Experience Rating: Insurance Versus Efficiency

by

Steeve Mongrain
Department of Economics
Simon Fraser University

Abstract

When unemployment insurance is publicly provided, firms' layoff decisions can be distorted. Unemployment insurance reduces the cost of laying off workers, thereby encouraging layoffs and leading to more unemployment. To dampen this increase in unemployment, it has been suggested that unemployment insurance should be financed with an experience rated tax. This paper examines the possibility that, despite that increasing the level of experience rating can reduce the level of unemployment, it can also reduce the wealth of unemployed workers. The reason is that, under high level of experience rating, firms may reduce their severance payments by more than the publicly provided unemployment insurance benefit.

We build a model where competitive firms offer long-term contracts to risk-averse workers. Asymmetric information about worker's productivity leads to over unemployment and incomplete private insurance against unemployment. This paper shows that an experience rated unemployment insurance program cannot increase the wealth of unemployed workers without increasing unemployment.

Key Words: Unemployment Insurance, Experience Rating, Layoffs

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* Corresponding author: Steeve Mongrain, Department of Economics, Simon Fraser University, 8888 University Drive, Burnaby, BC, Canada, V5A 1S6, e-mail: mongrain@sfu.ca, Phone: (604) 291-3547, Fax: (604) 291-5944. I would like to thank Robin Boadway, Gregory Caldwell, Steve Easton, Patrick Francois, Nicolas Marceau, Joanne Roberts, Tim Sargent, John Leach and Jano Zabojnik for useful comments. I would also like to thank SSHRC for financial support. All errors are mine.
1. Introduction.

The primary objective of unemployment insurance is to provide coverage to workers in the presence of risky future employment opportunities. However, implementing unemployment insurance introduces adverse distortions in the economy. One distortion is that unemployment insurance reduces the cost of laying off workers, and consequently firms increase the number of layoffs leading to more unemployment. One instrument to dampen or eliminate this perverse effect is experience rating in the way unemployment insurance is financed. In the presence of experience rating, firms have to pay a tax proportional to the total cost they are imposing on the unemployment insurance program from their layoff decisions. Without experience rating, unemployment insurance acts like a subsidy to firms for laying off workers. However, experience rating increases the cost of laying off workers and can counterbalance the negative distortion introduced by unemployment insurance. With perfect experience rating, Topel and Welch (1980) have demonstrated that firms have to pay the full cost of unemployment insurance, and subsequently, there is no increase in unemployment when the government introduces such a program. One important question that has been overlooked in the literature is how experience rating affect the government’s ability to better insure workers against unemployment. This paper shows that to better insure workers, a government will consequently increase the level of unemployment. Further, full experience rating is non-distortionary and yet does not increase the welfare of unemployed workers.

In practice, experience rating is not prevalent. The majority of countries only use payroll taxes to finance their unemployment insurance programs. Even countries like the United States which use experience rating, only use it partially. An important question is: if experience rating is efficient, why is it not a more prominent feature in unemployment insurance programs? Marceau (1993) and Burdett and Wright (1989) show that increasing the level of experience rating can lead to more unemployment. Under Cournot competition and free entry, or variable firm size, increasing the level of experience rating can increase unemployment by reducing the number of firms (or firm size).

This paper highlights another un-desirable effect of experience rating. When experience
rating increases, the unemployment insurance program loses the ability to increase the welfare of unemployed workers. Public insurance crowds out private insurance. With high levels of experience rating, some workers can be hurt \textit{ex post} by an increase in unemployment insurance because private insurance diminishes by more than public benefit.

In this model firms offer long-term contracts to risk averse workers. Firms have no prior information about the ability of workers hired. However, once working, firms learn their workers’ abilities. At some point, firms must decide whether to retain or layoff each worker on the basis of their observed quality. In this environment, all layoffs will be permanent. Oswald (1986) notes that from 1973-1976 in the United States, 40\% of layoffs in the manufacturing sector were permanent. More recently this proportion has fallen, but permanent layoffs are still an important source of unemployment in the manufacturing sector. The proportion of permanent layoffs is much larger in a sector less affected by seasonal or demand fluctuations. Because workers are risk-averse, firms provide long-term insurance contracts to workers by using severance payments. In reality, severance payments are only paid for permanent layoffs. Half of the workers in manufacturing (one quarter in non-manufacturing) receive severance payments. The introduction of unemployment insurance will allow firms to reduce their severance payments and consequently reduce the private cost of laying off a worker.

When a firm lays off workers, other firms infer the average ability of both the fired workers and the retained workers. Laid-off workers are tagged as low-ability and receive lower wage offers. If this wage offer is too low, these workers prefer to remain unemployed. This happens to unemployed workers with abilities above the average. Since they would be willing to accept a job paying their marginal productivity, a portion of unemployment can be somehow considered as involuntary. Gibbons and Katz (1991) show that post-displacement wage offers are lower for workers displaced by layoffs than for those displaced by plant closings. This suggests that there is important information contained in the layoff decisions of firms. As noted above, firms also infer the average ability of retained workers and subsequently make wage offers to attract them from their current employer. This implies that firms will have to make an initial wage offer to the workers they want to keep which is large enough to ensure loyalty. This constraint on wages prevent firms from being
able to perfectly insure workers.

In a similar framework Waldman (1984) shows that when a worker’s ability is the private information of the present employer, but other firms are able to observe promotions, firms will tend to promote too few workers. The reason is that a promoted worker has to be paid according to her inferred average ability, while firms make their promotions according to the marginal ability of the worker. Similarly we find that firms will lay off too many workers as compared to the efficient allocation. Workers with reservation wages which are less than their productivities will end up being unemployed. Laing (1994) has also examined the impact of asymmetric information on firms’ layoff decisions. He has shown that there are too many layoffs and that seniority rules could eliminate this distortion. Seniority rules eliminate some firm discretion in making layoffs, and make it harder for firms to infer the ability of workers after a layoff decision has been made. The American Bureau of Labor Statistic stated that in 1997 only 14% of American workers were unionized, suggesting that there is still a large amount of discretion in layoff decisions.

This paper focuses on the impact of introducing an unemployment insurance program in a similar environment that the one describe in Gibbons and Katz (1991). Since firms do not offer full insurance, government may want to introduce public unemployment insurance. On the other hand, a “laissez-faire” economy results in too many layoffs as compared to the efficient allocation so that the government has an incentive to reduce unemployment. We will show that an experience rated unemployment insurance system cannot achieve both objectives at the same time.

In the next section, we present the basic model. Then, we depict the sequences of events governing firms’ decisions. The characterization of the equilibrium is done in section 4. In section 5, we look at the impact of government intervention. Finally we provide some concluding remarks. All proofs are included in the appendix.

2. The Model

The economy is composed of a measure of $N$ workers, indexed by ability, born in each
period. Workers live for two periods. We call the workers young in the first period of life, and old in the second period. The ability of a worker is given by \( \theta \), where \( \theta \) is distributed uniformly on \([\theta^\ell, \theta^h]\).\(^1\) Workers are risk averse and do not discount the future. The per period, twice continuously differentiable, utility function is given by \( U = U(W + R) \) where \( W \) is the outside income and \( R \) is home production. This home production takes a value of zero if the worker is working, and a positive value of \( r \) if the worker is unemployed. It is also assumed that \( U'(W + R) \geq 0, U''(W + R) \leq 0 \) and \( U'''(W + R) \geq 0 \).\(^2\) Each worker supplies one unit of work inelastically to the firm that offers the highest wage. It is assumed that if wage offers are the same, workers prefer to stay with their current firm.

This economy is also composed of \( N \) identical firms that live forever.\(^3\) Firms do not discount the future, take prices as given and are risk neutral. The output price is normalized to one. It is assumed that firms have full discretion over the layoff decisions. If in any period, a firm is not able to attract new workers, the firm goes bankrupt and has to suffer a bankruptcy cost of \( C \).\(^4\) The productivity of a worker depends on that worker’s ability level, and is simply equal to \( \theta \). All firms know the distribution of \( \theta \). We assume that \( \theta^\ell < r < \theta^h \), and that \( \frac{\theta^\ell + \theta^h}{2} > r \). That is, some workers are more productive at home than at work but, on average, a worker is more productive at work.

The ability level of a young worker is not observable by firms or the worker himself.\(^5\) After the first period, both the firm and the worker learn the value of \( \theta \). No other firm learns about the ability level of the worker. This reflects the fact that firms have better

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1 For simplicity we assume that \( \theta^h - \theta^\ell = 1 \). However, this assumption does not change the nature of the results.

2 Assumptions about the third derivative of utility function are uncommon. Later on, this assumption will be used to derive a particular sufficient condition for some results to hold. It will be mentioned later that even if \( U'''() < 0 \) (but small) the results will still hold. Moreover, most common utility functions have this feature.

3 The fact that the number of firms is equal to the measure of workers ensures that in equilibrium, there is a measure of one worker per firm.

4 This cost can be interpreted in various ways. It could be legal costs. It can also be interpreted as costs related to bad credit records. Finally, it can be manager’s disutility from having to find a new job. Boadway and Marceau (1995) use the same kind of assumption.

5 This implies that separating contracts where firms offer a menu of contracts will not be possible.
information about their own workers than do other firms. The ability level of a worker is always unverifiable to a third party, so contracts with wages contingent on $\theta$ are not enforceable. Which party initiates separation is also not observable outside the firm. This is assumed since either party can take actions which induce the other party to initiate separation. This implies that contracts involving separation payments are not enforceable. However, workers in a firm are able to observe the reason for separation. By reputation, young workers know if, in the past, a firm agreed to pay a severance payment, and did not fulfill its obligation. Since firms need to hire new workers so they do not have to shutdown, reputation will ensure that in equilibrium, firms pay severance payments. On the other hand, workers who live for only two periods, are not able to commit themselves to staying with their present firm if they receive a better wage somewhere else.

3. Sequence of Events

The OLG framework implies that the sequence of events differs from the viewpoint of firms and workers. Workers are young in the first period and old in the second period. On the other hand, firms live forever. At time zero, firms have to choose between making an investment in capital or not. At the beginning of the first period, firms offer a long-term contract to young workers. Let $w_y$ be the wage offer to a young worker, $w_o$ be the wage offer to an old worker, and $s$ the severance payment offered to workers in the event they are fired. Workers then choose to work for the firm that offers the best contract. Production then takes place and wages are paid. Once the production phase has ended, the worker and the firm learn the ability level $\theta$. Finally, firms that cannot hire new workers shutdown and incur a bankruptcy cost $C$.

Now, consider the sequence of event for the second period. In the second period, firms know the ability level of all their old workers. The wage $w_o$ and the severance payment are already determined from the contract signed in the previous period. Although firms are

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6 Macleod and Malcomson (1989) and Carmichael (1983) used the same type of argument.
7 Workers in a firm are able to see the difference between a worker who quits voluntary and another one who quits because the firm pressured the worker to leave.
8 If all firms offer the same contract, workers will distribute themselves evenly among all firms.
not legally constrained to make severance payments, reputations will ensure they are paid. First, firms choose which workers to fire and choose to pay severance payments or not. If a firm chooses not to pay severance payments to their workers, no more young workers will want to work for that firm. Workers that are fired join the pool of unemployed, and can try to find a new job. Next, firms offer long-term contracts to the new generation of young workers. Firms also make a wage offer to unemployed workers. Firms are able to infer the average ability of a fired worker using the information contained in the layoff decisions and consequently make a wage offer \( w'_o \). Then unemployed workers choose between taking a new job or staying unemployed. Firms also try to attract workers that were not fired from other firms. Since workers die at the end of this period, there are no concerns about reputation that prevent workers from changing firms. Consequently, the wage \( w_o \) offered in the contract has to be generous enough to keep workers from leaving in the second period spot market. Finally, production take place, wages and cost of capital are paid, and the second period ends with bankruptcy of firms that do not pay their user cost of capital. The sequence is then respected for all generations.

In this paper, we concentrate on the steady state equilibrium. The fact that contracts offered to one generation of workers are not affected by the interaction with any other generation of workers makes this steady state equilibrium easy to compute.

4. Equilibrium Without Unemployment Insurance

4.1. Case Where Severance Payments Are Enforceable

First, we consider an equilibrium without public intervention. To begin, we analyze an equilibrium when workers can commit themselves to stay at the same firm despite the fact that they may receive a better offer from other firms. If severance payments are enforceable by a court, contracts can involve large fines when a worker leaves a firm without being fired. Firms compete with one another to attract young workers by offering long-term contracts \( \{w_y, w_o, s\} \) that maximize the lifetime expected utility of a young worker subject

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9 Young worker are able to identify instantaneously firms who do not pay severance payments. Allowing for a delay will not change the nature of the results.
to their constraints.

Firms will make a better offer to workers as long as they receive positive profits. Competition drives profit to zero, so the zero profit constraint will always be strictly satisfied. Let \( \bar{\theta} \) denote the worker’s ability level for which the firm is indifferent between keeping or firing her. To find the firms’ expected profits, we first solve for \( \bar{\theta} \). The benefit of keeping a worker with ability \( \theta \) is \( \theta - w_o \). On the other hand, the cost of firing the worker is \( s \). Firms will keep all old workers for which \( \theta \geq w_o - s \) so \( \bar{\theta} = w_o - s \). The firm’s expected operating profits from a young worker are:

\[
E[\Pi] = \frac{\theta^\ell + \theta^h}{2} - w_y + [\bar{\theta} - \theta^\ell](-s) + [\theta^h - \bar{\theta}] \left( \frac{\theta + \theta^h}{2} - w_o \right). \tag{1}
\]

However, the firms need to make operating profits large enough to cover the user cost of capital. It implies that the zero profit condition is given by \( E[\pi] \geq 0 \).

As mentioned earlier, firms offer contracts that maximize the expected utility of young workers. Let \( E[U] \) be the expected utility of a young worker, which is a function of the contract \( \{w_y, w_o, s\} \). Specifically:

\[
E[U] = U(w_y) + [\bar{\theta} - \theta^\ell]U(r + s) + [\theta^h - \bar{\theta}]U(w_o). \tag{2}
\]

The contract offered to young workers solves the following problem:

\[
\max_{w_y, w_o, s} \left\{ U(w_y) + [w_o - s - \theta^\ell]U(r + s) + [\theta^h - w_o + s]U(w_o) \right\},
\]

subject to:

\[
\frac{\theta^\ell + \theta^h}{2} - w_y + [w_o - s - \theta^\ell](-s) + [\theta^h - w_o + s] \left( \frac{w_o - s + \theta^h}{2} - w_o \right) \geq 0.
\]

\(^{10}\) Since there is no population growth and no discounting, the per period profits and the profits from one generation of workers are the same.
and $s \geq 0$

In other words, firms maximize the expected utility function of a young worker subject to the zero-profit condition, and a non-negativity constraint on severance payments.

**Proposition 1:** When severance payments are enforceable, firms are fully smoothing workers’ utility across periods and are fully insuring workers again unemployment.

Since workers can commit to stay at the same firm, firms only face the zero-profits constraint when they determine the wage of those old worker. Because workers are risk averse, firms equalize utility of workers across periods and across employment status. This implies that $w_y = w_o = s + r$.

**Corollary 1:** Since firms are fully insuring workers, layoffs decisions are efficient in the sense that $\bar{\theta} = r$.

Under perfect insurance, the layoffs decisions are efficient for the following reason. The last old worker to be kept is the one for which the firm is indifferent between firing or keeping her. The cost of firing her is the foregone production $\theta$, while the benefit is the difference between the wage $w_o$ and the severance payment $s$. Because workers are fully insured, the difference is simply $r$, so it implies that $\bar{\theta} = r$.

When severance payments are enforceable, workers can commit to stay at the same firm. Firms offer a full insurance contract to young risk averse workers. Under this full insurance contract, workers who are more productive at home are laid off and workers that are more productive at work are retained. Consequently, there is no reason for government intervention.
4.2. Case Where Severance Payments Are Not Enforceable

Now consider an equilibrium when severance payments are not enforceable in court. Again, firms compete with one another to attract young workers by offering long-term contracts. Firms offer a contract \( \{w_y, w_o, s\} \) that maximizes the lifetime expected utility of a young worker subject to their constraints. Since severance payments are not enforceable in court, and workers live for only two periods, the first constraint is \( s \geq 0 \).

A second constraint requires that the wage offer to old workers be large enough to ensure second period loyalty. Once a firm has fired some old workers, other firms acquire some information about the quality of the workers that the firm keeps. Firms can now make an offer to workers that were not fired. With competition among firms, we know that other firms will offer a wage equal to the average productivity of a retained worker. Let \( \tilde{\theta} \) denote the worker’s ability level for which the firm is indifferent between keeping or firing her. Since all workers with an ability level greater than or equal to \( \tilde{\theta} \) are kept, the average ability of these workers is given by \( \frac{\theta + \theta^h}{2} \). If firms wish to retain workers with ability greater than or equal to \( \tilde{\theta} \) they must set \( w_o \geq \frac{\theta + \theta^h}{2} \), the average ability of remaining workers.

Finally, firms will make a better offer to workers as long as the firm receives positive profits. Here again, the zero-profit constraint will always be strictly satisfied. As in the previous section, firms fire all workers for which \( \theta < \tilde{\theta} \) where \( \tilde{\theta} = w_o - s \). The firm’s expected profits from a young worker are given in (1).

Before looking at the contract offered to young workers, we first consider the problem of old workers who are fired. Workers who are fired receive an offer \( w'_o \) from other firms. From competition, we know that the offer made corresponds to the worker’s average ability conditional on being fired. This implies that \( w'_o = \frac{\theta + \theta^h}{2} \). If \( w'_o > r \), there is no unemployment because all fired workers accept an offer from another firm. Since the objective of this paper is to look at the impact of unemployment insurance in this type of environment, we focus on an equilibrium in which \( w'_o \leq r \).

The contract that will be offered to young workers solves the following problem:
\[
\max_{w_y, s} \left\{ U(w_y) + [w_o - s - \theta^\ell]U(r + s) + [\theta^h - w_o + s]U(w_o) \right\},
\]
subject to:

\[
\frac{\theta^\ell + \theta^h}{2} - w_y + [w_o - s - \theta^\ell](-s) + [\theta^h - w_o + s] \left[ \frac{w_o - s + \theta^h}{2} - w_o \right] = 0,
\]

\[w_o \geq \theta^h - s\] and \[s \geq 0\].

In other words, firms maximize the expected utility function of a young worker subject to the zero-profit condition, the second period wage restriction, and a non-negativity constraint on severance payments.

**Proposition 2:** There exists a long-term contract \(\{w^*_y, w^*_o, s^*\}\) which is a function of \(\{r, \theta^h, \theta^\ell\}\) that solves the firm’s problem.

Looking at the first order condition of this problem, one observation is that the constraint on the wage of an old worker is binding. The first order condition with respect to \(w_o\) is strictly negative, so that \(w_o = \theta^h - s\). Using this result, we find the cut-off level of \(\bar{\theta}\), by solving for the ability level of a worker that makes a firm indifferent between firing her or not. If \(\bar{\theta}\) is smaller than \(\theta^\ell\), the firm simply keeps all workers. Since the objective of this paper is to analyze UI system in this particular environment, we will restrict ourself to cases where some workers are laid-off, which corresponds to \(\bar{\theta} > \theta^\ell\). Firms fire all workers with \(\theta \leq \bar{\theta}\), where \(\bar{\theta} = \theta^h - 2s^*\).

Now that we have solved for the contract offered to young workers, we will verify that firms have an incentive to pay the severance payments to the laid-off workers. If a firm does not pay the severance payment to all workers, the firm gains \([\bar{\theta} - \theta^\ell]s^*\). However, when a firm does not pay the severance payment to fired workers, no more young workers choose to work for that firm. Consequently, the firm will go bankrupt after the end of the period. The cost of not paying severance payments is the bankruptcy cost \(C\). As long as \(C \geq [\bar{\theta} - \theta^\ell]s^*\), firms will prefer paying severance payments to workers they fire. For the
rest of the paper, it is assumed that the bankruptcy cost is large enough to ensure that firms have an incentive to pay severance payments in equilibrium. We will now study the properties of the contract \( \{w_y^*, w_o^*, s^*\} \).

**Lemma 1:** Firms are not able to fully smooth workers’ utility across periods and fully insure workers as long there exists some workers who are more productive at home.

The intuition behind Lemma 1 is the following. The only way that the firms are able to satisfy the zero-profit condition and simultaneously allow workers to receive the same level of utility in both periods, is if they do not fire any old workers, \( \bar{\theta} = \theta^\ell \). When firms do not fire any workers, the wage offered to old workers is \( \frac{\theta^\ell + \theta^h}{2} \). Firms can smooth workers’ utility by offering \( w_y = w_o = \frac{\theta^\ell + \theta^h}{2} - \frac{k}{2} \) and \( s = \frac{\theta^\ell + \theta^h}{2} - \frac{k}{2} - r \) and make zero profits since they never have to pay severance payments. However, the problem with this contract is that if \( r > \theta^\ell \), firms prefer to fire all workers with ability level \( \theta < r \). The reason is that if they keep one of these low-ability workers, they have to pay \( w_o \) and the worker only produces \( \theta < r \). However, if the firm fires these workers, it only has to pay \( w_o - r \). Firms will prefer to fire workers with \( \theta < r \) and not have to pay the value of the home production, required for keeping these workers. Since we already made the assumption that some workers are more productive at home, we know that perfect smoothing of utility across periods is not possible.

An implication which we state in Proposition 3 is that \( w_o^* > w_y^* \). This implies that the model predicts an increasing wage profile for retained workers as usually observed in the data.

**Proposition 3:** The equilibrium long-term contract schedule does not fully insure old workers, and the wage of a young workers is less than the total compensation of workers who are laid off, \( w_y^* < r + s^* < w_o^* \).

We already know that perfect smoothing is not possible. Proposition 3 shows that although perfect insurance for old workers is not an outcome in equilibrium, they are still guaranteed to be better off than young workers. In order better to understand this, we should examine the firm’s optimality condition. Competitive wages for young workers and
optimal severance payments are determined by equalizing the marginal utility of wealth when young with the expected marginal utility of next period’s wealth. A decrease in $w_y$ induces an increase in marginal utility when young. On the other hand, an increase in $s$ has three different effects. First, since severance payments increase, the marginal expected utility of wealth when old decreases due to a decrease in the marginal utility of being fired, “ceteris paribus”. Secondly, when $s$ increases, firms fire fewer workers, so the average ability of retained workers decreases, leading to a reduction in $w_o$. Since the wage given to old workers decreases, the marginal expected utility when old increases. Finally, since $s$ increases, the probability that an old worker will be fired decreases, so the expected utility when old increases. These opposing effects lead to an equilibrium long-term contract where young workers have lower utility than the old unemployed workers. It is cheaper for the firm to give workers higher utility in the old state.

**Corollary 2:** Since the firms do not fully insure old workers, the necessary conditions for unemployment in equilibrium are satisfied.

This ensures that in this type of equilibrium, unemployment can exist. Unemployment will exist in this economy only if $w'_o = \frac{\bar{\theta} + \theta^e}{2} \leq r$. Another way to state this condition is that there will be unemployment if $s^* > \frac{\theta^e + \theta^h}{2} - r$, which will be satisfied as long as $r$ is large enough. When the value of home production is small, unemployed workers will accept any new jobs. However, when $r$ increases, the average ability of the unemployed workers is much smaller than their home productivity, and thus, they prefer being unemployed. Although this is true on average, some of those unemployed workers would prefer to work if they were able to obtain a wage that corresponds to their ability. Since we are interested in the effects of unemployment insurance, we restrict ourselves to equilibria where this condition is satisfied.

**Proposition 4:** Since firms do not fully insure old workers, and given some workers are more productive at home, there will be too many layoffs compared to the efficient

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11 This condition is always satisfied when $r = \frac{\theta^e + \theta^h}{2}$ but never satisfied when $r = 0$. It is possible to find an $\bar{r}$ for which this condition will be satisfied because as we will see later, $s^* + r$ is increasing with $r$. 

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allocation.

It is interesting to see that competition introduces a distortion in equilibrium layoffs decisions. This distortion comes from the informational externality. Asymmetric information creates a rent to the incumbent firm because the incumbent firm has better information about the productivity of their workforce. By bidding away workers, firms do not care that they make it harder for other firms to ensure second period loyalty. In contrast, in section 4.1, there is no such externality because workers can commit themselves to stay at the same firm. Propositions 3 and 4 have important implications for government intervention. First, when firms do not fully insure their workers, there is too much unemployment in the economy. This leaves room for government intervention to try to lower the level of unemployment. Moreover, the fact that old workers are not fully insured against unemployment can increase political incentives for a publicly provided unemployment insurance. Before considering the implications of government intervention, we look at some comparative statics.

**Proposition 5:** Since firms do not fully insure old workers, severance payments will be decreasing with $r$, and given that workers do not exhibit too much decreasing absolute risk aversion, severance payments will be decreasing in a proportion less than one.

The reason why raising $r$ leads to reductions in severance payments is fairly intuitive and is a common feature in the literature. A more interesting result is the fact that there is not perfect crowding out of the severance payments. First, let us see what would happen if old workers were fully insured in equilibrium. We know from Proposition 3, that the wage when young has to be lower than $w_o$. Because workers prefer to have smoother utility, and they are already fully insured when old, any increase in $r$ will be transferred into first period wages. This implies that the reduction will be one-for-one. However, if workers are not fully insured, only part of the increase in $r$ will be transferred into first period wages, but the other part will be used to better insure old workers. This implies a reduction of less than one-for-one in severance payments.

If agents exhibit a high degree of decreasing absolute risk aversion, the above result can
change. We know that in equilibrium workers receive the lowest utility when they are young. If workers become a lot less risk averse when their income increases, any increase in $r$ may be used to increase utility when they are young.\(^{12}\) For instance, if workers become risk neutral after a certain level of income, and this level is smaller than $s^* + r$ and $w^*_o$, but larger than $w^*_y$, any increase in $r$ will be used to increase $w_y$. In this case, workers will not care about insurance against unemployment since they are risk neutral in this range of income, but will care about smoothing income between the first and the second period. However, for a reasonable degree of decreasing absolute risk aversion such type of problem should not occur. Changes in the level of severance payments also have some effects on $w_o$ and on $\bar{\theta}$. In equilibrium, the constraint on $w_o$ is binding. Old workers’ wages equal the average ability of a retained worker. Since in equilibrium $\bar{\theta}^* = \theta^h - 2s^*$, we know that $w^*_o = \theta - s^*$. When severance payments increase, firms fire fewer workers ($\bar{\theta}$ decreases) because the cost of firing a worker increases. Moreover, an increases in $s$ lowers $w_o$ because the average ability of retained workers decreases.

5. Government Intervention

As mentioned above, we restrict our attention to the equilibrium where, given $r$ is large enough, unemployment occurs in equilibrium. When there is competition to bid away old workers, the resulting equilibrium involves too much unemployment compared to the level of unemployment that maximizes the total surplus of the economy. This motivates government intervention. On the other hand, in equilibrium old workers are not perfectly insured against unemployment. Moreover, the unemployment can be interpreted as involuntary for at least some workers. All old workers that are fired and have an ability level $\theta > r$ would be willing to work if they were able to obtain a wage corresponding to their marginal productivity. However, firms are not willing to pay that wage because they are not able to differentiate these workers from workers with low ability. Consequently, all the fired workers prefer not taking a new job. This, combined with the fact that firms do not provide full insurance against job loss, introduces political incentives for a government to

\(^{12}\) Simulation with utility functions with considerably high level of increasing risk aversion still leads to a reduction of less that one of one of the severance payments when $r$ increases.
provide a public unemployment insurance program. We will analyze two questions separately. First, if a government wishes to provide some public unemployment insurance (UI), how can it finance the UI program without introducing further distortions in the firm’s layoff decisions. Then, we will consider how government intervention can reduce or eliminate the initial distortion inherent in a “laissez-faire” economy.

5.1. Unemployment Insurance System with Experience Rating

We will now introduce a government into this economy. The only role for the government will be to provide unemployment insurance. The benefit paid to unemployed workers is denoted \( b \). Further, assume that the government only taxes firms to finance this program.\(^{13}\)

Let \( T \) be the total tax imposed on firms per period. This tax is composed of two parts. The first part is a payroll tax \( \tau \) per worker, while the second is an experience rated tax, where firms pay a proportion \( e \) of the total cost they impose on the UI system due to their layoff decisions.\(^{14}\) The total tax per period imposed on any firm will be given by:

\[
T = \tau + (\theta^h + \theta)\tau + e(\bar{\theta} - \theta^e)b.
\]

The first term is a payroll tax on young workers, while the second is a payroll tax on old retained workers. The last term is the experience rating component. The government will be required to keep a balanced budget. However, since there is no population growth, we will assume that the government maintains a balanced budget every period. The government budget constraint is the following: \( [\bar{\theta} - \theta^e]b = \tau + [\theta^h + \theta]\tau + e[\bar{\theta} - \theta^e]b \). Under a full experience rating system \((e = 1)\), the payroll tax will be equal to zero.

5.2. Competitive Equilibrium with Unemployment Insurance

\(^{13}\) The reasons for only taxing firms are the following: (1) taxing both firms and workers will have the same effect as long as the government use only payroll taxes, and (2) in practice, this kind of experience rating taxes are always imposed only on firms.

\(^{14}\) In practice, experience rated taxes are implemented at the industry level to provide insurance against any idiosyncratic shocks. In the present problem, since firms are \textit{ex ante} and \textit{ex post} identical, industry and firm level experience ratings are the same.
To solve for the competitive equilibrium, we proceed exactly the same way as in section 4. Firms take as given \( \tau \) and \( e \). First, we solve for the constraint on \( w_o \). When a firm hires a worker, it has to pay the payroll tax, so firms will offer a wage equal to the average ability minus the tax. The old workers’ wages constraint is given by \( w_o \geq \bar{\theta} + \theta h - t \). Since, in equilibrium, this constraint is satisfied with equality, we can solve for \( \bar{\theta} \). If a firm keeps an old worker, the worker will produce \( \theta \), but the firm has now to pay the wage \( w_o \) plus the payroll tax. If the firm fires the worker, the firm has to make a severance payment plus pay an experience rated tax of \( eb \). Solving for the critical ability level at which the firm is indifferent between firing or retaining the worker, we find \( \bar{\theta} = \theta h - 2[s + eb] \).

We can see that, as before, an increase in severance payments, leads to a decrease in \( \bar{\theta} \) and subsequently a decrease in layoffs. The payroll tax has no direct effect because it simultaneously increases the cost of keeping a worker and decreases the wage. The level of experience rating has a direct positive effect on employment by increasing the cost of firing a worker. In equilibrium, changes in unemployment benefits have two effects. Taking the partial derivative of \( \bar{\theta} \) with respect to \( b \), we find \( \frac{\partial \bar{\theta}}{\partial b} = -2[\frac{\partial s}{\partial b} + e] \). When the government increases unemployment insurance benefits, as we will see later, severance payments decrease. This leads to more unemployment in the economy ceteris paribus. However, the cost of firing a worker increases because of the experience rated tax. Without any experience rating and as long as severance payments are strictly decreasing with \( b \), UI will increase unemployment in the economy. However, if \( e \) is positive the total effect of UI can be ambiguous.

We now solve for the competitive equilibrium contract \( \{w_y^*, w_o^*, s^*\} \), which is a function of \( \{\theta h, \theta^L, r\} \) and \( \{\tau, e, b\} \), and then look at how severance payments change when UI benefits change. The contract offered to young workers is given by the solution to the following problem:

\[
\max_{w_y, w_o, s} \left\{ U(w_y) + [\bar{\theta} - \theta^L]U(r + s + b) + [\theta h - \bar{\theta}]U(w_o) \right\},
\]

subject to:
\[
\frac{\theta^\ell + \theta^h}{2} - w_y - \tau + [\bar{\theta} - \theta^\ell](-s - eb) + [\theta^h - \bar{\theta}]\left[\frac{\theta + \theta^h}{2} - w_o - \tau\right] = 0,
\]

\[\bar{\theta} = \theta^h - 2[s + eb]\]

\[w_o \geq \theta^h - s - eb - \tau\]

and \(s \geq 0\).

Here again, firms maximize the expected utility function of a young worker subject to a zero-profit condition, a wage restriction, and a non-negativity of severance payments condition. This problem is similar to the one in section 4. The only difference now is that \(\bar{\theta}\) depends on \(b\) and \(e\). Also in the zero-profit condition, firms have to pay a payroll tax on all workers and a experience rated tax on any laid off workers. The long term contract \(\{w^*_y, w^*_o, s^*\}\) is a function of \(\{r, \theta^h, \theta^\ell\}\) and the unemployment insurance system \(\{\tau, e, b\}\).\(^{15}\)

**Lemma 2:** Given an unemployment insurance system \(\{\tau, e, b\}\), firms are not able to fully smooth workers’ utility across periods as long there exists some workers that are more productive at home.

As long as UI is not overly generous, the only way firms can fully smooth utility and satisfy the zero-profit condition is by not firing any workers. However, following the reasoning behind proposition 3, firms still prefer to fire all workers with ability level lower than \(r\). If UI is too generous, and since that \(\theta^h \leq (1 + e)b + \tau + r\), the severance payments will be equal to zero because of the non-negativity constraint on \(s\). However, even without any severance payments, firms are not able to provide a wage to young workers that is large enough to smooth utility across both periods. This is because taxes have to be large enough for the government not to run a deficit in any period.

**Proposition 6:** As long as firms still offer severance payments, the equilibrium long term contract will not fully insure old workers, and the wage of a young workers is less than the

\(^{15}\) The proof of the existence of an equilibrium is similar to the proof of proposition 2, and is consequently omitted.
total compensation of workers who are laid off, \((w_y^* < r + b + s^* < w_o^*)\).

When firms offer severance payments in equilibrium, they retain the right to reduce severance payments and give a higher wage to young workers. For the same reason as before, firms will try to smooth the utility of workers by not offering perfect insurance. However, if the UI system becomes too generous and the non-negativity constraint on severance payments is binding, then this allows workers to be fully insured. Because firms simultaneously try to smooth wealth across periods and insure old workers against unemployment, it is possible that UI can completely crowd out severance payments before old workers have been fully insured. At a minimum, severance payments will become completely crowded out when old workers become fully insured. We know that in equilibrium, \(w_o^* = \theta h - s - eb - t\) and that unemployed workers receive \(r + s^* + b\). This implies that if \(b = \frac{\theta h - r - t}{1+e}\), firms will definitively not offer any severance payments.\(^\text{16}\)

**Proposition 7:** Given that \(s^* > 0\), severance payments are decreasing in \(b\). Moreover, assuming that workers do not exhibit a high degree of decreasing risk aversion; if \(e < 1\), severance payments are decreasing in a proportion less than one; if \(e > 1\), severance payments are decreasing in a proportion more than one, and if \(e = 1\), severance payments are decreasing in a one-to-one proportion. However, if \(s^* = 0\) severance payments are independent of \(b\).

The result that severance payments are a decreasing function of UI benefits is not surprising and is consistent with the previous literature. However, the fact that the crowding out effect is a function of the level of experience rating has important implications. The intuition behind this result is as follows. When \(b\) increases, old workers are better insured against unemployment. Given positive severance payments, firms can now reduce them. However, since old workers are not perfectly insured, firms will consequently try to insure them better by diminishing the severance payments by less than the amount \(b\) increases. On the other hand, firms have to pay more tax when UI benefits increase because of experience

\(^\text{16}\) If government sets \(b > \frac{\theta h - r - t}{1+e}\), unemployed agents are better off than old workers. If it is the case that workers choose to leave their job, then we assume that the government sets \(b \leq \frac{\theta h - r - t}{1+e}\).
rating. When $e < 1$, the change in the experience rated tax is less than the change in $b$. Consequently, if the firms reduce $s$ by the same amount that $b$ increases, their profits will increase. Thus, by competition firms will be able to use part of this increase to smooth workers’ utility, but also use part to increase the wealth of old unemployed workers. When $e = 1$, severance payments and UI benefits become perfect substitutes, resulting in perfect crowding out. Finally when $e > 1$, increases in experience rated tax will be larger than the increase in UI benefit. Firms will reduce the severance payments by more than the increase in $b$ to finance the tax. Here again if workers exert too much absolute increasing risk aversion, they will not care that much of being insure against layoff but will prefer smooth the income between young and old, and this because ($w^*_y < r + b + s^* < w^*_o$).

5.3. Impact of Unemployment Insurance System

Having fully characterized the competitive equilibrium in the presence of UI, we now consider the impact of government intervention. To begin, we restrict our attention to the case where severance payments are positive in equilibrium. The first question is how a government can introduce UI without changing the level of employment in the economy. We know that without UI, old workers are under-insured, and this might create a political incentive for the government to give unemployment benefit to these workers. We will now find the level of experience rating that allows the government to provide UI without changing the level of unemployment. We know that $\bar{\theta} = \theta^h - 2s + eb$, and the larger $\bar{\theta}$ is, the more unemployment there will be. The change in unemployment with respect to $b$, is given by $\frac{\partial \bar{\theta}}{\partial b} = -2[\frac{\partial s^*}{\partial b} + e]$. Given Proposition 7, we can see that if $e = 1$, the level of employment does not change when UI benefits change. Moreover since the crowding out of $s$ is complete when $e = 1$, old workers are equivalently insured.

Proposition 8: Given that $s^* > 0$, the government cannot reduce inefficiency in the layoff decisions created by UI, and at the same time, increase the insurance provided to old workers.

When $e = 1$, the reduction in severance payments is equal to the increase in UI benefits. We also know that if $e < 1$, the reduction in severance payments is smaller than the
increase in UI benefits \(0 > \frac{\partial \theta}{\partial b} > -1\). More important, \(\frac{\partial \theta^*}{\partial b} + e < 0\) One implication is, since we know that \(\frac{\partial \theta}{\partial b} = -2[\frac{\partial \theta^*}{\partial b} + e]\), then unemployment in the economy will increase. There are already too many laid off workers in the “laissez-faire” equilibrium and UI makes it worst. However, a second implication is that old workers end up better insured against unemployment. When \(e > 1\), UI can reduce the inefficiency resulting from layoffs by reducing the number of layoffs. On the other hand, workers are less insured against unemployment. Consequently, the government faces the following trade off, increasing the level of experience rating increases the total surplus in the economy, but subsequently reduces old workers’ insurance.

### 6. Conclusion

We characterized an economy where firms hire workers without knowing their ability. Once firms learn the productivity of their workers, they can choose to lay some off. Retained workers, receive good wage offers from rival firms, and fired workers are not able to find a job with a wage higher than their reservation wage. This type of competition leads to too much unemployment and under-insurance of old workers.

We looked at the impact of unemployment insurance on this type of competition and found that as long as firms provide severance payments, the government cannot reduce unemployment while simultaneously increasing insurance against being laid off. The reasoning is that a low level of experience rating increases layoffs because firms do not face the full cost of laying off a worker. However, because severance payments are reduced by less than the increase in unemployment benefits when the level of experience rating is low, workers are better insured when the government provides unemployment insurance. On the other hand, high levels of experience rating make it more costly for firms to fire workers and thus reduce layoffs. This result has a positive effect on the economy, since there were too many layoffs compared to the efficient allocation of the “laissez-faire” economy. However, high levels of experience rating subsequently reduce the insurance that old workers receive.

In this version of the paper, we restrict our attention to cases where firms still provide severance payments when unemployment insurance is publicly provided. A possible ex-
tension will be to analyze the situation when severance payments are completely crowded out. When this occurs, the government now may be able to increase workers’ wealth while unemployed and simultaneously reduce unemployment in the economy.
7. Appendix A: Proofs

**Proof of proposition 1:** The first order conditions of the firm’s problem can be rewritten the following way:

\[
\begin{align*}
\left[U(r + s) - U(w_o)\right] + (\theta^h - w_o + s)\left[U'(w_o) - U'(w_y)\right] &= 0, \\
\left[U(w_o) - U(r + s)\right] + (w_o - s - \theta^\ell)\left[U'(r + s) - U'(w_y)\right] &= 0, \\
\frac{\theta^\ell + \theta^h}{2} - w_y - [w_o - s - \theta^\ell]s + [\theta^h - w_o + s]\left[\frac{w_o - s + \theta^h}{2} - w_o\right] &= 0.
\end{align*}
\] 

Equations (A1) and (A2) represents the trade-off between \( w_y, w_o \) and \( s \), while equation (A3) is simply the zero-profit condition. We can see that if \( w_y = w_o = r + s \) all the first order conditions are satisfied.

**Proof of Corollary 1:** When firms fully insure workers, \( w_o = s + r \). We also know that \( \bar{\theta} = w_o - s \), so \( \bar{\theta} = r \).

**Proof of proposition 2:** The first order conditions of the firm’s problem can be rewritten the following way:

\[
\begin{align*}
2\left[U(\theta^h - s^*) - U(r + s^*)\right] + [\theta^h - \theta^\ell - 2s^*]U'(r + s^*) \\
-2s^*U'(\theta^h - s^*) - [\theta^h - \theta^\ell - 4s^*]U'(w_y^*) + \psi &= 0. \\
\frac{\theta^\ell + \theta^h}{2} - w_y^* - [\theta^h - \theta^\ell - 2s^*]s^* &= 0, \\
\psi : & s \geq 0 \quad s\psi = 0 \quad \psi \geq 0,
\end{align*}
\] 

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where $\psi$ is the Lagrange multiplier on the constraint $s \geq 0$. Equation (B1) represents the trade-off between $w_y$ and $s$, while equation (B2) is simply the zero-profit condition. The Kuhn-Tucker condition (B3) implies that if $s > 0$ then $\psi = 0$. The old worker’s wages are given by (B4). First, we know that new workers are bid away by the market at some wage which does not depend on the worker’s ability. The wage for young workers will be determine by the zero profit condition (B2) for any given $w_o$ and $s$. Second, if a worker is kept in the second period, the firm need to offer a wage sufficiently large to ensure that the worker will stay. Consequently, using (B4) and (B1) we can solve for $w_o^*$ and $s^*$.

**Proof of Lemma 1:** When firms fully smooth workers’ utility $w_y = w_o = \frac{\theta^h + r}{2}$, $s = \frac{\theta^h - r}{2}$ and $\psi = 0$.

The first order conditions on the zero-profit constraint is given by:

$$
\left[\frac{\theta^\ell - r}{2}\right][1 + \theta^h - r] = 0 \quad (C)
$$

Note that equation (C) cannot be satisfied as long as $\theta^\ell < r$.

**Proof of Proposition 3:** First we will show that $w_o^* > r + s^*$. Suppose that $w_o^* = r + s^*$, then first order condition (B1) becomes 

$$
[\theta^h - \theta^\ell - 4s^*][U'(r + s^*) - U'(w_y^*)] = 0.
$$

This implies that $r + s^* = w_o^* = w_y^*$, but we know from Lemma 1 that this is not possible. Now, we will show that $w_y^* < r + s^*$ in equilibrium. If we reorganize equation (B1) we get:

$$
2[U(w_o^*) - U(r + s^*)] + [\theta^h - \theta^\ell - 2s^*][U'(r + s^*) - U'(w_y^*)] + 2s^*[U'(w_y^*) - U'(w_o^*)]. \quad (D)
$$

Since $w_o^* > r + s^*$, we can see that the first term is positive. Since $s > 0$, we can show, using the zero-profit condition, that $w_o^* > w_y^*$. The third term is also positive by concavity.
We also know that $\theta^h - \theta^\ell - 2s > 0$, because we already showed that $\frac{\theta^h - \theta^\ell}{2} > \frac{\theta^h - r}{2} \geq s$. This implies that the second term must be negative, and by concavity we know that it implies that $s^* + r > w_y$.

**Proof of Corollary 2:** Fired workers will decline new job offerings only if $w'_o = \frac{\bar{\theta} + \theta^h}{2} \leq r$. Substituting for $\bar{\theta}$, this condition becomes $s^* \geq \frac{\theta^h - \ell^h}{2} - r$. Since $\theta^h - \theta^\ell - 2s^* > 0$, we know that $s^*[1 + (\theta^h - \theta^\ell - 2s^*)] > s$. Now we will look at the condition under which $s^*[1 + (\theta^h - \theta^\ell - 2s^*)] \geq \frac{\theta^h - \ell^h}{2} - r$. We can reorganize these inequalities in the following way: $s^* + r > \frac{\theta^h + \ell^h}{2} - [\theta^h - \theta^\ell - 2s^*]s^*$. But given the zero-profit condition we know that $w_y \leq \frac{\theta^h + \ell^h}{2} - [\theta^h - \theta^\ell - 2s]s$, which implies that in order for $w'_o \leq r$ to be possible $s^* + r$ must be larger than $w^*_y$.

**Proof of Proposition 4:** We know from proposition 2 and 3 that $\frac{\theta^h - r}{2} > s > 0$. This implies that $\theta^h > \bar{\theta} > r$.

**Proof of Proposition 5:** Conducting comparative statics on the first order conditions we find that $\frac{\partial s^*}{\partial r} = \frac{\alpha}{\beta}$ where:

$$\alpha = [\theta^h - \theta^\ell - 2s^*]U''(r + s^*) - 2U'(r + s^*),$$

$$\beta = 4[U'(w^*_o) + U'(r + s^*) - U'(w^*_y)] - [\theta^h - \theta^\ell - 2s^*]U''(r + s^*)$$

$$- 2s^*U''(w^*_o) - [\theta^h - \theta^\ell - 4s^*]^2U''(w^*_y).$$

First, we can see that $\alpha$ is clearly negative. Moreover, assuming that the second order condition are satisfy $(-\beta)$ has to be negative so $\beta > 0$. This imply that $\frac{\partial s^*}{\partial r}$ is negative. We can also see that $-\frac{\partial s^*}{\partial r} < 1$ if $\beta + \alpha > 0$.

$$\beta + \alpha = 4[U'(w^*_o) + \frac{1}{2}U'(r + s^*) - U'(w^*_y)]$$

$$- 2s^*U''(w^*_o) - [\theta^h - \theta^\ell - 4s^*]^2U''(w^*_y).$$  \hfill (E1)

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If we look at $\beta + \alpha$ (E1) we can see that all the terms are definitively positive, except the first one which can be positive or negative. First add and subtract the term $2s^*U'''(w^*_y)$ in expression (E1), and we get:

$$
\beta - (-\alpha) = [4U'(w^*_o) - 2s^*U'''(w^*_y)] - [4U'(w^*_o) - 2s^*U'''(w^*_y)]
$$

$$
\frac{1}{2}U'(r + s^*) - 2s^*U'''(w^*_y) - [\theta^h - \theta^\ell - 4s^*]2U'''(w^*_y).
$$

(E2)

We can see that all the terms are definitively positive to the exception of $[4U'(w^*_o) - 2s^*U'''(w^*_y)] - [4U'(w^*_o) - 2s^*U'''(w^*_y)]$ This term will be positive if:

$$
U'''(w^*_o) \left[ \frac{1}{U''(w^*_o)} - s^* \right] > U'''(w^*_y) \left[ \frac{1}{U''(w^*_y)} - s^* \right]
$$

(E3)

Because $u'''(w) < 0$ we know that $U'''(w^*_o) > U'''(w^*_y)$. If $U'''(w)$ the absolute risk aversion coefficient is increasing then $\beta + \alpha$ is positive. By continuity, even if $U'''(w)$ is decreasing but not excessively decreasing, $\beta + \alpha$ will remain positive. Note that the same argument hold for $U'''(w)$, by continuity $\beta + \alpha$ will remain positive even if $U'''(w) < 0$ as long it is small in absolute term. This implies that if agent do not exhibiting high level of decreasing absolute risk aversion, then $\beta + \alpha > 0$. Since $\alpha < 0$ it implies that $\frac{a}{\beta} < 1$, so $|\frac{\partial s^*}{\partial r}| < 1$.

**Proof of Lemma 2:** In the proof of Lemma 2 we can proceed directly the same way as with Lemma 1. The only difference is that here the non-negativity constraint on s can become binding. Full smoothing implies that $r + b + s^* = w^*_o = \theta^h - s^* - eb - \tau$. The non-negativity constraint on s will become binding only if $\theta^h \leq (1 + e)b + \tau + r$. First, consider the case where $s^* > 0$. If we reorganize the zero-profit condition to where young and old workers get the same utility ($s^* = \frac{\theta^h - (1+e)b - \tau - r}{2}$ and $w^*_o = w^*_y = \frac{\theta^h + (1-e)b - \tau + r}{2}$) we get:

$$
- \left[ \frac{r - \theta^\ell}{2} - \frac{(1-e)b + \tau}{2} \right] [1 + \theta^h - r - (1-e)b - t] \geq 0.
$$

(F1)

We also know that government has to balance it’s budget. Using the government budget constraint, the zero-profit condition (F1) become:
\[-\left[\frac{r - \theta^\ell}{2} - \frac{(1 - e)b + \tau}{2}\right]^2\frac{(1 - e)b}{t} \geq 0.\] (F2)

However, when \(r > \theta^\ell\) condition (F2) cannot be satisfied given that the government needs to balance its budget. The only way that this condition can be satisfied is if \(e > 1\) and \(t > 0\), but this implies a surplus. However, if \(e < 1\) and \(t < 0\), then there will be a deficit. Similarly, we can show that when \(s^* = 0\), full smoothing is not possible without violating the zero-profit condition or the government’s balanced-budget condition.

**Proof of Proposition 6:** As long as \(s^* > 0\), Proposition 6 can be proved the same way as Proposition 3.

**Proof of Proposition 7:** If we take the first order conditions, reorganize them and perform some comparative statics, we find \(\frac{\partial s^*}{\partial b} = \frac{\eta}{\beta'}\) where:

\[
\eta = -4e[U'(w_o^*) - U'(w_y^*)] - 2(1 + e)U'(r + s^* + b) + [\theta^h - \theta^\ell - 2(s^* + eb)]U''(r + s^* + b) \\
+ 2e(s^* + eb)U''(w_o^*) + e[\theta^h - \theta^\ell - 4(s^* + eb)]2U''(w_y^*).
\]

\[
\beta' = 4[U'(w_o^*) + U'(r + s^* + b) - U'(w_y^*)] - [\theta^h - \theta^\ell - 2(s^* + eb)]U''(r + s^* + b) \\
- 2(s^* + eb)U''(w_o^*) - [\theta^h - \theta^\ell - 4(s^* + eb)]2U''(w_y^*).
\]

If the second order conditions are satisfy \(\eta\) is negative and \(\beta'\) is positive. This implies that \(\frac{\partial s^*}{\partial b} < 0\). We can also see that if \(\eta + \beta > 1\) then \(-\frac{\partial s^*}{\partial b} < 1\), if \(\eta + \beta = 1\) then \(-\frac{\partial s^*}{\partial b} = 1\) and if \(\eta + \beta < 1\) then \(-\frac{\partial s^*}{\partial b} > 1\).

\[
\eta + \beta = 4(1 - e)[U'(w_o^*) - U'(w_y^*)] + [4 - 2(1 + e)]U'(r + s^*) \\
- 2(1 - e)s^*U''(w_y^*) - (1 - e)[\theta^h - \theta^\ell - 4s^*]2U''(w_y^*).\] (G1)
If we look at \((G1)\) we can see that if \(e = 1\) then \(\eta + \beta' = 1\) so \(-\frac{\partial s^*}{\partial b} = 1\). Moreover, if \(e < 1\) all terms are multiplied by \(1 - e > 0\) \((U'(r + s^*)\) is multiplies by \([4 - 2(1 + e)]\) which is also positive. The problem is essentially the same that in Proposition 5 so agent do not exhibiting high level of decreasing absolute risk aversion \(\eta + \beta' > 1\) so \(-\frac{\partial s^*}{\partial b} < 1\).

An increase in \(e\) will lead to a reduction in \(1 - e\) and in \(4 - 2(1 + e)\) so \(\eta + \beta'\) will decrease and \(-\frac{\partial s^*}{\partial b}\) will increase. On the other hand, if \(e > 1\) then if agent do not exhibiting high level of decreasing absolute risk aversion \(\eta + \beta' < 1\) so \(-\frac{\partial s^*}{\partial b} > 1\).

**Proof of Proposition 8:** First we know that \(\frac{\partial \bar{\theta}}{\partial b} = -2\left[\frac{\partial s^*}{\partial b} + e\right]\). From Proposition 7, we know that \(\frac{\partial s^*}{\partial b} = \eta \beta\). It is possible to show that \(\frac{\partial s^*}{\partial b} + e < 1\) when \(e < 1\), \(\frac{\partial s^*}{\partial b} + e = 1\) when \(e = 1\), and \(\frac{\partial s^*}{\partial b} + e > 1\) when \(e > 1\).
References.


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