{DG}}} \cdot (\text{Cost} - 50)(\text{TypG}((x)))
\]

\[
\Rightarrow \exists z \cdot \text{ Shoe}(z) \land |z| = 2 \rightarrow \exists y \cdot \text{ Shoe}(y) \land y \neq z \land z = \exists y \cdot (\text{Cost} - 50)(\text{TypG}(y))\]

For example, assume that \[\text{Shoe} = \{a, b, c, d, e, f\}\]. In (27a) the interpretation of (4a) is as expected: the quantifier all picks pure atoms x in the extension of \[\text{Shoe}\] and asserts that each of those individuals costs $50. An LF for the interpretation where each pair of shoes cost $50 is represented in (27b). The function (26) reveals that \[\Theta_{\text{Shoe}}(2) > \Theta_{\text{Shoe}}(1)\], and so the operation ↑ applies to pairs: ↑(a ⊕ b), ↑(c ⊕ d) and ↑(e ⊕ f), for example. What is important to note here is that all the rest remains the same: quantifiers can only have access to individual entities, and so the difference between (27a) and (27b) amounts to a difference in the entity that the quantifier picks as its argument. We can then see plural NPs like shoes to be ambiguous between an i–sum interpretation and a typical group interpretation (just like Landman 2000 suggests for the case of plural DPs like the shoes).

5. Conclusions

This paper suggests that some problematic cases of non–atomic distribution can be better handled if we adopt some notion of typicality. If we allow plural NPs to be ranked on a scale of typicality according to the cardinalities that are conceptually more salient, and we adopt a principle in the vein of the Typical Cardinality Hypothesis, we understand why quantifiers allow for non–atomic distribution: the grammar allows to build impure atoms out of the most typical cardinalities for each concept (if any), and therefore, quantifiers and other kinds of operators gain access to them.

One of the advantages of precluding the operator ↑ to apply freely is that not any sub–plurality of shoes will be accessible for quantification. Letting typicality mediate in the availability of impure atoms has the further advantage of relegating the role of context to a secondary plane and make context independent non–atomic distribution available. Context–dependent intermediate interpretations of plural DPs can still be handled by covers (or your theory of choice); the important consequence is that we can dispense the theory of distributivity from having to give an account of these problematic cases, all the rest being equal.

Some aspects of the account require still further elaboration, and they will have to remain as open questions. One challenge is that relying on typical cardinalities may still not be sufficient. For instance, the typical group function in (26) predicts that any group of two shoes should qualify as a typical group. But it is not clear that this is correct. Should two left shoes qualify as a typical group? In this sense, typical groups resemble substantive groupings, in the sense of Kratzer (2003). Since, as discussed above, substantive groupings alone are not sufficient to account for the data in §1, it may be that ingredients from both approaches are needed.

An anonymous reviewer is concerned that some quantifiers may not have access to typical groups, on the basis of examples like the following:

28) There are three shoes here.

Sentence (28) seems to lack an interpretation where it refers to the presence of three pairs of shoes, rather than three individual shoes. Under the approach presented here, nothing precludes this option. In some cases, however, typical groups seem to be accessible for numeral quantifiers too.

29) Likewise, you will be able to stick to my famous Three Shoe Rule. Now, I am not saying you can only bring three shoes. But I am limiting you to three pairs of shoes.8
Summing–up, three important facts about non–atomic distributive readings have been established: (i) sentences of the form $Q\text{-Det} A B$ are predicted to have intermediate readings only if $A$ can denote a typical group, (ii) non–atomic distributive interpretations over typical groups are context–independent, and (iii) $Q\text{-Dets}$ must have access to typical groups. The general theory of distributivity is relieved from the need to account for cases like (4b), since the reason why non–atomic distribution can survive quantification is independent from any version of $D$.

References