Differential Geometry: Curves – Surfaces – Manifolds

• Spring 2020 • MATH 495 (Selected Topics)

Differential geometry is a core subject in mathematics. The tools of differential geometry are routinely used in most subfields of mathematics, while at the same time they have wide applications in various theoretical as well as applied disciplines such as physics (theory of relativity, mechanics), econometrics, computer graphics, signal processing and statistics.

The course is a mathematically rigorous introduction to the fundamental notions in differential geometry: curves, surfaces and abstract differentiable manifolds. About half of the course focuses on curves and surfaces in three-dimensional Euclidean space and introduces key concepts such as first and second fundamental forms, Gauss curvature, covariant derivative and geodesics. The second part of the course centres on Riemannian manifolds, and presents important concepts such as Riemannian connection and the curvature tensor. Towards the very end of the class we will discuss about the General Theory of Relativity, a remarkable application of differential geometry.

Outline:

1. The Local Theory of Curves: Tangents, normals and bi-normals, curvature and torsion, Frenet formulas

2. The Local Theory of Surfaces: Tangent plane, first fundamental form, Gauss map, second fundamental form, curvature of surfaces (normal curvature, Gauss and mean curvatures)

3. The Intrinsic Geometry of Surfaces: Covariant derivative, parallel displacement and geodesics, the Gaussian equation and the Theorema Egregium

4. Manifolds: The notion of a manifold, tangent space, Riemannian metrics, Riemannian connection, tensors, the curvature tensor

5. Selected Introductory Topics in General Relativity: Gravity as a space time curvature, the geometry of curved space time (geodesics, the field equations), orbits in general relativity, the bending of light.

Prerequisites: Vector Calculus (Math 252), Linear Algebra (Math 240 or Math 232) and familiarity with ordinary differential equations, or permission of the department.