Detection of Exoplanet $\tau$ Boötis b at the Simon Fraser University Trottier Teaching and Outreach Observatory

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**Date:** September 1, 2016

**Abstract**

This is a report on the first detection of an exoplanet at Simon Fraser University’s Trottier Observatory, based on radial velocity (RV) measurements of the host star. The observatory is teaching and public outreach facility with a 0.7-m aperture telescope under suburban skies. Measurements were done using a commercially manufactured, off-the-shelf, echelle spectrograph that is comparable in cost to high-end imaging systems, and that can be used to study a remarkable variety of astrophysical objects. The detected exoplanet is $\tau$ Boötis b, one of the first to be discovered by professionals, and is a “hot Jupiter.” The exoplanet host star, $\tau$ Boötis A, is of magnitude 4.5, and spectral class F7V, and has one of the largest orbital RVs of any known host. Nevertheless, the Doppler shifts are tiny, and careful control of systematic uncertainties, and sophisticated data analysis methods, are necessary for a successful detection. The RV was measured on ten nights, between mid-March of 2016 and the end of June. The results cover most of the orbital cycle, and are in excellent agreement with the RV determined by professionals. This study provides one example of the many types of advanced spectroscopy projects that can be undertaken by undergraduate students at university teaching observatories, and by amateur astronomers, with increasingly affordable and easy-to-use equipment.

1) **Introduction**

Astronomical spectroscopy has long been a part of amateur astronomy, though until recently it has been practiced by relatively few enthusiasts, generally using home-made equipment with limited capabilities. However, we may now be at the threshold of a new era of amateur spectroscopy, thanks to commercially-available equipment of varying degrees of cost and sophistication. These include systems that can produce results of professional quality, and that are available at costs which are comparable to high-end astronomical imaging systems.

This situation seems reminiscent of the revolution in amateur astrophotography that began about twenty years ago, when CCD cameras first became widely available to hobbyists, opening up vast new realms of imaging possibilities. Amateur spectroscopists can now make measurements of astonishing variety and precision, and in addition to investigations done for personal education and reward, can participate in research collaborations with professionals (for a recent introduction to amateur spectroscopy, see Ref. [1]).

![Image 1: Nighttime view of the Trottier Observatory and Science Courtyard at Simon Fraser University. Image courtesy of PWL Landscape Architects, Inc.](image)

This also seems like an opportune time for universities with small teaching observatories to re-consider the educational opportunities associated with astronomical spectroscopy. Students from both the arts and sciences can engage in sophisticated scientific observations, with only a modest background in the relevant
astrophysics, and after relatively straightforward equipment training. A remarkable variety of fascinating astrophysical systems can be studied, many of which possess distinctive spectral signatures that can yield very interesting quantitative results, after straightforward data analysis and interpretation (a largely qualitative introduction to astronomical spectra can be found in Ref. [2]). Such activities can provide educational experiences that closely emulate the practice and excitement of cutting-edge science.

Curriculum of the aforementioned nature is being developed at the Simon Fraser University (SFU) Trottier Observatory. The observatory, which opened in April of 2015, is the anchor of a high-profile public space devoted to science, situated near the centre of the Burnaby campus (see Figure 1). The observatory houses a PlaneWave Instruments CDK700 alt-az telescope system, with a 0.7-m aperture operating at f/6.5, under a 20’ Ash dome. The observatory is used for public outreach, student education, and as a resource for local amateur astronomers, especially members of the Vancouver Centre of the Royal Astronomical Society of Canada.

A central component of the earliest plans for development of the observatory was to equip it with a large-format astronomical imaging camera system, and a high-resolution spectrograph. We settled on a Finger Lakes Instruments PL16803 16-Megapixel imaging camera, and a high-resolution “echelle” spectrograph made by a small French company, Shelyak Instruments. The spectrograph is marketed as the eShel, at a cost that is comparable to our imaging system, and is intended for use primarily by advanced amateurs.

The camera was up and running before the observatory was officially opened, and some of our first images have been published [3], [4]. The spectrograph was installed in early summer of 2015, and an extensive set of initial studies were completed by the end of the year; an informal report on some of that work can be found on the observatory website [5].

The eShel is mainly intended for high-resolution spectroscopy of brighter stars, owing to a trade-off between resolution and sensitivity that is characteristic of all spectrometers, although many diffuse objects are bright enough to be studied.

High-quality data for naked-eye stars can be collected in minutes, and in less than an hour for stars down to about eighth magnitude, even with modest apertures under suburban skies. An impressive set of measurements made with a wide range of amateur spectrographs, including the eShel, has been compiled by a leading French astronomer, Christian Buil, and are available on his website [6].

Measurements that have been done at SFU so far include: radial velocities of stars in double-lined spectroscopic binaries, including Mizar; properties of circumstellar disks (radial velocities and radii of the inner and outer rims), including that of the companion to the giant star VV Cephei A [7], and of the stars in the Pleiades; stellar-wind speeds, including those of Wolf-Rayet 140 (one of the brightest stars of that class), and P Cygni (the prototype of an entire class of stars with “optically-thick” winds); rotational speeds of many stars, including Vega, and the extremely fast rotator ζ Aquilae (which has an equatorial radial velocity about of 0.1% of the speed of light!); Jupiter’s rotational speed; and the emission spectrum of the Ring Nebula.

Even one night of data taking was enough to yield high signal-to-noise spectra in the cases listed above, with quantitative results that were generally within about 10% of known values, and several interesting systems could often be measured in one night.

This situation is very different from acquiring a high-quality astronomical image, which requires many nights of telescope time. And, unlike astronomical image processing, which demands many hours of effort, with lots of trial-and-error, to obtain the best possible final image, the processing of spectroscopic data requires relatively little time and intervention by the user, once the initial setup for a particular spectrometer/telescope system has been made.

This report details the most ambitious spectroscopic study that has been done at SFU to date: the detection of the exoplanet τ Boötis b, from measurements of the radial velocity (RV) of its host star, τ Boötis A. This star has one of the largest known orbital RVs of any host, but even so, this measurement is near the limits of
spectrographs that are likely to be available at university campus teaching observatories and to amateur astronomers, including the eShel (an exhaustive on-line compilation of known exoplanets can be found in Ref. [8]).

To set the scale for this measurement, the orbital RV of τ Boötis A generates fractional wavelength shifts of less than 1 part in 600,000. The detection of τ Boötis b requires careful control of systematic uncertainties in the data acquisition process, and a sophisticated approach to the data analysis; moreover, measurements must be done over many nights, in order to map out a decent portion of the host star’s orbital cycle.

τ Boötis b was one of the first exoplanets to be discovered, in 1997 [9], and was the first to be detected by amateur astronomers from RV measurements, in 2004 [10] (the detection has since been repeated by a handful of other amateurs, see, for example, Ref. [6]). It is a so-called “hot” Jupiter, with a mass of about 6 times Jupiter’s mass, an orbital semi-major axis of only 0.046 AU (compared with 0.39 AU for Mercury), and an orbital period of just over 3 days (compared with 88 days for Mercury). The host star is a 4.5-magnitude main-sequence star of spectral type F7, and it has a maximum orbital radial velocity of 470 ± 15 m/sec. Interestingly, spectral lines in the infrared radiation from the planet itself have recently been detected, and show that water exists in its atmosphere [11].

The rest of the report is organized as follows. Section II provides an overview of the spectrograph, including its configuration at SFU, and an example of a stellar spectrum acquired with it. Section III briefly surveys the two techniques that amateurs have used to detect exoplanets, namely photometric detection of transiting exoplanets, and RV measurements, and quantifies the challenges in obtaining sufficiently precise results in the latter case. Section IV outlines the strategy used for the data acquisition and analysis, and Section V presents a detailed analysis of the data, including estimates of the systematic and statistical uncertainties. Section VI presents a fit to the results for the orbital motion of the host star, compares the results with professional measurements, and extracts the mass of the exoplanet. The report concludes in Section VII, which includes a brief outlook for future work. An Appendix provides a qualitative introduction to the basic principles of echelle spectrometers for interested readers (a fuller introduction to spectrographs can be found in Ref. [1], while Ref. [12] provides a treatise for advanced undergraduates and graduate students).

Note: The report contains equations that are included for readers with sufficient mathematical background. However, the text is written for a non-mathematical readership, and the equations can be skimmed, or skipped entirely, without loss of continuity.

II) The spectrograph

Echelle spectrometers, such as the Shelyak Instruments eShel, are advantageous compared to other designs because of their inherently superior resolution. Spectral resolution is characterized by the smallest possible wavelength interval δλ between two neighbouring spectral features that can be individually identified, as illustrated in Figure 2. Resolution is usually specified in terms of a fractional measure R that is defined by

$$ R \equiv \frac{\lambda}{\delta \lambda} $$

The eShel has a resolution R of about 10,000, meaning that it can distinguish between visible-light spectral features separated by about 0.5Å. Many professional echelles by comparison have resolutions above 100,000.

![Figure 2: Three sets of emission lines separated in wavelength by different amounts. The dotted curves plot the total intensity of overlapping lines.](image)

The eShel layout when in use at the SFU observatory is shown in Figure 3. The spectrograph is remarkably compact, and sits on
top of a small dolly that is wheeled into the dome area to collect data. A light-weight acquisition/guiding unit is attached to one of the telescope’s two ports. The acquisition unit houses a small plane mirror with a hole: the mirror deflects almost all of the incoming starlight to a guide camera mounted on the side of the unit, while a fibre optic cable mounted behind the hole in the mirror feeds light to the spectrometer. The spectrometer’s diffraction grating disperses the light, and a CCD camera (the Atik 460ex, in our case) that is mounted on the spectrometer images the resulting spectrum. USB cables connect the equipment to the control room computer.

The spectrograph has calibration sources, and the light that they produce when activated is fed to the acquisition unit at the telescope through a separate fibre optic cable; a flip mirror in the acquisition unit redirects the calibration light to the pickup fibre, and back to the spectrograph, so that the calibration is done along the same optical path that is followed by incident starlight.

The fibre optic cable that transmits the incoming starlight is only 50 microns in diameter! The small size of the fibre is necessary for the high resolution of the spectrometer, but also necessarily limits its sensitivity. The images of the Ring Nebula and Vega in Figure 3 (the latter is highly over-exposed!) were taken through the guide camera, and show the tiny region occupied by the hole in the mirror. This illustrates why this unit is suited to stellar spectroscopy, and not so much to extended sources, though brighter nebulæ can be measured. These images also demonstrate the importance of auto-guiding, to keep the target starlight on the hole!

An acquisition image of the spectrum of Vega obtained with our eShel is shown in Figure 4; Vega is so bright that this spectrum required less than three minutes of total exposure. As is evident from this image, echelle spectra take the form of a set of narrow bands stacked one on-top of the other. Each band corresponds to the output from one order of the diffraction grating (the bands have no colour here because we use a monochrome acquisition camera). This characteristic pattern gives rise to the adjective “echelle” (French for ladder).

Figure 3: Spectrograph layout at the SFU observatory.
In the case of the *eShel*, some 25 orders cover the entire optical band, from just below 4000 Å to well beyond 7000 Å. Some of the absorption lines that stand out in Fig. 4 are identified with coloured boxes. On the left side of the image, running from top to bottom are: the Hydrogen-alpha line at 6562Å (red box); the Hydrogen-beta line at 4861Å (green); and a Magnesium line at 4481Å (cyan). On the right side, again from top to bottom, there is a very dense set of absorption lines due to oxygen molecules in our own atmosphere, running from about 6850-7000Å (orange box), and the Hydrogen-gamma line at 4340Å (blue).

![eShel acquisition image for Vega. The yellow text identifies the central wavelength in each of the displayed orders. The coloured boxes identify some significant spectral features, as detailed in the text.](image)

Figure 4: *eShel* acquisition image for Vega. The yellow text identifies the central wavelength in each of the displayed orders. The coloured boxes identify some significant spectral features, as detailed in the text.

![Acquisition image produced by the eShel’s thorium-argon hollow cathode lamp.](image)

Figure 5: Acquisition image produced by the *eShel*’s thorium-argon hollow cathode lamp.

Data acquisition with the *eShel* is conveniently handled using a freeware package that has specially-designed utilities for this purpose [13]. Analysis of raw acquisition images, like the one in Figure 4, is done by professionals using very powerful but arcane command-line software packages [14], [15]. A relatively user-friendly GUI freeware package better suited to non-experts has been written by Christian Buil [6], and was used to process the data in this study.

To turn an image like the one in Figure 4 into a graph of intensity versus wavelength requires calibration of the spectrometer/camera system. Calibration is needed for four reasons: to map the geometry of the echelle rungs; to correct for intrinsic variations of the echelle output within each rung; to map the pixel position at which light reaches the imaging CCD to the corresponding wavelength; and to correct for overall variations in the response of the spectrometer and CCD chip.
The eShel is equipped with continuous sources (a tungsten lamp and blue LEDs) that are used to characterize the echelle rungs. A hollow cathode lamp containing a mixture of thorium and argon gases is used for wavelength calibration: when a voltage is applied to the tube, atomic transitions are induced that emit light at hundreds of precisely known wavelengths. Figure 5 shows the spectrum produced by the cathode lamp.

![Figure 5: Spectrum produced by the cathode lamp.](image)

**Figure 6: Spectrum of Vega without correction for the overall response of the eShel/CCD chip, compared with a standardized spectrum of a star of the same spectral type. The dashed line is the computed response function of the spectrometer.**

![Figure 6: Vega spectrum with and without correction.](image)

It is often more convenient to divide out the smooth underlying trend in a star’s spectrum, which is produced by the continuous radiation emitted by the body of the star (and that is usually of less interest than the absorption lines produced by the star’s atmosphere), to obtain a so-called “rectified” spectrum. Figure 7 shows the fully-calibrated spectrum of Vega, and the result of a fit to the underlying trend (referred to as the pseudo-continuum), along with the resulting rectified spectrum. If only the rectified spectrum is required, it can be obtained without need of the spectrometer/CCD response function.

**Figure 7: Fully calibrated spectrum of Vega, and a fit to the underlying trend (upper panel); Rectified spectrum (lower panel). The four optical hydrogen Balmer lines are labelled.**

![Figure 7: Vega spectrum with and without correction.](image)

To correct for the overall response of the spectrometer/CCD chip, one can measure the spectrum of a star and compare the result with a standard reference spectrum of the same star, or of another star of the same spectral type; the ratio between the uncorrected and reference spectra yields the response function of the spectrometer, which can then be used to produce corrected spectra of other stars. Figure 6 shows an uncorrected spectrum of Vega obtained with the eShel, and a standardized spectrum [16] for another star of the same spectral type (A0V), along with the computed response function.

### III) Exoplanet detection

Exoplanets have been discovered by at least five techniques (a non-specialist overview of these techniques can be found in Ref. [17], while Refs. [18] and [19] provide introductions for advanced undergraduates and graduate students). Two of these methods are accessible at small university teaching and amateur observatories: photometric measurement of an exoplanet transit across the host star; and spectroscopic measurement of a host star’s motion (projected along the line of sight of the observer), due to the gravitational influence of an exoplanet.

The general principles of transit and radial velocity measurements are illustrated in Figure 8 (other methods include gravitational microlensing, direct imaging, and astrometry). Transits are inferred from a drop in the apparent brightness of the host star, and can be used to determine the size of the exoplanet: the larger the exoplanet diameter, the deeper the drop. Radial velocities are measured from Doppler shifts of absorption lines in the star’s spectrum, which vary in wavelength as the star moves around the centre-of-mass of the system, and the results can
be used to infer a lower bound on the mass of the exoplanet: the greater the planet mass, the faster the host star moves in an orbit of a given size.

Only a fraction of exoplanets transits their host star along our line of sight, and relatively few produce an RV motion in the host star that is large enough to be measured. In those rare cases where both measurements can be done, one can infer the density of the exoplanet; this is how professionals have identified exoplanets that are likely to be gaseous in composition, and those that are likely to be rocky terrestrials.

Photometric detection of a transiting exoplanet was done at the SFU Observatory shortly after it opened, by one of the observatory’s most proficient users, local amateur astronomer Oleg Mazurenko, who has since detected several additional systems; a report on Oleg’s results is available at the observatory website [5].

RV detection of an exoplanet on the other hand requires an exacting combination of carefully-controlled measurements, and painstaking data analysis, owing to the very small orbital RVs to be measured. Even the “easiest” case is near the limits of the capabilities of spectrographs that are likely to be available at small observatories, including the eShel, and only a handful of exoplanets are within the reach of such instruments.

To quantify the challenge of these measurements, consider the formula for the Doppler shift:

$$
\Delta \lambda = \lambda \frac{v_r}{c}
$$

where $\Delta \lambda$ is the shift in wavelength of light emitted or absorbed by a source at wavelength $\lambda$ in its rest frame, $v_r$ is the radial velocity (the component along the line of sight) of the source relative to the observer, and where $c = 300,000$ km/sec is the speed of light. τ Boötis A has one of the largest known orbital RVs due to an exoplanet, about 470 m/sec (roughly only 1.5 times the speed of sound in air), which produces a maximum fractional wavelength shift $\Delta \lambda / \lambda$ of about 1 part in 600,000. To make a convincing detection however will require the Doppler shift to be measured at the level of one part in a few million, so that statistical fluctuations in the measured RVs will be smaller than maximum RV by a reasonable margin.

For another indication of the challenge of RV exoplanet detection, let’s suppose that the uncertainty in any one measurement of the RV is required to be about 10% of its maximum value, or about 50 m/sec. Since it turns out that the eShel/camera pixel scale is about 0.15Å/pixel, a velocity shift of 50 m/sec is equivalent to a shift in the position of a spectral line of only 1/200 of a pixel! By comparison, professionals measure RVs equivalent to shifts of 1/1000 of a pixel!

*Figure 8: Exoplanet detection by transit of the host star (upper panel, NASA image [20]), and from Doppler shift measurements of the host star’s radial velocity (lower panel, ESO image [21]).

Detection of transiting exoplanets is relatively straightforward, with dozens of cases within the reach of modest telescopes and cameras (for an introduction to transit detection for amateurs, see Ref. [22]). In the most favourable cases, the drop in apparent brightness during the transit is about 0.02 magnitudes, which can be measured using routine differential photometry, not unlike variable star measurements. Data reduction is also routine, thanks to an on-line compendium to which amateurs can upload their data [23].
Since it is not possible to directly measure such tiny positional shifts, the RVs are actually inferred from changes in the intensities of spectral lines, rather than in their wavelengths. If the intensity of a spectral line is measured with sufficient precision (i.e., with sufficient signal-to-noise), and if the intensity varies sufficiently rapidly as a function of wavelength (i.e., if the line is sufficiently deep), then a Doppler shift will result in a measurable change to the intensity of the line [24], [25], even if the pixel spacing is much larger than the wavelength shift.

![Figure 9: Rectified spectra of Vega and τ Boötes. For clarity, the spectrum of Vega was shifted upwards by the indicated amount.](image)

Cooler host stars, which have many deep absorption lines, are favoured for exoplanet detection. Most professional studies have been done for spectral types ranging from about F5 to M5, which have up to several thousand well-separated absorption lines; hotter stars on the other hand have too few lines, while the lines in cooler stars are too overlapped. The advantages of stars of appropriate spectral types is evident from Figure 9 and Figure 10, which compare rectified spectra taken at SFU of Vega (spectral type A0V), τ Boötes A (type F7V), and 45 Boötes (type F5V), a reference star used in this study.

A survey of results from a variety of spectrometers [19] shows that a device with a resolution comparable to the eShel can produce RV measurements with a precision of around 50 m/sec, if the target spectrum has a few thousand deep absorption lines, and is acquired with a signal-to-noise of about 100:1 (which in our case takes less than an hour for stars down to about 8th magnitude). However, dealing adequately with potential thermal or mechanical changes to the spectrometer is essential, since even tiny uncontrolled shifts in pixel positions can be enough to swamp the effects of the small Doppler shifts to be measured.

**IV) Measurement and analysis strategies**

As detailed in the previous section, detection of an exoplanet will stretch the capabilities of our spectrograph. A convincing detection will therefore require many observations, spread out over a good portion of the orbital cycle, owing to the relatively large uncertainties that will be present in any one measurement. In this study, data were taken on 10 nights that covered most of the orbital cycle.

Moreover, to reach meaningful conclusions, one must estimate the scientific uncertainties in the quantitative results. To estimate the uncertainties, between 6 and 9 measurements were made on each night, and the resulting averages and standard errors were computed. Examples of the dispersion in individual measurements will be shown below.

One of the most difficult challenges in any scientific measurement is to estimate the systematic uncertainties that are associated with
any apparatus and technique. As indicated in the last section, it is critical to deal with possible systematic effects that can arise from thermal and mechanical changes in the spectrometer over the course of the night, or from one night to the next; systematic effects of this sort are often referred to as spectrometer or calibration “drift” [19].

The setup currently in use at the SFU observatory makes spectrometer drift very challenging to deal with (though only at the high-level of precision required for exoplanet detection). The limited space available in the rest of the observatory requires that the spectrograph be operated in the open environment of the dome area, which can produce significant thermal drift over the course of the night; the spectrograph must be also be moved into storage when not in use for extended periods, which can cause significant mechanical and thermal changes from one night of observations to the next.

To control for spectrometer drift, a two-part strategy was used. First, calibration data were taken at regular intervals over the course of the night, generally before and after each data set was acquired (professional setups normally take object and calibration data simultaneously, by feeding object and calibration light to the spectrometer at the same time, through separate fibres). This helps to reduce the effects of thermal drift over the course of the night.

The second part of the calibration strategy was to take the spectrum of a reference star, either immediately before or immediately after each measurement of the spectrum of τ Boötis. This further reduces the effects of thermal drift, and controls for mechanical changes when the spectrograph is stored between nights.

A good choice for a reference star is one that is of similar spectral type to the target, and that is bright, and nearby on the sky. For this study, 45 Boötis was used: it has a visual magnitude of 4.9 and spectral class F5V, compared with magnitude 4.5 and class F7V for τ Boötis, and the two stars are reasonably close on the sky (separated by about 20°).

The standard approach used by professionals to estimate a radial velocity is to compute a so-called cross-correlation function (to be defined precisely below), which compares the measured spectrum of the target object with a reference spectrum [18], [19]. The reference spectrum is usually an idealized “template” of the spectra of objects of similar spectral type as the target. In this study, the reference spectrum will be the measured spectrum of 45 Boötis.

A total exposure of ten minutes was used for each spectrum acquisition (five two-minute exposures were added together, in order to avoid saturation in the camera response), while each calibration data set took one or two minutes. It typically required about 30 minutes to complete one measurement cycle (consisting of data acquisition for the two stars, spectrometer calibration, and target slewing); measurement cycles will be labelled in the following by a sequential “bin” number.

The cross-correlation between the two measured spectra will be denoted by \(C_{12}(v_r)\), and will be evaluated as a function of the relative radial velocity \(v_r\) between the two stars. The best estimate of \(v_r\) will be the value that produces the closest possible match between the two spectra. The velocities will be evaluated separately for each data bin, and an overall average value and standard error will be estimated from the ensemble of measurements taken each night.

This combination of data acquisition and analysis yields very precise results, since in essence one exploits the difference between two measurements, taken with the same instrument, and at nearly the same time, to largely cancel out systematic uncertainties.

[Before considering the precise form of \(C_{12}(v_r)\), the reader who wishes to skip the mathematical details need only be aware that the function is bounded by “1” in absolute value, and will precisely equal “1” only for a perfect match between the two spectra. If desired, the reader can now skip to the paragraph following Eq. (5).]

The cross-correlation function is defined by

\[
C_{12}(v_r) = \frac{\sum_\lambda I_1(\lambda)I_2(\lambda \left(1 + \frac{v_r}{c}\right))}{\sqrt{(I_1^2)(I_2^2)}}
\]  

(3)
where \( I_1 \) and \( I_2 \) are the two stellar spectra, and where the wavelength in one spectrum is Doppler shifted relative to the other, here using the standard non-relativistic formula, cf. Eq. (2). The angle brackets in the denominator in the above expression are normalizing factors, defined by

\[
\langle I_{1,2}^2 \rangle = \sum_{\lambda} I_{1,2}^2(\lambda) \tag{4}
\]

One can show that the cross-correlation satisfies the so-called Schwarz inequality

\[-1 \leq C_{12} \leq -1 \tag{5}\]

If \( C_{12} = 1 \) then the two spectral functions are said to be completely correlated, while if \( C_{12} = -1 \) they are said to be completely anti-correlated. The best-fit value for the relative velocity \( v_r \) is the one that is found to numerically maximize the cross-correlation function.

Figure 11 shows the cross-correlation between the spectra of \( \tau \) and 45 Boötes, using averages of the spectra extracted from the individual data bins acquired on the indicated date. The RV at which the cross-correlation is maximized is indicated on the plot (this includes barycentric corrections, which are described in the next section). The cross-correlation function typically has a Gaussian shape around its maximum, as illustrated by the solid line in Figure 11, which is the result of a fit to the data points in that region.

Evaluating uncertainties in the estimates of the radial velocities is done in the next section. Before doing so, it is very useful to first get a visual impression of the quality of the data. Figure 12 zooms into a small section of the spectra of the two stars,\(^7\) from the same data set that was used to compute the cross-correlation function in Figure 11. The spectrum of 45 Boötes is plotted twice, with different Doppler shifts applied arithmetically to the data before making the plot; one plot has been Doppler shifted by 2 km/sec less than the best fit value for the relative RV between the two stars, and the other by 2 km/sec more than the best fit. This comparison demonstrates that a shift of only a few km/sec can easily be discerned by eye over a small part of the spectrum. This makes it plausible that radial velocities estimated from a cross-correlation analysis which, for appropriate spectral types, simultaneously compares many thousands of absorption lines, can have uncertainties of much less than one km/sec.

V) Data analysis

As described in the previous section, one of the principal challenges in this study is to control for drift in the spectrometer calibration. In particular, if the data acquisition strategy was successful, then no systematic change in the RV between the two stars, as inferred from the set of sequential measurements from a given night, should exist.

In addition to analyzing the relative RV, it is also instructive to analyze the RV of each star separately. The latter can be done by maximizing an “auto”-correlation function, where the spectrum of a star is compared with itself; specifically, the first spectral function in Eq. (3)
is fixed to the star’s spectrum from the first data bin, while the second function is taken from one of the other bins. In this way, the RV inferred from one bin is computed relative to the first.

Figure 12: A small section of the spectra of τ and 45 Boötes, from the same data set that was used to compute the cross-correlation plotted in Figure 11. The spectrum of 45 Boötes is plotted twice, after being Doppler shifted arithmetically by the indicated amounts relative to the best-fit value for the relative RV. The dotted vertical lines are drawn through some min/max points in the spectrum of τ Boötes, to facilitate comparison of the three plots.

In order to extract any RV, one must take account of the observer’s motion relative to the solar system’s centre-of-mass, also known as its barycentre (which, to a first approximation, is the rest frame of the Sun). Earth’s orbital velocity around the Sun is about 30 km/sec, and its component in the direction of a point on the celestial sphere can change significantly, relative to the accuracy of the measurements in this study, over the course of a few days. Perhaps more surprising is that, for exoplanet detection, Earth’s rotational motion must also be taken into account, even over the few hours needed to complete one night of measurement; at our latitude, a point on the surface rotates at about 0.3 km/sec, which is almost as large as the orbital RV of τ Boötes A! The observer’s net velocity around the solar system barycentre, in the direction of any point on the celestial sphere, can be computed using a number of software packages [6], [14].

The importance of barycentric corrections is illustrated by the plots in Figure 13, which show changes in the apparent radial velocity of τ Boötes over the course of one night of observations. The plot labelled “raw” shows the change in radial velocity without taking account of barycentric corrections. The barycentric correction, and the corrected radial velocities, are also plotted. The measured data clearly exhibit the overall trend due to motion of the observatory relative to the barycentre, even over this short five-hour period of observations.
From here on, all results will be presented with barycentric corrections taken into account.

Returning to consideration of potential drift in the spectrometer, Figure 13 already provides one example of monitoring for drift in the RV of an individual star: no systematic trend is discernable, after the barycentric correction is applied, with the changes from one bin to the next being consistent with random statistical fluctuations.

Systematic changes in the RVs of both stars are evident; the two trends also appear to be almost identical, as would be expected from spectrometer drift, which should induce roughly the same shift in all measurements taken at roughly the same time.

Fortunately, the drift appears to cancel out when the spectra of the two stars are cross-correlated, as shown in Figure 15. The strategy of measuring the target and reference stars one right after the other, and comparing the two spectra, was apparently successful in controlling for drift.

![Figure 13: Change in the radial velocity of τ Boötis, and in the velocity of the observer relative to the solar system barycentre, over the course of a night.](image)

![Figure 14: Change in the apparent radial velocities of the two stars on the indicated date. The dashed lines are linear fits to the data. Barycentric corrections have been taken into account.](image)

Noticeable drifts in the spectra occurred on just two or three of the ten nights of observations. The auto-correlations from the night with the most significant drift are shown in Figure 14.

Figure 15: Relative RV between the two stars, as determined from the cross-correlation between their spectra. The data is from the same set used in Figure 14. The average over the nine bins is also shown; the error bar is the standard error.

The final results are obtained by computing an average, denoted by $\bar{v}_r$, of the relative radial velocities extracted from each data bin

$$\bar{v}_r = \frac{1}{N} \sum_{i=1}^{N} v_{r,i}$$

(6)

where $N$ is the number of bins, and $v_{r,i}$ is the estimate from the $i$-th bin. To estimate the uncertainty in the average, the so-called standard error was used, denoted here by $e_{v_r}$, and given by

$$e_{v_r} = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^{N} (v_{r,i} - \bar{v}_r)^2}$$

(7)

Table 1 lists the results for all ten nights of observations, and Figure 15 shows the average/uncertainty of the data for that particular night.
Equation (7) provides a valid error estimate only if the variations in the individual measurements are due to purely statistical fluctuations. It is possible that there are small residual systematic errors due to spectrometer drift, or other unidentified systematic effects, but the consistency of the error estimates obtained from Eq. (7) will be supported by an overall fit to the data, which will be detailed in the next section.

Table 1: Relative RVs between τ and 45 Boötes on the ten nights of observations. The values in parenthesis are the standard error uncertainties in the last two digits. The date and time is the mid-point of the interval over which the data was acquired, which varied between about 3 and 5 hours.

<table>
<thead>
<tr>
<th>Date (UT 2016)</th>
<th>No. of Bins</th>
<th>Relative RV (km/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18-Mar 10:15</td>
<td>8</td>
<td>-6.831(41)</td>
</tr>
<tr>
<td>1-Apr 9:30</td>
<td>9</td>
<td>-6.805(41)</td>
</tr>
<tr>
<td>8-Apr 9:45</td>
<td>9</td>
<td>-6.865(44)</td>
</tr>
<tr>
<td>9-Apr 8:15</td>
<td>7</td>
<td>-7.597(46)</td>
</tr>
<tr>
<td>10-Apr 8:15</td>
<td>8</td>
<td>-7.092(64)</td>
</tr>
<tr>
<td>20-Apr 7:15</td>
<td>7</td>
<td>-7.133(93)</td>
</tr>
<tr>
<td>7-Jun 7:15</td>
<td>7</td>
<td>-7.179(30)</td>
</tr>
<tr>
<td>16-Jun 7:00</td>
<td>7</td>
<td>-6.712(25)</td>
</tr>
<tr>
<td>17-Jun 7:30</td>
<td>6</td>
<td>-7.201(56)</td>
</tr>
<tr>
<td>28-Jun 6:15</td>
<td>6</td>
<td>-7.460(37)</td>
</tr>
</tbody>
</table>

Regarding systematic uncertainties, it turns out that activity in a target star, such as convective spots in its photosphere, can induce shifts in the star’s spectral lines, which can mask the true radial velocity, a phenomenon known as stellar “jitter” [18], [19]. Jitter varies in magnitude with a star’s spectral type, rotation period, and other attributes. For main-sequence F-class stars, like those measured here, jitter can induce shifts as large as 30 m/sec, on times scales comparable to the star’s rotation period (typically a few days for F-class main-sequence stars, see e.g. Ref. [26]).

A related concern in a study such as this one, which relies on relative RV measurements, is the possibility that the RV of the reference star is not sufficiently stable, perhaps due to the influence of an unknown stellar companion, or unusually large intrinsic jitter. A professional study that monitored 45 Boötes over a period of about 100 days found no significant systematic variation in its RV [27].

The upshot is that the statistical uncertainties obtained here are nearly as small as can be useful for exoplanet detection, unless one controls for the effects of jitter.

VI) Orbital motion of τ Boötes A and the mass of its exoplanet

In order to detect an exoplanet, one looks for a periodic variation in the RV of the host star. With enough measurements, one can find the orbital period, eccentricity, semi-major axis (projected along the line of sight), and the maximum orbital RV of the host star [18], [19].

Given the limited number and precision of the measurements obtained here, a more modest validation will be considered, by taking the period and eccentricity of the orbit as inputs from previous professional studies (see Ref. [8] for a compilation of results). A fit to the data will be used to determine the maximum radial velocity, usually denoted by the symbol K, and the result will be compared with the accepted value of (470 ± 15) m/s. The accepted orbital period is 3.31249(3) days, and the accepted eccentricity is 0.08 ± 0.03, which is small enough to ignore within the accuracy of this study.

RV data is most usefully plotted against the orbital phase, which is defined by

\[ \text{phase} = (t_{\text{obs}} - t_{\text{ref}}) / \text{period} \]  

where \( t_{\text{obs}} \) is the time of an observation, and \( t_{\text{ref}} \) is an arbitrary fixed reference time. In the following, the reference time will be specified by an instant \( t_{\text{max}} \) at which the RV has its maximum value.

The time that it takes for light to travel from a target object to the Earth depends on the Earth’s position in its orbit, and a correction for this effect should be made when computing the phase. For this purpose, time is conventionally specified in terms of the Heliocentric Julian Date.
(HJD), which is the Julian Date corrected for the difference between the light travel time from the target object to the Sun, and to the Earth. Conversions to HJD can be done with a number of utilities [6], [14]. A published value for an epoch of maximum radial velocity for τ Boötis A is \( t_{\text{max}} = (2450235.4 \pm 0.2) \) HJD [9].

For a circular orbit, the radial velocity will follow a simple cosine curve, given by

\[
RV = K \cos(\text{phase}) + K_0
\]  

(9)

where \( K \) is the maximum orbital radial velocity, and \( K_0 \) is an additive constant related to the overall translational motion of the system. Since the RV of τ Boötis has here been measured relative to that of 45 Boötis, \( K_0 \) should be equal to the difference in the translational RVs of the two systems. Previous measurements of systemic RVs can be found from a huge on-line database of astronomical data known as SIMBAD [28], from which an accepted value of \( K_0 = (-4.8 \pm 1.2) \) km/sec can be extracted.

The values of the three constants \( K, K_0, \) and \( t_{\text{ref}} \) in Eq. (9) can be estimated by fitting the cosine curve to the data. A standard least-squares fit to the data in Table 1 produces the following values

\[
K = (425 \pm 18) \text{ m/sec} \\
K_0 = (-7.13 \pm 0.14) \text{ km/sec} \\
t_{\text{ref}} = t_{\text{max}} = (2450235.55 \pm 0.02) \text{ HJD}
\]

(10)

which are all in agreement with the accepted values, within the uncertainties.

The measured radial velocities and the fitted cosine curve are plotted in Figure 16, with the overall additive constant \( K_0 \) subtracted from the data points, and removed from the fit curve, for convenience. In order to get a visual impression of the agreement between the data and the accepted value of the maximum orbital RV, the cosine curve of Eq. (9) is also plotted using the accepted central value of 0.47 km/sec for \( K \), and using a value of 0.44 km/sec, which is two standard deviations below the accepted central value. As is already evident from the results in Eq. (10), the agreement between the data and the known RV is very good.

Having determined the orbital RV of the host star, one can set a lower bound on the exoplanet’s mass; the actual mass can only be found if the angle of inclination \( i \) of the orbital plane relative to our line of sight is known, since the radial component of the orbital velocity will always be less than the true velocity, unless it happens that \( i=90^\circ \).

For the special case of a circular orbit of an exoplanet with a mass \( M_p \) that is much less than the host star’s mass \( M_\star \), the following formula applies [18], [19]

\[
K = 28.4 \text{ m/s} \cdot \left( \frac{P}{1 \text{ yr}} \right)^{-\frac{1}{3}} \left( \frac{M_\star}{M_\odot} \right)^{-\frac{2}{3}} \left( \frac{M_p \sin i}{M_J} \right)
\]

(11)

where \( P \) is the orbital period, \( M_\odot \) is the mass of the Sun, and \( M_J \) is the mass of Jupiter. It is instructive to note that in the case of our solar system, Jupiter induces an orbital speed in the Sun of only 12.5 m/s, about the same as a car turning a corner, while the Earth generates a tiny speed of 9 cm/s, only a few times faster than a garden snail moving at top speed!

Equation (11) can be solved for the mass of the exoplanet, given the host star’s mass and its orbital RV. In the case of τ Boötis b, the host star mass is about 1.3\( M_\odot \), yielding \( M_p \sin i \sim 4M_J \), with an uncertainty of about 10% [8].

**VII) Conclusions**

The exoplanet τ Boötis b was detected from measurements of the radial velocity of its host star, τ Boötis A, at Simon Fraser University’s teaching and outreach observatory. The spectrograph used is a commercially manufactured, off-the-shelf unit of echelle design, and is available at a cost that is comparable to high-end imaging camera systems. Although τ Boötis A has one of the largest orbital RVs of any known host, this was a very challenging study, requiring Doppler shifts of the star’s absorption spectrum to be measured to 1 part in a few million. Measurements were made on ten nights, which covered most of the orbital cycle, and a fit to the data gave a value for the
maximum orbital RV in agreement with the known value, to within about 10%.

Figure 16: Orbital radial velocity of τ Boötis A versus its phase. Each data point is labelled by the date of the observation, and the horizontal error bar expresses the amount of time over which data was collected, typically between about 3 and 5 hours. The solid line is a best fit to the data for a circular orbit. The dashed and dotted curves illustrate the uncertainty in the accepted value of the orbital RV, as described in the text.

This study was done in connection with a course in observational astronomy that is under development at SFU, that will be offered as a breadth course for students from the arts and sciences. While exoplanet detection may only be suitable for more advanced students, there are many interesting astrophysical systems that can be studied by spectroscopy with far less effort, and in many cases interesting quantitative results can be extracted with simple analysis methods and only a basic understanding of the relevant astrophysics. These kinds of studies can provide educational experiences that closely emulate the practice and excitement of cutting-edge science.

Only a few other exoplanets can be detected using spectrographs that are likely to accessible at a campus teaching observatory or by amateur astronomers. One particularly interesting case is the star HD 189733 A; it has a transiting hot Jupiter that generates an easily-measured light curve, and that produces an orbital RV in the host star that can be detected with the spectrograph used in this study. This would be a particularly exciting project for students, who could measure both the size and mass of the exoplanet, and thereby determine its density, confirming for themselves that it is likely to be a gas giant. An exploratory study of this system is under way at the SFU observatory.
Appendix: Rudimentary principles of an echelle spectrometer

Most spectrometers, from cheap to state-of-the-art, are made using diffraction gratings as the principle dispersive element. Diffraction gratings consist of reflective or transparent surfaces that are etched with many parallel grooves, as illustrated in Figure 17 in the case of a reflection grating. The intensity of the reflected light is reinforced in specific directions, and vanishes in others, resulting in a so-called interference or fringe pattern, as seen when white light is reflected from DVDs and other compact disks, producing a spectrum of colours.

![Figure 17: Reflection of two incident light rays from a diffraction grating. The small rounded dips represent non-reflective etchings in the surface.](image)

To understand the origin of this behavior, first consider that the distances travelled to the reflecting surfaces by neighbouring incident light rays will generally differ by some amount \( d_{\text{inc}} \), and similarly the distances travelled after reflection will differ by some amount \( d_{\text{ref}} \), as shown in Figure 17. If the overall path difference \( d_{\text{ref}} - d_{\text{inc}} \) (which varies with the angles of incidence and reflection) is a whole number of wavelengths \( \lambda \), then the reflected waves will add constructively, to produce a net reflected wave of maximum intensity, as illustrated in Figure 18. If, on the other, the path difference equals a “half-integer” number of wavelengths, then the reflected waves will cancel each other out completely, also as illustrated in Figure 18.

The so-called grating equation expresses the condition for constructive interference (see e.g. Ref. [1])

\[
d_{\text{ref}} - d_{\text{inc}} = a (\sin \theta_{\text{ref}} - \sin \theta_{\text{inc}})
\]

\[
= n\lambda, \quad n = 0, 1, 2, \ldots
\]

Maxima and minima in the intensities of different wavelengths occur at different reflection angles \( \theta_{\text{ref}} \) (for a given angle of incidence \( \theta_{\text{inc}} \)), producing the characteristic spread of colours. Successive interference bands are referred to by the order in the sequence, i.e. by the number \( n \) in the grating equation.

A key property of diffraction gratings is that the spread, or dispersion, in wavelength increases with the order, as illustrated in Figure 19. This means that to maximize the resolving power of the grating (that is, to split apart neighbouring spectral features to the greatest extent possible), one would like to use the reflected light from the highest possible order.

![Figure 18: Examples of constructive and destructive interference (graphic adapted from a Creative Commons source [29]).](image)

However, there are two critical drawbacks with using ordinary diffraction gratings.

One drawback is that most of the light energy is concentrated in the zeroth-order fringe \((n = 0)\), where all wavelengths emerge at the same angle, and the intensity also drops rapidly at higher orders. It is useful to note that at zeroth order the angles of incidence and reflection are equal, a situation known as specular reflection; this is the
familiar situation encountered when light is reflected from a large mirror.

The second drawback with ordinary diffraction gratings is illustrated in Figure 19: light of a given wavelength in one order is overlapped by shorter wavelength light from higher orders. In practice, there will be a range of wavelengths at a given order that does not suffer from an overlap, which is referred to as the free spectral range; this occurs because any detector that records the spectrum will be sensitive only to a limited range of wavelengths. The free spectral range is illustrated in Figure 19 in the case of a sensor that detects mainly optical light, such as a typical CCD chip. Unfortunately, the free spectral range decreases with increasing order, as can also be seen in Figure 19.

![Figure 19: Dispersion in reflected light from the first three orders of a diffraction grating.](image)

Echelle spectrometers use a two-part strategy to alleviate these drawbacks.

One part of the strategy is to use a so-called “blazed” reflection grating, where the reflecting facets are inclined at an angle with respect to the mounting surface, as shown in Figure 20. The bias in favour of specular reflection from the inclined facets, combined with interference due to the underlying grating pattern, concentrates the reflected energy at higher orders. It turns out that most of the energy in a particular wavelength emerges in one or two high-order diffraction bands, as suggested in Figure 20 by the reflected red ray (in the case of the eShel, for example, the red Hα line is near the middle of order 34).

![Figure 20: Illustration of a blazed grating. “Blaze normal” identifies the direction perpendicular to the reflecting facets, while “Grating normal” identifies the direction perpendicular to the grating substrate.](image)

To deal with the overlap of wavelengths from different orders, another dispersive element is introduced, known as a cross-disperser. The cross-disperser is used to spread apart the light that emerges from the primary blazed grating, in a direction perpendicular to the initial dispersion, as illustrated in Figure 21; the cross-disperser can be a low-resolution reflection grating, as in the
diagram, or a prism (which is the case with the eShel).

As a result of the cross-disperser, the spectrum produce by an echelle consists of rows of dispersed light, as depicted in Figure 21, and in an actual image taken with an eShel, shown in Figure 4 on page 5. Each row in the output corresponds to the spectrum of one order produced by the blazed grating.

As seen in Figure 4, the intensity of light is not uniformly distributed across the rungs of an echelle, with the reflected energy at each order concentrated in a specific wavelength of light. In the case of a prism cross-disperser, the bands are curved, since the index of refraction in glass varies with wavelength.

References


[28] SIMBAD Astronomical Database.  
http://simbad.u-strasbg.fr/simbad/.

[29] Haade (2010). Interference of Two Waves,  

Endnotes

1 Interestingly, beginning in fall 2017 the disk of VV Cephei B and the star itself will undergo an eclipse by its giant partner VV Cephei A, and the progress of the eclipse can be monitored with amateur spectrographs. Information on a pro-am campaign to monitor the eclipse can be found in Ref. [7].

2 The CDK700 is usually installed at ground level, however at SFU we mounted it on a pedestal in order to minimize the observatory's footprint. The unusual device seen in Figure 3, hanging from the telescope's visual observing port, effectively acts as a “periscope”. It is comprised of two Televue NP101s, connected at their objectives, and is the elegant means by which eyeballs are brought to the eyepiece. The periscope was designed by Nick Seiflow, owner of the Vancouver Telescope Centre, in consultation with Al Nagler of Televue, and its components were manufactured by PlaneWave Instruments.

3 To compute the response function, it is usual to first interpolate over deep absorption lines in the spectra, and to apply a low-pass filter, to eliminate fine features in the spectra that are unrelated to the smooth instrumental response. Early A-class stars are commonly used, since they have relatively few absorption lines. Note that the response function obtained in this way depends on the altitude at which the comparison star is observed, due to differential absorption of light of different wavelengths by the Earth’s atmosphere, and is not solely a property of the instrument. Note also that the overall scale of the corrected spectrum is arbitrary, and the result is a plot of relative intensity versus wavelength.

4 Equation (2) is an approximation that is valid for source speeds much smaller than the speed of light.

5 Although routines to evaluate cross-correlation functions are available in the analysis packages used by professionals [14, 15], and also in the more user-friendly package written by Buil [6] (though not in a very convenient form), I wrote my own.

6 Rectified spectral functions are used, since the smooth continuum behaviour in a star’s spectrum is very insensitive to a Doppler shift. Also, an overall constant is subtracted from the spectral functions, so that the resulting averages are zero, \( \langle i_{1,2} \rangle = 0 \), since overall shifts in the rectified spectra are of no physical consequence.

7 The absorption lines in the spectrum of 45 Boötis are wider and shallower than those of τ Boötis, which indicates that 45 Boötis has a much larger rotational speed. This was not anticipated, and in retrospect another choice of reference star might have been preferred.

8 A number of techniques for dealing with stellar jitter have been studied by professionals; see e.g. Refs. [18] and [19].

9 The chi-squared measure of the goodness-of-fit is about 1.5 per degree-of-freedom, indicating a reasonably reliable fit; this provides a posteriori evidence that that the statistical errors have been reliably estimated.

10 The orbital angle of inclination has been inferred from measurements of the RV of the exoplanet itself, from absorption lines in its atmosphere, and is about 45° [11], which implies that the exoplanet mass is approximately 6 times the mass of Jupiter.

11 For example, the \( n = 1 \) maximum for a given wavelength \( \lambda \) overlaps with the \( n = 2 \) maximum for wavelength \( \lambda/2 \).