Tax Evasion and Trust*

by

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Abstract

Tax evasion is typically analyzed in a principal/agent framework, the government (principal) trying to provide agents with the incentives to pay their taxes. However, evading sales, excise or trade taxes requires the cooperation of at least two taxpayers. When individuals evade taxes, they face two potential costs. One is that tax evasion may be detected and sanctioned; the other is that their partner in crime might cheat. An increase in the sanction for tax evasion leads to a direct increase in the expected cost of a transaction in the illegal sector. However, it may also reduce the incentive to cheat. It may then be that a small increase in the sanction reduces the total cost of transacting in the illegal sector. Tax evasion may increase as a result.

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1. Introduction

Most of the literature on tax evasion is presented in a principal/agent framework, with the government (principal) trying to provide the right incentives to each taxpayer (agent).\(^1\) Income taxes are generally thought to be suitable for this type of analysis because the main strategic interaction is between the taxpayer and the government. However, there are many types of tax evasion which involve the participation of more than one taxpayer. Taxes on transactions, such as sales taxes, excise taxes on tobacco or alcohol, and taxes on trade are examples of taxes for which evasion often involves the collaboration of at least two taxpayers — a buyer and a seller. In fact, even income tax evasion might require at least the complicity of second parties, as when labour services are supplied in the untaxed sector. The purpose of this paper is to investigate the determinants of tax evasion in settings where agents to a transaction must collaborate to determine whether to undertake it in the illegal sector.\(^2\)

When two individuals decide to undertake a transaction in the illegal sector to avoid paying a tax, they forego the option of using the legal system to resolve any dispute that could arise. Consequently, they face two potential costs. The first one is that the government may detect and sanction a tax evader, while the second one is caused by potential cheating by the partner in crime.\(^3\)

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\(^1\) The classic analysis is by Allingham and Sandmo (1972). For general reviews of the traditional literature, see Cowell (1990) and Myles (1995). Tax evasion has been incorporated into an optimal non-linear income tax setting by Cremer and Gahvari (1996), Marhuenda and Ortuño-Ortín (1997), and Chandar and Wilde (1998). Some recent analysis has departed from the principal-agent setting by allowing taxpayers and tax collectors to collude. See, for example, Flatters and MacLeod (1995) and Hindriks, Keen and Muthoo (1999). In this literature, there is no cooperation among taxpayers, which is the focus of our analysis.

\(^2\) Governments are well aware that tax evasion may require the collaboration of several individuals. In an attempt to reduce underground transactions, the Canadian federal government has launched an advertisement campaign to deter house renovation paid for ‘under the table’. (Globe and Mail, 9 August 1999, page A6).

\(^3\) When transacting in the illegal sector, individuals can cheat in several ways. One possibility is for an individual to provide less care than was agreed on in avoiding
particular, the chances of getting caught evading taxes depend jointly on the evasive activities of the two transacting partners. An increase in the sanction for tax evasion leads to a direct increase in the expected cost of a transaction in the illegal sector. However, a higher sanction may also facilitate cooperation between criminals by reducing the incentive to cheat. It may then be the case that a small increase in the sanction reduces the total cost of a transaction in the illegal sector and therefore increases tax evasion.

We construct a model in which a continuum of infinitely lived agents, differing only in their aversion to dishonesty, decide whether to undertake their transactions in the legal or the illegal sector. All agents undertake a large number of transactions each period — for simplicity, one with every other agent in the economy. This ensures that pairs of agents form lasting repeated relationships, and that relationships span all combinations of honesty-types. For each transaction in each period, the pair of agents involved can choose which sector to use. Those who choose the legal sector in a given period obtain a sure benefit from the transaction, but have to pay a tax. Those who choose the illegal sector avoid paying taxes, but may be caught and sanctioned. They receive an uncertain benefit depends on their aversion to dishonesty and on the level of crime enforcement undertaken by the government. The chances of getting caught engaging in an illegal transaction depend partly on the amount of costly avoidance effort that is provided jointly by the two parties to the transactions.

Two agents transacting in the illegal sector can potentially increase their detection by the authority. This is the kind of cheating we are focusing on in this analysis. Of course, such cheating increases the probability of detection for all individuals involved in the transaction. Examples of such cheating are that an individual may publicly (rather than privately) consume a good, or that he may openly discuss the 'low' price he paid for the good. Another example is that individuals that have transacted in the illegal sector should also provide care so as to avoid being caught for other crimes, because observing one crime may reveal that other crimes have been committed.
payoff by simultaneously providing a high, or cooperative, level of avoidance effort. However, they will then expose themselves to potential deviation by their partner. Because contracts in the illegal sector are not enforceable, reputations and punishments are the only mechanisms that can be used to enforce higher levels of effort. The possibility of cooperation enhances the payoffs from illegal activity. To enforce cooperation, agents will punish each other. Depending on the agents involved, the punishment may occur either in the legal or the illegal sector with low (non-cooperative) avoidance effort levels. We assume that no agent can force another one to transact in the illegal sector. For some agents — the ones with higher aversion to dishonesty — the non-cooperative equilibrium in the illegal sector yields a lower expected payoff than that of the legal sector. Consequently, if one of them prefers the legal sector, they will transact in the legal sector for the duration of the punishment phase. On the other hand, if both prefer the illegal sector, they will keep on evading taxes with non-cooperative levels of avoidance for the duration of the punishment phase.

Under the assumptions we make, the resulting equilibrium takes the following form. Agents with high aversion to dishonesty pay taxes on all their transactions. Agents with low or medium aversion avoid taxes by transacting in the illegal sector with all agents willing to do so. Agents in the illegal sector cooperate with other agents in the illegal sector until one of them deviates. When one partner deviates, they enter the punishment phase of the strategy. Pairs of agents with low aversion to dishonesty remain in the illegal sector for the punishment phase, while those in which at least one of the two agents has a medium aversion to dishonesty go back to the legal sector. Because an agent’s aversion to dishonesty is observable, and because agents are not willing to make a transaction in the illegal sector if they know that their partner will cheat, some agents who would prefer to trade in the illegal sector simply cannot. Indeed, some agents are unable to commit to behaving cooperatively in the illegal sector and, consequently, have to trade in the legal sector with every other agent. This
implies that in equilibrium, there is no deviation from cooperative behaviour in the illegal sector. In contrast with the standard literature, it is not solely the willingness to participate in the illegal sector that determines which agents evade taxes, but also their ability to commit not to cheat. Some agents are left out of the illegal sector despite their desire to transact in it, simply because they cannot commit to providing the cooperative level of avoidance effort.

When the government changes the level of the sanction, all payoffs in the illegal sector decrease, but in different proportions for different types of participants. An increase in the sanction can lead to a larger reduction in the deviation payoff than in the cooperation payoff. Consequently, an increase in the sanction can increase cooperation, thereby increasing tax evasion. Despite the direct impact of an increase in the sanction on the expected payoff of transacting in the illegal sector, tax evasion can increase with an increase in sanction because it is the ability to commit not to cheat that determines which agents evade taxes. By the same token, an increase in the tax rate can lead to an increase in tax evasion.

In the following section, we formulate the model and our assumptions, and set out the types of equilibria in avoidance effort and their payoffs that can occur in the illegal sector. In section 3, we analyze which levels of dishonesty will be sufficient to enable agents to commit to cooperative transactions in the illegal sector repeatedly. Then, in section 4 we can establish precisely how transactions divide themselves between the legal and illegal sectors according to the aversion to dishonesty of the partners. We show that all transactions in the illegal sector will be accompanied by cooperative avoidance effort levels—no one will deviate in equilibrium. Moreover, we show that the number of transactions carried out illegally will increase in the sanction as well as in the tax rate provided the discount rate is high enough. We conclude in Section 5.
2. The Model and Static Equilibrium

The economy consists of a continuum of infinitely-lived agents, who differ only in their tolerance for engaging in illegal transactions—those that involve evading taxes. Denote this tolerance for dishonesty by \( \theta \), with \( \theta \in [0, 1] \) and distributed according to cumulative distribution function \( F(\cdot) \) with \( F'(\cdot) > 0 \) everywhere. The total population is normalized to unity. Agents engage in many bilateral transactions with one another, and these may be in the illegal sector \( u \) or the legal one \( \ell \). Our analysis focuses on representative types of transactions that can occur in each sector. To facilitate the analysis, we make the extreme assumption that each agent engages in a large number of transactions per period, one with every other agent in the economy. This allows us to treat each transaction as an infinitely repeated game in which lasting relationships determine the nature of the transactions. In particular, since the payoffs from transactions depend upon whether agents behave cooperatively or not, repeated relationships can give rise to cooperative behaviour being sustained in equilibrium. An alternative more complex model would assume that transactions occur randomly with other agents in the economy. It would be more difficult to maintain cooperative behaviour in this setting because punishment for non-cooperative behaviour would be less effective. But Kandori (1992a,b) has shown that the folk theorem for repeated games can be generalized to the case of a large community of individuals who are matched randomly in pairs each period. Even if two individuals are matched only once, cooperation can be enforced if their behaviour in previous matches is observable. In that case, an individual may want to cooperate because cheating now will trigger retaliation from future partners, whoever they are.

We abstract from production and simply suppose that each agent receives a before-tax benefit of \( v \) per legal transaction in each period.\(^4\) An agent of type

\(^4\) The total pre-tax surplus from a legal transaction is therefore \( 2v \), which is
\(\theta\) who makes a transaction in the illegal sector only gets a benefit of \(\theta v\), as well as incurring the chance of being caught. All agents are risk neutral. They can undertake any given transaction in the legal or the illegal sector, provided the agent with whom they are transacting agrees. In our model, there will be some agents who conduct a portion of their transactions in sector \(\ell\) and the rest in sector \(u\).

For transactions in sector \(\ell\), a tax \(t\) per transaction is levied on both agents. Thus, the total tax per transaction is \(2t\) assumed to be shared equally. (Alternative assumption regarding the incidence of the tax would have no significant influence on our results.) The net benefit each agent obtains per transaction in sector \(\ell\), denoted \(\pi^{\ell}\), is therefore \(\pi^{\ell} = v - t\).

Agents transacting in sector \(u\) pay no tax. Those who are detected evading the tax have a sanction \(s\) imposed on them. Agents can reduce the likelihood of detection by providing some costly effort. Denote by \(\alpha_i\) the avoidance effort provided by agent \(i\). For simplicity, we assume that \(\alpha_i\) can take only two values, 0 or 1, and that the cost associated with each of these choices is 0 and \(c\), respectively. The effort levels of the two individuals engaged in an illegal transaction combine to yield a probability that their transaction will be detected. If both choose \(\alpha = 1\), then the probability is \(p_2\); if both choose \(\alpha = 0\), the probability is \(p_0\); and if only one chooses \(\alpha = 1\), the probability is \(p_1\). It is natural to assume that as total avoidance effort increases, the probability that an illegal transaction will be detected decreases, so \(p_0 > p_1 > p_2\). Note the important point that the probability of detection depends only on current avoidance effort. It does not depend either on past avoidance effort or on whether illegal behaviour has been detected in the past. This is obviously a strong assumption — it is conceivable that enforcement agencies monitor

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sumed to be equally shared in the current analysis. Results similar to those obtained below could be obtained in a generalized version of the current model in which there is unequal sharing of the surplus, provided that side payments are possible.
past criminals more intensively than they monitor those who have never been convicted. Nonetheless, the assumption is not uncommon in the literature and we adopt if for simplicity. By the same token, we assume that the sanction s is independent of past convictions.

We assume that agents undertaking an illegal transaction choose their avoidance effort levels $\alpha$ simultaneously. We will say of two individuals providing maximal avoidance effort that they cooperate. It is useful to denote effort under cooperation as $\alpha^c$, with $\alpha^c = 1$. Under cooperation, agent $i$ obtains a payoff $\pi^c_i = \theta_i v - p_2 s - c$. Alternatively, the two individuals may not cooperate and provide minimal avoidance effort, denoted $\alpha^n$, with $\alpha^n = 0$. Under no cooperation, the payoff of agent $i$ is $\pi^n_i = \theta_i v - p_0 s$. Because effort is chosen simultaneously, an individual may fool his cooperating partner and deviate from maximal to minimal avoidance effort. We denote effort under deviation by $\alpha^d$, with $\alpha^d = 0$. Because the fooled partner provides $\alpha^f = \alpha^c = 1$, the payoff of agent $i$ who deviates is $\pi^d_i = \theta_i v - p_1 s$. That of the fooled partner, say $j$, is $\pi^f_j = \theta_j v - p_1 s - c$.

The payoffs $\pi^z_i$, $z = c, n, d, f$ are those of a two-player stage game in which each player chooses between cooperating and not cooperating (deviating). In Figure 1, we present the stage game payoffs.

<table>
<thead>
<tr>
<th></th>
<th>Cooperate</th>
<th>Deviate</th>
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<tbody>
<tr>
<td><strong>Player i</strong></td>
<td></td>
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<tr>
<td>Cooperate</td>
<td>$\pi^c_i = \theta_i v - p_2 s - c$</td>
<td>$\pi^d_i = \theta_i v - p_1 s - c$</td>
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<tr>
<td>Deviate</td>
<td>$\pi^n_i = \theta_i v - p_0 s$</td>
<td>$\pi^f_i = \theta_j v - p_1 s - c$</td>
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<tr>
<td><strong>Player j</strong></td>
<td>$\pi^c_j = \theta_j v - p_2 s - c$</td>
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<tr>
<td>Cooperate</td>
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<tr>
<td>Deviate</td>
<td>$\pi^n_j = \theta_j v - p_0 s$</td>
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**Figure 1**

For our purposes, it is useful to assume that the payoffs are structured as
in a prisoner’s dilemma. We also want to ensure that payoffs are such as to lead to interior solutions with transactions divided between the legal and illegal sectors in equilibrium. We therefore make the following assumptions.\(^5\)

**Assumption 1:**

(a) \(p_1 s + c > p_0 s\)
(b) \(p_2 s + c < p_0 s\)
(c) \(p_2 s + c > p_1 s\)
(d) \(t > p_0 s > p_2 s + c\)
(e) \(p_1 s + c > t\)

Part (a) implies that the best response to non-cooperation is to not cooperate. Part (b) says that the payoff under cooperation is larger than that under no cooperation. Part (c) ensures that there is an incentive to fool one’s cooperating partner and deviate. Part (d) implies \(\pi^c_i(\theta = 1) > \pi^l_i(\theta = 1)\), which says that for some agents — those with high enough values of \(\theta\) — it is better to evade taxes if they can cooperate than to transact in the legal sector. This assumption ensures that there will be some tax evasion in equilibrium. Part (d) also implies that \(\pi^n_i(\theta = 1) > \pi^l_i(\theta = 1)\), meaning that for agents with sufficiently high values of \(\theta\), evading taxes under no cooperation dominates a transaction in the legal sector. The implication of part (d) is that cooperative and non-cooperative outcomes in the illegal sector are both possible. Finally, part (e) states that all individuals prefer to transact in the legal sector than to be fooled.

In every period, all agents make one transaction with every other agent. Since no agent can force another agent to engage in an illegal transaction, a transaction will be undertaken in sector \(u\) only if both agents prefer to do so.

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\(^5\) Mongrain (2000) shows that the same prisoner’s dilemma structure for the payoffs can be obtained in a continuous effort model.
given their own tolerance for dishonesty and what they observe. At the beginning of each period, agents can observe for all other agents with whom they transact: (i) their level of tolerance to dishonesty $\theta$; and (ii) the level of avoidance effort $\alpha$ they exerted whenever they transacted in the illegal sector in any previous period. Given that information, agents choose which of their transactions to undertake in the legal or the illegal sector. Then, for all transactions that are undertaken in the illegal sector, both agents decide simultaneously and non-cooperatively their levels of effort $\alpha$. Finally, transactions occur. Those in the legal sector bear taxes, and some of the transactions in the illegal sector are detected and sanctions are imposed. Since the probability of detection depends only on current period avoidance effort, the sequence of transactions constitute a repeated game in which the only link between periods is the ability of agents to observe the past behaviour of their partners.

We now turn to the determination of which pairs of agents transact in the illegal sector and which in the legal sector. This involves specifying the circumstances that must apply for (repeated) equilibrium transactions in the illegal sector to entail cooperative behaviour. To anticipate our results, we shall show that in equilibrium, only two sorts of transactions will occur — those in the legal sector and cooperative outcomes in the illegal sector. There will be a marginal agent with $\theta = \tilde{\theta}$ such that transactions will be in the illegal sector only if both agents have $\theta \geq \tilde{\theta}$. If at least one agent has $\theta < \tilde{\theta}$, the transaction will be a legal one. Thus, there will be no deviations from cooperative behaviour in equilibrium. Having characterized $\tilde{\theta}$, we can then show how government policies affect the volume of illegal transactions. Paradoxically, increasing the sanction level $s$ can actually increase the number of illegal transactions. At the same time, increases in the tax rate $t$ will also increase the size of the illegal sector.

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6 Agents may provide different levels of avoidance effort in different transactions.
3. Cooperative Equilibrium in the Illegal Sector

We begin by considering how a pair of agents can sustain a level of effort $\alpha^c = 1$, given that both choose to undertake the transaction in the illegal sector. The strategies for each agent and for each transaction in the illegal sector are the following. We assume that all agents use the same trigger strategy with infinite punishment. This is one of an indefinite number of strategies that would lead to similar results. We choose to concentrate on this particular strategy because it is simple and standard in the literature on repeated games. Thus, in any time period, for each transaction in the illegal sector, agent $i$ chooses the level of effort $\alpha^c = 1$ with every agent $j$ who never deviated in any transaction with him in the past. At the same time, for each agent $i$ and each transaction in the illegal sector, agent $i$ punishes every agent $j$ who deviated in any transaction with him in the past. Equivalently, for any transaction between agent $i$ and $j$ in the illegal sector, if agent $j$ deviates in any time period, agent $i$ will punish agent $j$ in all subsequent periods.

Punishments can take two possible forms. After agent $j$ deviates, $i$ and $j$ can make all their subsequent transaction in the illegal sector, playing the no-cooperation equilibrium with $\alpha^n = 0$. However, it is possible that one of the two (or both) agents obtains a higher payoff by transacting in the legal sector instead. If so, that agent refuses to deal in the illegal sector so both agents will make all their subsequent transactions in the legal sector. The following lemma indicates how transactions in the punishment phase divides between the two sectors. The proof of this and other lemmas, as well as all propositions are in the Appendix.

Lemma 1: Let $\bar{\theta} = (v - t + p_0 s)/v$. For any agents $i$ and $j$, if both agents have $\theta \geq \bar{\theta}$ they will transact in the illegal sector for the punishment phase, while if at least one of the two agents has $\theta < \bar{\theta}$ they will go back to the legal sector for the punishment phase.
It can be seen that \( \bar{\theta} \) is increasing in \( s \), so a higher sanction induces more agents to go back to the legal sector for the punishment phase. At the same time, a tax increase causes more agents to stay in the illegal sector for the punishment phase. Since we have assumed that \( t > p_0 s \), some agents will want to stay in the illegal sector for the punishment phase.

Given these strategies and punishment phases, we now establish the conditions under which agents \( i \) and \( j \) can commit in an infinitely repeated series of transactions to exerting the cooperative level of effort \( \alpha^c = 1 \). An agent who can commit to cooperation is one who is better off being in a cooperative equilibrium indefinitely rather than deviating now and subsequently being punished forever, where the punishment can be either in the legal or the illegal sector depending on the two partners to the transaction. Let \( \delta \) be the discount factor for all agents. Consider first the case where both agents have \( \theta \geq \bar{\theta} \), so they would choose to make all their transactions in the illegal sector for the punishment phase. It is straightforward to determine the circumstances under which they could commit to cooperate for all future periods.

**Lemma 2:** For any pair of agents \( i \) and \( j \) with \( \theta \geq \bar{\theta} \) (so punishment will be in the illegal sector), both agents can commit to cooperating in every period if

\[
\delta > \frac{p_2 s + c - p_1 s}{(p_0 - p_1)s}
\]

We assume in what follows that condition (1) is satisfied. Thus, all pairs of agents \( i, j \) with \( \theta \geq \bar{\theta} \) can commit to cooperating.\(^7\) Next, consider the case where at least one of the transacting agents has \( \theta < \bar{\theta} \), so both agents would be obliged to go back to the legal sector for the punishment phase. Some of these agents could also commit to cooperating.

\(^7\) Assuming that condition (1) is not satisfied implies that in equilibrium there are no cooperative transactions in the illegal sector. The results will parallel those of the traditional literature in this case.
Lemma 3: For any pair of agents $i$ and $j$ for which at least one of the two agents has $\theta < \bar{\theta}$ (so punishment will be in the legal sector), agent $i$ can commit to cooperating forever if and only if $\theta \geq \tilde{\theta}$, where $\tilde{\theta}$ is given by:

$$\tilde{\theta} = \frac{\delta (v - t) + (p_2 s + c) - (1 - \delta)p_1 s}{\delta v}$$ (2)

It is straightforward to show that if condition (1) is satisfied, then $\tilde{\theta} \leq \bar{\theta}$. Since (1) is assumed to be satisfied, then the marginal agent with $\tilde{\theta}$ is one that can commit to cooperating forever if transacting in the illegal sector. Therefore, cooperation in the illegal sector will occur for transactions between all pairs of agents $i$ and $j$ such that $\theta \geq \tilde{\theta}$. Conversely, if at least one agent to a transaction has $\theta < \tilde{\theta}$, then at least one agent would not cooperate in an illegal transaction.

We are now in a position to state our first main result.

**Proposition 2:** If $\delta < (p_1 - p_2)/p_1$, then an increase in the sanction leads to an increase in the number of agents who can commit to cooperating forever in the illegal sector. The same obtains, without the restriction on $\delta$, when the tax rate increases.

Proposition 2 indicates that if the discount factor is low enough, an increase in the sanction can increase the number of agents who can commit to cooperating forever in the illegal sector, that is, $d\tilde{\theta}/ds < 0$. The intuition is as follows. When $s$ increases, the payoff from cooperating forever in the illegal sector decreases, while the payoff from taking the punishment phase in the legal sector stays the same. This makes it harder for an agent to commit to cooperating. However, an increase in $s$ leads to a larger reduction in the payoff from deviating than that in the payoff from cooperating. This can lead to an increase in cooperation if the discount factor is low enough. In the same way, when $t$ increases, the payoff from the punishment phase decreases so it becomes more attractive for an agent to cooperate.
Having established which agents can commit themselves to cooperating if they choose the illegal sector, we now examine which agents will in fact prefer to make their transactions in the illegal sector.

**Lemma 4:** An agent $i$ would prefer to cooperate in the illegal sector rather than transacting in the legal sector if $\theta \geq \hat{\theta}$, where $\hat{\theta}$ is given by:

$$\hat{\theta} = \frac{v - t + p_1 s + c}{v}$$  \hspace{1cm} (3)

It can be seen that $\hat{\theta}$ is decreasing in $t$ and increasing is $s$.

**Lemma 5:** $\hat{\theta} < \tilde{\theta}$.

Lemma 5 is important because it implies that the number of agents who would like to make their transactions in the illegal sector if they could cooperate (those with $\theta \geq \hat{\theta}$) is larger than the number of agents who can commit to cooperate in the illegal sector (those with $\theta \geq \tilde{\theta}$). This leave us with some agents who would be better off in the illegal sector in a cooperative equilibrium, but cannot commit themselves to cooperating. Because we ensured, in Assumption 1, that no agent wants to make a transaction in the illegal sector if he knows that his partner will cheat, the capacity to commit to cooperate is a key factor: it is this capacity that ultimately determines who transacts in the legal or the illegal sector.

We also know that agent $i$ of type $\theta$ prefers the illegal sector without cooperation relative to the legal sector if $\pi^n(\theta) \geq \pi^\ell$. This condition will be satisfied only for agents with $\theta \geq \bar{\theta}$. Moreover, for those agents, cooperation is sustainable (assuming that condition (1) is satisfied) and leads to higher payoffs. Consequently, all those agents will prefer to cooperate. This implies that only those agents with $\theta \geq \bar{\theta}$ who deviated will choose not to cooperate in the illegal sector (and in equilibrium none will deviate).
4. The Equilibrium Number of Legal and Illegal Transactions

We now have all the elements to describe completely the equilibrium. Given our assumptions that: (i) the least honest agent prefers the illegal sector without cooperation to the legal sector \((\pi_c(\theta = 1) > \pi^e)\); and (ii) the discount factor is high enough to ensure that all pairs agents who would take the punishment phase in the illegal sector can commit to cooperating (condition (1)), the \(\theta\)'s satisfy the following chain of inequalities:

\[0 \leq \hat{\theta} \leq \tilde{\theta} \leq \bar{\theta} \leq 1\]

where

\[\hat{\theta} = \frac{v - t + p_2s + c}{v}; \quad \tilde{\theta} = \frac{\delta(v - t) + p_2s + c - (1 - \delta)p_1s}{\delta v}; \quad \bar{\theta} = \frac{v - t + p_0s}{v}\]

The equilibrium can be described as follows. All agents with \(\theta < \hat{\theta}\) prefer to trade in the legal sector, so make all their transactions there. (Recall that they cannot be forced to trade in the illegal sector.) All agents with \(\hat{\theta} \leq \theta \leq \tilde{\theta}\) would prefer to cooperate in the illegal sector rather than trading in the legal sector. However, they are not able to commit to cooperating in equilibrium, so any illegal trades must involves a non-cooperative equilibrium \((\alpha^n = 0)\). But in these circumstances, they prefer the legal sector over the non-cooperative equilibrium in the illegal sector, so they all choose to transact in the legal sector. Finally, all agents with \(\theta \geq \tilde{\theta}\) prefer to cooperate in the illegal sector rather than transacting in the legal sector, and they can also commit to cooperating in equilibrium. They will make their transactions in the legal sector with all agents for which \(\theta < \tilde{\theta}\), and will make their transactions cooperatively in the illegal sector with all agents for which, like themselves, \(\theta \geq \tilde{\theta}\). This equilibrium is summarized in the following proposition and in Figure 2.\(^8\)

\(^8\) Recall that all those with \(\theta \in [\hat{\theta}, \tilde{\theta}]\) will punish a deviation in the legal sector while those with \(\theta \in [\tilde{\theta}, 1]\) will prefer to do so in the illegal sector. Since all agents transacting in the illegal sector do cooperate, punishment is never observed.
**Proposition 3:** All agents with \( \theta \geq \tilde{\theta} \) make their transactions cooperatively in the illegal sector with all agents for whom \( \theta \geq \tilde{\theta} \) and make their transaction in the legal sector with all agents for whom \( \theta < \tilde{\theta} \). All agents with \( \theta < \tilde{\theta} \) make all their transaction in the legal sector.

![Figure 2](image.png)

Proposition 3 has an implication that cannot be found in the standard literature on tax evasion. Some agents who would like to evade taxes are not able to do so (those with \( \theta \) between \( \hat{\theta} \) and \( \tilde{\theta} \)). The reason is that these agents are too ‘honest’ (have a low tolerance for dishonesty \( \theta \)) and are not able to commit to cooperating: they would always deviate from a cooperative equilibrium. Because honesty is observable, no agents want to trade with them in the illegal sector. In contrast with the standard literature, it is not solely the difference between the payoffs in the legal and illegal sectors that determines who is the marginal evading agent, but the ability of this agent to commit to cooperating in equilibrium. This difference has important implications for the effects of policy, as the following proposition demonstrates.
Proposition 4: If \( \delta < (p_1 - p_0)/p_1 \), then an increase in the sanction and an increase in the tax rate both lead to an increase in the number of transactions that are made in the illegal sector.

This counter-intuitive result can be interpreted in the following way. When the sanction increases, the ability of an agent to commit increases as long as the discount factor is small enough. Since it is not the preferences of an agent but his ability to commit that determines his choice of sector, an increase in \( s \) can lead to an increase in the number of transactions for which taxes are evaded. It follows that an increase in tax also leads to an increase in tax evasion because \( \tilde{\theta} \) decreases. This latter result is counter to that obtained in the standard model, where an increase in the tax rate cause a reduction in evasion if absolute risk aversion is decreasing with income (Myles, 1995). It is also important to notice that any policy which affects the willingness to evade tax \( \hat{\theta} \) but does not affect the ability of agents to cooperate \( \tilde{\theta} \) will have no impact on tax evasion, at least as long as \( \tilde{\theta} > \hat{\theta} \) continues to apply.

5. Conclusion

The two key results of this paper are as follows. First, when tax evasion requires the complicity of two agents (e.g., a buyer and a seller), the main determinant of which transactions are in the illegal sector is the ability of each of the participating agents to commit to undertaking the cooperative level of avoidance activity. Indeed, some agents would like to evade taxes, but cannot because of their inability to commit to cooperate. In our model, ability to commit is determined by an agent’s tolerance for dishonesty. More dishonest agents are better able to commit since their pay-off from illegal activity is higher. Second, when the discount factor is low enough, an increase in the sanction can increase the ability of an agent to commit to cooperate, and lead to more tax evasion.

Our analysis could be extended in several ways. The current model assumes
that aversion to dishonesty ($\theta$) as well as past deviations are observable. As is shown in Mongrain (2000), making these things costly to observe — in the limit unobservable — can help explain the dynamics of recidivism. Indeed, individuals may then go back and forth from the legal to the illegal sector. In the same vein, individuals could search for partners in the illegal sector rather than meet them randomly. If search costs are large enough, cheating is less likely to occur because individuals willing to transact in the illegal sector would find it more difficult to seek each other out. It would also be possible to endogenize the size of the surplus that agents obtain when transacting. For example, an individual with a low aversion to dishonesty may decide to carry out relatively large transactions in the illegal sector. Finally, the current analysis is a positive one. It would be interesting to compare optimal deterrence policy — optimal probability of detection and sanctions — in the current multi-agent framework with those that obtain in the standard tax evasion model.
Appendix

Proof of Lemma 1: If $\pi^n_i(\theta_i) \geq \pi^\ell$ and $\pi^n_j(\theta_j) \geq \pi^\ell$, punishment after deviation is in the illegal sector because both agents prefer the no-cooperation outcome. If $\pi^n_i(\theta_i) < \pi^\ell$ and/or $\pi^n_j(\theta_i) < \pi^\ell$, punishment after deviation is in the legal sector. We can find a $\theta$ which is the solution to $\pi^n(\theta) = \pi^\ell$, where $\theta$ is given by $\theta = (v - t + p_0s)/v$. QED

Proof of Lemma 2: For any two agents with $\theta \geq \hat{\theta}$, the discounted payoff of cooperating forever is $\pi^c(\theta_i)/(1 - \delta) = (\theta_i v - p_2 S - c)/(1 - \delta)$, while the discounted payoff from deviating is $\pi^d(\theta_i) + \delta \pi^n(\theta_i)/(1 - \delta) = \theta_i v - p_1 s + \delta(\theta_i v - p_0 s)/(1 - \delta)$. An agent with $\theta_i$ will choose to not deviate if $\pi^c(\theta_i)/(1 - \delta) \geq \pi^d(\theta_i) + \delta \pi^n(\theta_i)/(1 - \delta)$ which implies: $(\theta_i v - p_2 s - c)/(1 - \delta) \geq \theta_i v - p_1 s + \delta(\theta_i v - p_0 s)/(1 - \delta)$. Simplifying, we get that agent $i$ prefers to cooperate forever if: $\delta > (p_2 s + c - p_1 s)/(p_0 - p_1 s)$. QED

Proof of Lemma 3: For any pair of agents $i$ and $j$ for which at least one of the two agents has $\theta < \hat{\theta}$, the discounted payoff of cooperating forever is given by $\pi^c(\theta_i)/(1 - \delta)$, while the discounted payoff of deviating is $\pi^d(\theta_i) + \delta \pi^n(\theta_i)/(1 - \delta)$. Agent $i$ will prefer cooperating forever if $\pi^c(\theta_i)/(1 - \delta) \geq \pi^d(\theta_i) + \delta \pi^n(\theta_i)/(1 - \delta)$. We can find a $\hat{\theta}$ for which all agents with $\theta \geq \hat{\theta}$ will want to cooperate, and for which all agents with $\theta < \hat{\theta}$ will want to deviate, where $\hat{\theta}$ is given by: $\hat{\theta} = (\delta(v - t) + (p_2 s + c) - (1 - \delta)p_1 s)/(\delta v)$. QED

Proof of Proposition 2: Using $\hat{\theta}$ from Lemma 3 we obtain: $\partial \hat{\theta}/\partial s = (p_2 - (1 - \delta)p_1)/\delta v$. It is easy to show that $\hat{\theta}$ is decreasing in $s$ if $\delta < (p_1 - p_2)/p_1$. Recall that we have assumed that $\delta > (p_2 s + c - p_1 s)/(p_0 - p_1 s)$. However, it is still possible to find a range where this condition is satisfied and at the same time $\delta < (p_1 - p_2)/p_1$, as long as $p_1 c < (p_1 - p_2)p_0 s$. As for tax $t$, using equation (2) yields $\partial \hat{\theta}/\partial t = -1/v < 0$. QED

Proof of Lemma 4: An agent $i$ of type $\theta$ will prefer to cooperate in the illegal sector rather than transacting in the legal sector if $\pi^c(\theta) \geq \pi^\ell$. It is easy to see that agent $i$ will prefer the illegal sector if $\theta \geq \hat{\theta}$, where $\hat{\theta}$ is given by: $\hat{\theta} = (v - t + p_1 s + c)/v$. QED

Proof of Lemma 5: Using (2) and (3), $\hat{\theta} < \hat{\theta}$ if $(v - t + p_2 s + c)/v < (\delta(v - t) + p_2 s + c - (1 - \delta)p_1 s)/(\delta v)$, or $p_2 s + c > p_1 s$, which we have assumed to be satisfied. QED
Proof of Proposition 3: This is an immediate implication of Lemma 5 and of the description of the equilibrium. QED

Proof of Proposition 4: We know from Proposition 2 that it is \( \tilde{\theta} \) that determines which transactions are made in the legal or in the illegal sector. We also know from Proposition 1 that if \( \delta < (p_1 - p_0)/p_1 \), then \( \partial \tilde{\theta} / \partial s < 0 \). Also note that from the definition of \( \tilde{\theta} \), \( \partial \tilde{\theta} / \partial t < 0 \). Since \( F'(\cdot) > 0 \), and since the number of transactions in the illegal sector is \( (1 - F(\tilde{\theta}))^2 \), it follows that anything that decreases \( \tilde{\theta} \) also increases the number of illegal transactions. QED
References


Mongrain, S. (2000), ‘Recidivism’, Simon Fraser University, Canada, mimeo

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