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The Ultimate Control Group

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Abstract. Empirical research on the organization of firms requires that firms be classified on the basis of their control structures. This should be done in a way that can potentially be made operational. It is easy to identify the ultimate controller of a hierarchical organization, and the literature has largely focused on this case. But many organizational structures mix hierarchy with collective choice procedures such as voting, or use circular structures under which superiors are accountable to their subordinates. I develop some analytic machinery that can be used to map the authority structures of such organizations, and show that under mild restrictions there is a well-defined ultimate control group. The results are consistent with common intuitions about the nature of control in some familiar economic settings.
The Ultimate Control Group

1. Introduction

Power, authority, and control are notoriously slippery concepts, but indispensable in many areas of social science. Virtually all large organizations, for example, have formal lines of authority and a set of ultimate controllers who delegate some of their authority to others. Everyone has seen hierarchical organization charts describing systems of authority and responsibility, which presumably bear some relationship to patterns of behavior at the level of day-to-day operations (Dow, 1988). Although informal behavior often deviates from the official structure, lines of formal authority surely have economic significance.

Much of the literature on economic organization over the last few decades has taken it for granted that firms have a hierarchical structure. Investigation has centered around the costs of alternative hierarchical systems (Radner, 1992; Katzner, 1992), control loss (Qian, 1994), and the performance of hierarchies relative to polyarchies or committees as ways of screening projects (Sah and Stiglitz, 1986, 1991). While this work has been illuminating, few economists have explored decision procedures that supplement or replace hierarchy in allocating the resources contributed by input suppliers.

Many firms have authority structures that do not map nicely onto a conventional tree diagram. For example, in corporations shareholders have ultimate formal control by virtue of their right to elect the board of directors. Even if the rest of the firm has a conventional administrative hierarchy, there is not literally a single peak controller, but rather a group of shareholders who arrive at decisions through voting. Similar remarks
apply to partnerships or any other organization where resources are ultimately allocated by committee.

There are also examples where authority structures seem to have circular elements rather than the unidirectional form associated with a tree diagram. In labor-managed firms, for example, there is generally a managerial hierarchy under which workers are supervised and coordinated in the usual fashion, but top managers are hired and fired by a council that represents the workforce as a whole. Such firms also commonly rely on mutual monitoring where each worker’s performance is monitored by other workers. This sort of peer review will be familiar to members of most university departments. For these cases the standard hierarchical organization chart does not adequately summarize lines of authority.

For a variety of reasons it is necessary to devise ways of describing and classifying organizational structures that have non-hierarchical elements, preferably in a manner which has some operational content. To take one example: a labor-managed firm can be defined as an enterprise in which ultimate control rights (typically, votes) are awarded in proportion to the supply of labor, while a capital-managed firm is defined as one that awards ultimate control rights in proportion to the supply of capital. A key question is why capitalist firms are common while labor-managed firms are rare (Dow and Putterman, 2000; Dow, 2000a). But in order to test proposed explanations for the prevalence of capitalist firms, one must first know how to distinguish these kinds of firms empirically, which means identifying the ultimate controllers in a given firm and discovering what inputs they supply. Since control is exercised by groups of agents in
each case, we need some procedure for mapping control structures which can accommodate non-hierarchical elements.

A generalization of control rights to non-hierarchical settings might also be helpful in thinking about shared ownership of physical assets. The standard notion of ownership involves a right to exclude: thus, if a single person owns all of the physical assets used by a firm, that person can in effect ‘fire’ everyone else by withdrawing these assets. But in the real world, ownership arrangements are seldom so clear-cut. In the corporation, owners of common stock have a shared claim on the firm’s assets in the sense that if the firm is ever liquidated, the value of these assets will be distributed on a per-share basis after all other claimants are satisfied. No shareholder, even one having a voting majority, can unilaterally extinguish the claims of other shareholders. In other situations, however, a subset of asset owners may be able to expel a fellow owner. This can occur in labor-managed firms if one owner’s work performance is unacceptable, or in condominiums if one owner refuses to pay prescribed repair or maintenance costs. Issues involving shared asset ownership have often surfaced in the theory of the firm (Hart and Moore, 1990; Dow, 2000b).

Bowles and Gintis (1993: 376-77) offer the following definition of power: “Agent A has power over agent B if, by imposing or threatening to impose sanctions on B, A is capable of affecting B’s actions in ways that further A’s interests, while B lacks this capacity with respect to A.” The original context in which they formulated this definition was the employment relationship, where A is a boss, B is a worker, A wants to extract effort from B, and A threatens to impose the sanction that if B does not work hard enough, B will be fired. This specific framework is familiar from efficiency wage theory
(Shapiro and Stiglitz, 1984; Bowles, 1985). Clearly, however, the underlying concept extends to a wide range of other social, economic, and political settings.

Here I generalize this definition of power to organizations that can have any number of members (e.g. firms viewed as production coalitions), and allow power to be exercised by groups rather than just individuals. There is no a priori assumption that lines of control must be hierarchical. The fundamental issue is whether it still makes sense in this context to talk about the ‘ultimate’ controllers of an organization, and if so how this ultimate control group can be characterized. I show in section 2 that under mild axiomatic restrictions there is a unique control group which is complete and minimal. Section 3 applies these results to some familiar organizational structures and argues that the minimal complete control group conforms to ordinary intuitions about the locus of ultimate control.

2. Analysis

Let the set of firm members be \( N = \{1 \ldots n\} \) and let \( S \subseteq N \) be a subset of agents. Choose any individual \( i \notin S \). If the coalition \( S \), acting together, can exercise control over \( i \), I write \( S \rightarrow i \). When \( S \rightarrow i \) does not hold, I write \( S \rightarrow\neg i \).

My notion of control is similar to that of Bowles and Gintis: the ability to influence someone’s behavior. This involves the transmission of instructions from group \( S \) to agent \( i \), backed up by a threat that sanctions will be imposed if \( i \) does not follow orders. I will generally assume the sanction for non-compliance is expulsion from the firm, for several reasons: this is the penalty posited in efficiency wage theory, it is the most severe sanction normally available in market economies, and it is likely to provide
an operational criterion for control across a wide range of organizations. However, other rewards or penalties can be imagined and in situations where incentive issues are not paramount one can fall back on communication channels as an alternative criterion for defining lines of authority.

Since I want to allow for the possibility that control is exercised by groups, some collective choice procedure must be adopted by S to formulate the instructions given to an agent i, and to determine rewards or penalties if necessary. This procedure is not modeled explicitly, but would usually involve voting or bargaining among the members of S. Some concrete examples are discussed in section 3. The notation “S → i” should be read as “the members of S, if they so decide, can expel agent i from the firm even when all members of N-S are opposed”. The novelty of this approach is that control is defined as a relationship between a group and an individual rather than as a binary relationship between two agents, a superior and her immediate subordinate. When S is a singleton, the usual interpretation applies. But the relation “→“ need not generate a conventional hierarchy.

**Monotonicity Axiom.** If S ⊆ T where i ∉ T, then S → i implies T → i.

This says that if S can fire i, then so can any larger coalition of which i is not a member. It is important to keep in mind that this is a statement about what could potentially occur if all members of T act together in firing agent i. Monotonicity would not apply if the members of S agree that agent i should go, but the members of T are unable to reach such a decision.
The set $S$ is called a control group if the following two conditions are satisfied.

(A) Power over outsiders. $S \rightarrow i$ for all $i \in N - S$.

(B) Defense against outsiders. $N - S \rightarrow i$ for all $i \in S$.

Condition (A) captures the idea that a control group can exercise power over everyone who is not a member. (B) captures the asymmetry of power: no member of a control group can be sanctioned by outsiders even if everyone outside the control group acts in unison. Note that $S - \{i\} \rightarrow i$ may occur, in which case agent $i$’s peers within the control group can expel $i$ even though no coalition of outsiders can. $N$ is a control group by definition, so at least one such group exists. I assume control groups are non-empty.

**Lemma 1.** Assume monotonicity. If $S$ and $S'$ are both control groups then $S \cap S' \neq \emptyset$.

**Proof.** Suppose $S \cap S' = \emptyset$ and both are control groups. Let $i \in S'$ which implies $i \notin S$. Since $S$ is a control group $S \rightarrow i$ by condition (A). Furthermore $i \notin N-S'$ and $S \subseteq N-S'$ implies $N-S' \rightarrow i$ by monotonicity. But $S'$ is also a control group and condition (B) implies $N-S' \rightarrow i$. This is a contradiction.

**Lemma 2.** Assume monotonicity. If $S$ and $S'$ are both control groups then $S \cup S'$ is also a control group.
Proof. Consider any \( i \notin S \cup S' \). This implies \( i \notin S \). Condition (A) gives \( S \rightarrow i \) and by monotonicity \( S \cup S' \rightarrow i \). This establishes condition (A) for \( S \cup S' \). Now consider any \( i \in S \cup S' \). Either \( i \in S \) or \( i \in S' \) (or both). Suppose \( i \in S \), and assume \( N - \{S \cup S'\} \rightarrow i \). Due to monotonicity \( N-S \rightarrow i \) which contradicts condition (B) for \( S \). Thus \( N - \{S \cup S'\} \rightarrow i \). The argument is the same if \( i \in S' \). This establishes condition (B) for the set \( S \cup S' \) and this set is therefore a control group.

Lemma 1 shows that control groups cannot be disjoint, and Lemma 2 establishes that they can always be combined to obtain a larger control group. However, our goal is to go in the opposite direction by finding the smallest group which can reasonably be regarded as controllers of the organization. At this stage it is not clear whether this idea is even well-defined: perhaps there are many overlapping control groups of equal size, with the property that none contains any smaller set of agents which is also a control group.

To visualize the problem, start from any control group \( S \). Consider all sets that can be formed by deleting a single agent from \( S \) (if \( S \) has \( s \) members, there are \( s \) such sets, each with \( s-1 \) members). Under what conditions would \( S-\{i\} \) also be a control group? Several things would have to be true:

(a) \( S-\{i\} \) must retain control over everyone in \( N-S \) in order to satisfy condition (A).
(b) In addition, \( S-\{i\} \) must have control over agent \( i \), again to satisfy (A).
Expanding the complement from N-S to N-[S-{i}] must not enable the complement to control anyone who remains in S-{i}, in order to satisfy condition (B).

For a specific i ∈ S, all three conditions may hold, in which case S-{i} is a control group, or one or more of the conditions may be violated. It is possible that two or more members of S meet all three criteria, so several distinct ‘daughter’ control groups might be obtainable by deleting various agents from S. In the latter case we could delete further members from each daughter group and apply the same tests to see whether any ‘granddaughter’ control groups are generated, and so on.

There is a further complication: even if S-{i} is not a control group, deleting further agents might eventually yield a control group. For instance S-{i}-{j} might satisfy (A) and (B) even though S-{i} does not. We need an additional axiom to prevent this.

**Protection Axiom.** Suppose S → i for some i ∉ S. Then N-S-{i} →| i.

This says that if the group S can sanction a non-member i, it can also protect that person from sanctions by acting in concert with i, even if all other agents N-S-{i} want to impose sanctions. If the sanction is expulsion, for example, the axiom says that any group S with the power to expel an agent has the power to retain that person in the face of unanimous opposition. This rules out “witch hunts” where a person or group can denounce an agent but cannot defend her against denunciation by others.
Theorem 1. Assume monotonicity and protection hold. Consider any non-empty $S$ and $T$ with $T \subseteq S$ and any $i \in T$. If $S\{i\}$ is not a control group then $S-T$ is not a control group.

Proof. Since $S\{i\}$ is not a control group at least one of the following must be true. Either

(A') $S\{i\} \rightarrow j$ for some $j \in N - [S\{i\}]$

(B') $N - [S\{i\}] \rightarrow j$ for some $j \in S\{i\}$

Suppose (A') is true. If $S-T$ is a control group then $S-T \rightarrow j$ because $j \in N - [S\{i\}] \subseteq N-(S-T)$. But by monotonicity $S-T \rightarrow j$ implies $S\{i\} \rightarrow j$. This contradiction of (A') shows that $S-T$ is not a control group. Now suppose (B') is true. There are two cases: (1) $j \notin T$; or (2) $j \in T$. Consider case (1). Combining $j \notin T$ and $j \in S\{i\}$ we have $j \in S-T$ and thus $j \notin N-(S-T)$. From (B') we have $N-[S\{i\}] \rightarrow j$ which by monotonicity implies $N-(S-T) \rightarrow j$. But $N-(S-T) \rightarrow j$ for $j \in S-T$ implies that $S-T$ is not a control group. The remaining case is (2), where $i \in T$ and $j \in T$. This implies $j \notin S-T$. If $S-T$ is a control group then $S-T \rightarrow j$. By monotonicity this implies $S\{i\}-\{j\} \rightarrow j$. From the protection axiom, $S\{i\}-\{j\} \rightarrow j$ implies $N - [S\{i\}-\{j\}] - \{j\} \rightarrow j$, or $N-[S\{i\}] \rightarrow j$. This contradicts (B') so again $S-T$ cannot be a control group.

Q.E.D.

Now start from the grand coalition $N$, which is a control group by definition, and consider each possible deletion of one agent $i$. This can be pictured as an initial node at
N, with n branches leading to n sets of the form N-\{i\}. Each new set is either a control group or not. If not, Theorem 1 implies that no further deletions can yield a control group, and therefore no further steps along this branch need be considered. If some N-\{i\} is a control group, the same process is repeated starting from the N-\{i\} node, yielding n-1 branches, and so on. This procedure can generate at most n! distinct paths, and must end after at most n-1 rounds because at this point any remaining set is a singleton.

D1. A control group S is said to be terminal if no proper subset of S is a control group.

Theorem 1 establishes that if a control group cannot be obtained by deleting any individual member of S, then S is terminal.

D2. A control group S is said to be complete if for any other control group T the set S \cap T is also a control group.

The motivation for definition D2 is as follows. First observe from Lemma 1 that if S and T are both control groups their intersection is non-empty. Suppose S is not complete so there is a control group T such that S \cap T is not a control group. Since it is impossible to have S = T in this situation, either S has some members who are not in T or vice versa. Assume the former (the argument is the same in the latter case). An ambiguity now arises because the ability of S to defend its members, required by condition (B) in the definition of a control group, depends on whether the coalition T
becomes active. If T forms, then T has the power to sanction any agent i who is in S but not T.

This does not contradict the assumption that S is a control group since condition (B) only requires that N-S cannot sanction any agent in S. But agents in the intersection $S \cap T$ may be essential to T’s power to sanction $i \in S$. If so, it may be true that $T \rightarrow i$ while at the same time $N-S \rightarrow |i$ as required by (B).

The ambiguity about whether S can defend all its members becomes irrelevant if the intersection $S \cap T$ is a control group. Then one set contains the other or neither is terminal, or both. In this situation the search for a locus of ultimate control should focus on $S \cap T$.

**D3.** Let M be the union of all terminal control groups.

**Theorem 2.** Assume monotonicity and protection hold. Then M is a control group, it is complete, and any other complete control group contains M.

**Proof.** Due to Lemma 2, M is a control group because it is the union of control groups. If M is not complete there is a control group T such that $M \cap T$ is not a control group. In this case Theorem 1 implies that no subset of $M \cap T$ can be a control group. To see why, start from the grand coalition N and choose any sequence of deletions that arrives at $M \cap T$. By assumption N is a control group while $M \cap T$ is not, so at some step in the sequence an agent was deleted from a control group which led to a subset that was not a control group. From Theorem 1, further deletions after this point cannot yield a control
group, and this includes all subsets of \( M \cap T \). Moreover \( T \) is not terminal because otherwise \( T \subseteq M \) which implies \( M \cap T = T \) and this is a control group. Because \( T \) is not terminal there is a terminal control group \( S \subset T \). But \( S \subseteq M \) and \( S \subset T \) implies \( S \subseteq M \cap T \). This contradicts the fact that no subset of \( M \cap T \) can be a control group. Hence \( M \) is complete. Finally, suppose there is another complete control group \( Z \) but \( M \) is not a subset of \( Z \). Then there is some \( i \in M \) with \( i \notin Z \). By the definition of \( M \), \( i \in S \) for some terminal control group \( S \). Due to \( i \in S \) and \( i \notin Z \), \( S \cap Z \) is a proper subset of \( S \). Because \( S \) is terminal, \( S \cap Z \) is not a control group and therefore \( Z \) is not complete. This shows that \( M \) is a subset of \( Z \).

\[ \text{Q.E.D.} \]

3. **Applications**

Theorem 2 showed that there is a complete control group \( M \) which is minimal in the sense that all other complete control groups contain it. Any subset of \( M \), though perhaps a control group, is incomplete because its members are vulnerable to the vagaries of coalition formation: if the wrong coalition forms, the control group will be unable to protect some of its members. For brevity I refer to \( M \) as the **ultimate control group**.

The concept of ultimate control groups helps in describing the authority structures of organizations where collective decision-making is important, or there is some circularity in lines of authority due to feedback mechanisms that make superiors accountable to their subordinates. At the same time the analysis gives standard results for ordinary hierarchies. These points are best explained through examples.
Dictatorship. The simplest type of hierarchy occurs when one agent has comprehensive decision-making powers and does not delegate these powers to anyone else. A firm might have this structure if it is a sole proprietorship. Control is straightforward: the boss can fire anyone but no coalition excluding the boss can fire anyone. Formally, if the boss is agent 1 then for any coalition S and any agent i \notin S, S \rightarrow i if and only if S contains agent 1. The only terminal control group is \{1\}, and this is also the ultimate control group M.

Bureaucracy. Consider an organization where anyone can fire an immediate subordinate as well as all of their indirect subordinates, perhaps by successively replacing subordinates or revoking delegated authority. For example, in the Saturday night massacre Richard Nixon wanted to fire Archibald Cox, who was not a direct subordinate. Nixon removed Cox by firing both the attorney general and the deputy attorney general. In this case the coalition S can fire agent i if and only if S includes someone who is a direct or indirect hierarchical superior to i. The grand coalition is not a terminal control group because when any bottom-level subordinate is deleted from N, the resulting set N-\{i\} is also a control group: N-\{i\} can expel \{i\} and \{i\} cannot expel anyone in N-\{i\}. After deleting all bottom-level agents, agents one level up can be deleted without violating the definition of a control group. After a finite number of iterations we arrive at the peak agent in the hierarchy. This singleton is the only terminal control group and is the ultimate controller of the organization.
Democratic. Consider an egalitarian firm where the only input is labor, and decisions are reached on the basis of one worker, one vote. A strict majority is necessary and sufficient to expel any individual member. Thus for any $S$ and $i \notin S$, $S \rightarrow i$ if and only if $S$ is a strict majority. The monotonicity and protection axioms hold because votes are additive. The only terminal control groups are majorities which would have 50% of the votes or less if one worker was deleted. Every worker is a member of some such group, so the ultimate control group $M$ is the grand coalition $N$. No smaller control group is complete. To see why, choose some $S$ which is not the grand coalition. For instance let $N = \{1 \ldots 100\}$ and $S = \{1 \ldots 99\}$. Despite its size, $S$ cannot defend its members $\{51, 52, \ldots 99\}$ against the alternative control group $T = \{1 \ldots 50, 100\}$. For any control group that falls short of the grand coalition there is an alternative majority which can expel some of its members, and such control groups are incomplete. Intuitively, the symmetry of the voters means that no one can be omitted from the description of the control structure. However, if the firm also hires wage laborers who cannot vote, the ultimate control group is not the grand coalition $N$, but the smaller set $M$ consisting only of voting members. In particular, this is true if the worker cooperative hires a manager who is ineligible to vote, as long as the manager cannot fire individual workers. If the manager can fire individual workers while the workers as a group can fire the manager, the firm involves mutual control (see the discussion of circular structures below). While lacking a vote in the latter case, the manager’s power to expel team members implies that once again $M = N$.

Partnerships. Professional partnerships usually have a core group of senior members who reach decisions by majority vote, along with a set of more junior members. I assume that
a strict majority of senior partners is necessary and sufficient to expel any junior member (for now, assume senior members can never be expelled). Then for any $S$ and $i \notin S$, $S \rightarrow i$ if $i$ is junior and $S$ contains a majority of senior members; otherwise $S \rightarrow \emptyset$. The set $S$ is thus a control group if and only if includes all of the senior members. Should any senior member be omitted, this person would be a member of $N-S$ but could not be expelled by $S$, which violates condition (A) in the definition of a control group. It makes no difference whether $S$ includes any junior members since a majority of senior members can fire a junior member without the help of other junior members. The only terminal control group is the set of all senior members, and this is automatically the ultimate control group $M$. If senior partners can be expelled by a strict majority of their peers then every coalition with a strict majority of the senior members is a control group, and there are many terminal control groups. But as in the discussion of democracy, $M$ is still the set of all senior partners.

Circles. Consider a special partnership without junior members. The partners are arranged in a circle and everyone monitors the person to their immediate left. The firm holds an annual meeting at which individual performance is reviewed. Accusations of shirking by one’s designated monitor lead to expulsion but accusations by other parties are ignored. In this organization $S \rightarrow i$ if and only if $S$ includes the partner to $i$’s right. The monotonicity and protection axioms apply. For any group of the form $N-\{i\}$, the outsider $i$ can fire some member of $N-\{i\}$, namely the person to the left of $i$. Therefore $N-\{i\}$ is not a control group because it fails to satisfy requirement (B). Theorem 1 establishes that there are no control groups other than the grand coalition so $M = N$ as in
the case of democracy, despite the absence of any voting mechanism. More generally, all parties to a system of mutual control must be included in the description of a complete control group.

**Corporations.** In a corporation a shareholder has voting rights that are proportional to the number of shares owned. No shareholder can be expelled from the firm involuntarily, but a strict majority vote suffices to replace the board of directors. Indirectly this enables such a majority coalition to replace managers and even employees. As in a bureaucracy, no set that includes direct or indirect subordinates of the top manager is a terminal control group. But now there is the further complication that this top manager can also be removed by a majority of shareholders. The situation is identical to the partnership analyzed above where it was impossible for senior partners to be expelled. As in that case, the firm’s entire set of shareholders is the only terminal control group and thus (at least formally) the shareholders as a group ultimately control the firm.

These examples have been kept simple in order to convey the core ideas, but more generally Theorems 1 and 2 make it possible to identify a unique set of ultimate controllers for any authority system that satisfies the monotonicity and protection axioms. Because the control relation involves groups rather than individuals, collective choice mechanisms such as voting and bargaining can be included in the description of an authority structure along with the more conventional relation of bilateral control between superiors and subordinates. As long as one can identify all of the groups which have the
power to expel a given agent, these two theorems can be applied to large organizations or social networks that combine voting, mutual control, and hierarchy in complex ways.

Reassuringly, the analysis yields the expected result for conventional hierarchical architectures. But if managers are supervised by committees whose decisions are based on majority voting, completeness requires that all voters be incorporated into the description of the ultimate control group. The same is true for cases of mutual control, unless the agents engaged in mutual monitoring are also subject to control from hierarchical superiors. These conclusions do not resolve any of the deep problems that surround coalition formation or cycling majorities. However, they do provide a logically coherent foundation for efforts to classify firms and other organizations according to their authority structures. This should facilitate theoretical and empirical research aimed at explaining why ultimate control rights are vested in one class of agents rather than another.
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