Deterrence in Rank-Order Tournaments

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May 2007
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May 8, 2007

Abstract

In a tournament, competitors may engage in undesirable activities, or “cheating”, in order to gain an advantage. Examples of such activities include the taking of steroids, plagiarism, and “creative accounting”. This paper considers the problem of deterrence of these activities and finds that there exist special considerations that are not present in a traditional model of law enforcement. For example, an agent’s returns to cheating depend on the cheating decisions of others, and so there may exist multiple equilibria. The problem of multiple equilibria can be reduced when the first-place prize is awarded to the person that performed best without cheating. Moreover, we show that re-awarding prizes reduces the amount of monitoring required to ensure compliance. We also demonstrate that monitoring costs can be further reduced by monitoring the winner of the tournament more than the loser, and by manipulating prizes, including through the introduction of prizes for non-winners.

Key Words: Enforcement; Cheating; Tournament

JEL: K42; J41

∗We would like to thank Rick Harbaugh, Anke Kessler, Tilman Klumpp, Eric Rasmusen and Joanne Roberts and participants at the American Law and Economics Association conference (2005), the Canadian Public Economics meeting (2006) and Indiana University seminar series. Steeve Mongrain would like to thank SSHRC for financial assistance. email: pcurry@sfu.ca and mongrain@sfu.ca
“We didn’t get beat, we got out-milligrammed. And when you found out what they were taking, you started taking them.”

– Tom House (former MLB pitcher) in USA Today

1 Introduction

Tournaments are a commonly used mechanism for the allocation of resources. Examples include promotion tournaments, sporting events, patent races, and the classroom environment. Indeed, rank-order tournaments have many desirable features, particularly in the workplace. In a tournament, however, competitors may have the incentive to engage in activities undesirable to the tournament organizer in order to gain an advantage. Examples of such activities include the taking of steroids, plagiarism, ballot box stuffing, sabotage, falsification and creative accounting. Recently, scandals in tournament environments have made headlines, from steroid use in baseball, international cycling and the US Olympic track team to Tom Delay’s indictment for conspiracy to violate election laws in 2002. How to prevent such behaviors has been deemed of paramount importance, with even the US Congress getting involved with the issue of steroids in sports.

This paper considers the problem of deterrence of undesirable activities in tournaments, which we shall simply call “cheating”, and finds that there exist special considerations that are not present in a traditional model of law enforcement. Notably, in a tournament an agent’s payoff from cheating depends on the cheating decisions of others, and so there may exist multiple equilibria. Whether multiple equilibria exist depends in part on the penalty scheme adopted. Specifically, the nature of the externality caused by cheating is influenced by whether an individual finishing second

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1 See, for example Bognanno (2001), Lazear and Rosen (1981), Prendergast (1999), and Choi and Gulati (2004).


3 Prendergast (1999) engages in a general discussion of the last two topics, while Jacob and Levitt (2003) analyze a special case involving teachers.
receives the first place prize after the winner is found to have cheated. We call such a penalty scheme re-awarding. If prizes are not re-awarded, then cheating behaviors are strategic complements. If prizes are re-awarded, then agents’ decisions to cheat may become strategic substitutes. It is worth mentioning that many tournaments do re-award prizes. Promotion tournaments and elections often require that somebody claim the first place prize, but the Olympics, which does not require that somebody receive the gold, also typically re-awards medals\textsuperscript{4}. Thus, while re-awarding prizes may appear to be costly, it can be optimal for the tournament organizer because it reduces enforcement costs. Re-awarding prizes is not the only way to reduce costs. We also discuss the benefits of monitoring winners and losers differently, as well as awarding prizes for losers.

The incentive for agents to engage in undesirable activities was noted early on in the tournament literature. Lazear (1989) remarked that the ability of agents to sabotage each other reduces the attractiveness of tournaments. This spawned works specifically on sabotage. Konrad (2000) drew the parallels between sabotage and public goods provision. As the number of contestants increases, the free rider problem becomes more severe, and the amount of sabotage decreases. Chen (2003) considered equilibrium decisions to sabotage and demonstrated that more able contestants are the target of more sabotage\textsuperscript{5}. While Chen considered modifications to the tournament design, such as pay equity, seniority-based promotions, and group compensation which can reduce the incentive to sabotage, neither of these papers consider enforcement decisions explicitly. Similarly, Epstein and Hefeker (2003) and Konrad (2005) modeled lobbying and doping in a tournament framework as an alternative form of legal input. There are some papers that consider enforcement in their analysis. Berentsen (2002) and Haugen (2004) both look at doping incentives in tournament settings, but with

\textsuperscript{4}In a rather interesting example, Becky Scott, a Canadian bi-athlete initially won the bronze medal at the 2002 Winter Olympics. After one of the Russian women ahead of her tested positive for a banned substance, she was upgraded to silver. When the other Russian woman tested positive as well, she (after a long legal battle) was awarded the gold, marking the first time an athlete received the bronze, silver and gold in the same event. This case is of interest, because the Olympic Committee debated for a long time whether it was worthwhile to issue another gold medal. This paper suggests that it was.

\textsuperscript{5}Experimental results by Harbring and al. (2004) provide support for this.
the exclusion of effort choices. Finally, in a paper most closely related to ours, Kräkel (2007) introduces effort, in addition to the cheating decision via a modification of the Lazear and Rosen (1981) tournament. By concentrating exclusively on a tournament with “re-awarding”, he characterizes the conditions for the existence of an equilibrium without cheating without focusing on optimal deterrence, including such issues as multiple equilibria.

This paper combines issues of tournament design with a more traditional law and economics approach to deterrence. That is, we consider a tournament in which the organizer is able to monitor the contestants to try to detect cheating. If any contestants are caught, then the principal can impose some penalty.

In the classical models of crime, as in Becker (1968), an individual’s decision to commit crime depends only on the benefit derived from the crime, the probability of being caught, and the penalty, and does not depend on the decisions of others to commit crime. This need not always be the case. For example, if there exist increasing returns, then the payoff to crime depends on the choices of others, and multiple equilibria can be present. Increasing return can be attributed to congestion effects in deterrence as in Sah (1991), or due to social norms and stigma associated with crime as described by Rasmusen (1996), or due to coordination issues surrounding occupational choice like in Burdett, Lagos and Wright (2003) and Murphy, Shleifer and Vishny (1993).

This paper presents a model in which two identical agents compete in a tournament. They not only provide productive effort, but can also engage in cheating activities. This paper does not consider why such activities are considered undesirable, but assumes that they exist and that they are beneficial to the agent because they increase the chance of winning the tournament. The tournament organizer deters cheating by monitoring the players and can impose a penalty when agents are found to have cheated. We assume limited liability, so that the maximum penalty the organizer can impose is the stripping of any prize awarded. We consider two possible schemes for penalties. In the first, all cheaters are simply stripped of their prize, and losers do not get moved up in rank. In this scheme, cheating on behalf of one agent reduces the expected prize for other contestants, thereby reducing the
cost of cheating. Conditions are found for the existence of multiple equilibria, specifically equilibria in which either both cheat or neither cheat. In the second scheme, the first-place prize is awarded to the person that performed best in the tournament and was not found to have cheated. In this case, one agent’s decision to cheat may increase the expected prize for everyone else, thereby reducing the incentive to cheat. We find that the minimal amount of monitoring necessary is lower when prizes are re-awarded than when they are not whenever the benefit to cheating is not simply a reduction in effort.

The notion of ensuring that neither agent cheats allows for a positive analysis of the model that does not require specifying social costs of cheating and tradeoffs between effort, cheating and prizes. We examine the benefits and costs of manipulating prizes, with a particular focus on second-place prizes. We find that second-place prizes reduce the amount of monitoring required, but that this comes at an additional cost of reduced effort on behalf of the contestants. However, we find that an increase in both the first- and second-place prizes such that the difference between them does not change decreases the amount of monitoring required while not affecting the effort exerted by contestants. Kräkel (2007) also finds the first result, but does not consider the second. He points out that a smaller spread between the winner and loser prizes leads to less cheating. Thus both our and Kräkel’s paper provide a reason for tournaments to not be winner-take-all. The fact that effort is determined by the gap between prizes was pointed out by Lazear and Rosen (1981), which thus implied that the tournament organizer would want to minimize the second-place prize as much as possible. There have been other reasons which we discuss later on, for tournaments not to be winner-take-all. This paper provides another reason: the deterrence of cheating. We also discuss the implication of the limited liability assumption in the context, and explore the use of an entry fee as a way to get around limited liability.

This paper also considers the issue of differential monitoring. In particular, it considers a scheme whereby the probability that a contestant is caught cheating depends on their outcome in the tournament. Many tournaments, athletic events in particular, monitor the winners to a greater degree than the losers. We find that when this is done, less total monitoring is required.
Finally, this paper considers the possible benefit to cheating when contests are asymmetric. Specifically, it considers a tournament in which the greatest effort is expended when it is a close competition, as in Lazear and Rosen (1981), and notes that if an equilibrium could be reached in which only low ability agents cheat, this could lead to greater effort being expended. We consider the case in which the tournament organizer is not able to identify which agents are low ability, and find conditions for levels of monitoring to exist such that only low ability agents cheat in equilibrium.

2 The Model

We consider the following model of a rank-order tournament. Two agents, 1 and 2, both of whom are risk neutral, compete for a first place prize, \( A \). We allow for a second place prize, \( B \), that may be greater than zero. The probability that agent \( i \) wins the tournament is partially determined by the effort exerted by each of the competitors, \( e_1 \) and \( e_2 \), respectively. In addition, there exists another activity that can increase a player’s chance of winning. This other activity is not desired by the tournament organizer in the same way as effort and so shall be referred to as “cheating”. This activity is assumed to be a binary decision so that a competitor either cheats or does not, there is no question as to how much to cheat.

The probability that 1 wins the tournament, called the tournament success function, is given by \( P(e_1, e_2; \theta_1, \theta_2) \), where \( \theta_i \) is an indicator function that takes the value 1 when \( i \) cheats and 0 when he/she does not. Thus \( P : \mathbb{R}_+^2 \times \{0, 1\}^2 \to [0, 1] \). We assume that the tournament is symmetric so that \( P(e, e'; \theta, \theta') = 1 - P(e', e; \theta', \theta) \) for all \( e, e', \theta, \theta' \), although the relaxation of this assumption is discussed in Section 7. Let \( P_1(\cdot) > 0 \) and \( P_2(\cdot) < 0 \) denote the marginal effects of the efforts of 1 and 2, respectively. We assume that \( P_1(\cdot) = \infty \) whenever \( e_1 = 0 \) and \( P_{11}(\cdot) < 0 \), as well as corresponding assumptions on \( P_2(\cdot) \).

Cheating increases the probability of winning for the cheater, so we have that \( P(e_1, e_2; 1, \theta_2) > P(e_1, e_2; 0, \theta_2) \) and \( P(e_1, e_2; \theta_1, 0) > P(e_1, e_2; \theta_1, 1) \). Further, it is assumed that cheating and effort are separable so that an agent’s marginal effect of
effort on $P(\cdot)$ does not depend on either $\theta_1$ and $\theta_2$.\footnote{As will become clear later on, one feature of our model is that cheating displaces effort. If cheating and effort were to be substitutes in the tournament success function, our results would only be reinforced. It is possible that cheating and effort are complementary. For example, some steroids reduce the recovery time and enable the athletes to train more efficiently. If what we call cheating was to induce more effort, the principal may actually want to encourage those activities, and no enforcement would be necessary. Of course, these activities could impose other costs for the principal. For example, even if competition is fierce, cheating may degrade the enjoyment fans derive from a game, and consequently generate less revenue for the principal. We are abstracting from all those issues.} Finally, it is assumed that effort comes at a per unit cost of 1, while cheating is costless (aside from any potential penalty).

Since cheating is assumed not to be a productive activity, the tournament organizer may wish to discourage it. This paper is agnostic with regards to the determination of the activities that are considered cheating and to their social costs. The organizer can monitor the participants to detect any cheating. Denote by $\pi$ the probability that the principal catches an agent who cheats. The principal can also set the penalty for cheating. We assume limited liability on behalf of the contestants so that the tournament organizer is not able to impose a penalty larger than the removal of the prize. We consider two possible sanction schemes. The first scheme dictates that if the winner of the tournament is caught, then they are stripped of any award. The second penalty scheme also strips the cheater of their prize (if caught cheating), but in the case that the winner is caught, then awards the first-place prize to the other contestant (provided that they did get caught cheating). If both agents are caught cheating, no prize is awarded. This latter scheme shall be referred to as “re-awarding”.

The timing is as follows. First, the tournament organizer announces and commits to the prizes, the probability of detection, and the penalty scheme. Agents decide their effort and whether to cheat simultaneously. A winner is determined, prizes are awarded and the organizer audits the players for cheating. If cheaters are detected, then the penalty is imposed. In the analysis that follows, we focus on the minimal levels of monitoring required for each sanction scheme that induce an unique equilibrium in which neither agent cheats. This stems in part from the desire to remain
agnostic about the social costs of cheating as well as issues of equilibrium selection.

3 No Re-Awarding

We begin by considering the case in which the winner of the tournament is stripped of the prize if they are discovered to have cheated, but the prize is not re-awarded. For given awards and probability of detection, $A$, $B$ and $\pi$ respectively, each competitor’s expected utilities are given by

$$EU^N_1 = (1 - \theta_1 \pi) [B + P(e_1, e_2; \theta_1, \theta_2) (A - B)] - e_1,$$

$$EU^N_2 = (1 - \theta_2 \pi) [A - P(e_1, e_2; \theta_1, \theta_2) (A - B)] - e_2.$$

Note that since cheating and effort decisions are separable, the marginal return to effort is lower when a contestant cheats. This is because if the agent is caught, effort is wasted; effort is productive only with probability $1 - \pi$. Consequently, there is less incentive to provide effort when cheating. Denote by $e^N_i(\theta_1, \theta_2)$ the optimal level of effort by agent $i$ for a given set of cheating decisions.

**Lemma 1:** Each agent provides less effort when both of them cheat compared to the case in which neither cheat. That is, $e^N_i(0, 0) \geq e^N_i(1, 1)$, where the inequality is strict when $\pi > 0$.

Let us now consider the equilibria to this game. First, note that each agent has a dominant strategy to cheat when the principal does not monitor ($\pi = 0$), since it increases the probability of winning, and does not cost anything. Further, when $\pi = 1$ each agent has a dominant strategy to not cheat. Of interest is the minimum level of $\pi$ such that neither agent chooses to cheat as the unique equilibrium. For simplicity, it shall be assumed that an agent does not cheat when indifferent between cheating and not.

We begin by considering the values of $\pi$ for which there exists an equilibrium in which neither agent cheats. This equilibrium may or may not be unique. In such an equilibrium, the symmetry condition implies that each agent chooses $e^N_i(0, 0)$ as their effort and each agent has a probability a half of winning the tournament. Thus the
expected payoffs for each agent are

\[ EU_i^N(0, 0) = \frac{A + B}{2} - e_i^N(0, 0). \]

In order for this to be an equilibrium, it must be that neither agent prefers to cheat instead. Since the game is symmetric, we focus on agent 1 in the following analysis, but the same reasoning can be extended to the other contestant. The payoff associated with deviating from this strategy is given by

\[
\max_{e_1} (1 - \pi) \left[ B + P(e_1, e_2^N(0, 0); 1, 0) (A - B) \right] - e_1.
\]

Denote by \( \hat{e}_1^N(1, 0) \) the level of effort that solves this maximization problem. We therefore have that participant 1 prefers not to cheat if and only if

\[
(1 - \pi) \left[ B + P(\hat{e}_1^N(1, 0), e_2^N(0, 0); 1, 0) (A - B) \right] - \hat{e}_1^N(1, 0) \leq \frac{A + B}{2} - e_1^N(0, 0).
\]

Denote by \( \pi(0, 0) \) the level of monitoring that solves the above equation with equality. Note that \( \pi(0, 0) \) is the minimal level of monitoring that supports an equilibrium in which neither agent cheats. An equilibrium in which both agents abstain from cheating will exist for all \( \pi \geq \pi(0, 0) \). Rewriting the above equation we have that there exists an equilibrium without cheating when

\[
\pi \frac{A + B}{2} \geq (1 - \pi) \left[ P(\hat{e}_1^N(1, 0), e_2^N(0, 0); 1, 0) - \frac{1}{2} \right] (A - B) + [e_1^N(0, 0) - \hat{e}_1^N(1, 0)] \tag{3.1}
\]

Now consider an equilibrium in which both agents cheat. In such an equilibrium, each agent expends effort \( e_i^N(1, 1) \). Thus each agent’s expected payoff is given by

\[ EU_i^N(1, 1) = (1 - \pi) \frac{A + B}{2} - e_i^N(1, 1). \]

There exists an equilibrium in which both competitors cheat as long as neither agent has incentive to deviate and not cheat. The payoff associated with this type of

\[ \text{One should note that when one contestant deviates, the effort level provided by the other agent remains unchanged. This is because we assume that both effort and cheating decisions are simultaneous. This need not be the case; it is possible to imagine an environment in which an agent’s cheating decision is observable to the other contestant before the choice of effort. This alternative environment would yield qualitatively similar results.} \]
deviation is given by
\[
\max_{e_1} B + P \left( e_1, e^N_2(1, 1); 0, 1 \right) (A - B) - e_1.
\]

Denote by \( \hat{e}^N_1(0, 1) \) the level of effort that solves this maximization problem. There exists an equilibrium in which both agents cheat for all \( \pi \) such that
\[
\pi \frac{A + B}{2} \leq \left[ \hat{e}^N_1(0, 1) - e^N_1(1, 1) \right] + (A - B) \left[ \frac{1}{2} - P(\hat{e}^N_1(0, 1), e^N_2(1, 1); 0, 1) \right].
\]
Let \( \pi^N(1, 1) \) denote the level of monitoring that solves the above equation when it holds with equality.

Whenever \( \pi(0, 0) < \pi^N(1, 1) \), there exists a range of monitoring, \( \pi \in [\pi(0, 0), \pi^N(1, 1)] \), such that an equilibrium in which neither agent cheats and an equilibrium in which both agents cheat co-exist. Before considering the existence of multiple equilibria formally, it should be noted that there are two potential types of benefits one derives from cheating. The first is the increase in the probability of winning directly due to cheating. For comparable levels of effort, cheating gives an edge. However, cheaters also reduce their effort, and so it is possible for the probability of coming in first to fall once adjusting for the change in effort. Whenever the probability of winning the tournament increases with cheating (after allowing for effort to adjust), a particular form of externality is present. When one agent cheats, it reduces the expected prize for the other agent, and by doing so reduces the expected penalty associated with cheating for this second agent. Thus, the benefit of cheating is not independent of the decision of the other player. In this case, cheating decisions are strategic complements, and multiple equilibria exist for some levels of monitoring.

We now consider the condition for multiple equilibria formally. Using equations 3.1 and 3.2, we have that \( \pi(0, 0) < \pi^N(1, 1) \) if and only if
\[
\left[ \frac{1 - \pi(0, 0)}{2} - P(\hat{e}^N_1(0, 1), e^N_2(1, 1); 0, 1) \right] (A - B) - \left[ e^N_1(1, 1) - \hat{e}^N_1(0, 1) \right] > \left[ (1 - \pi(0, 0)) P(\hat{e}^N_1(1, 0), e^N_2(0, 0); 1, 0) - \frac{1}{2} \right] (A - B) - \left[ \hat{e}^N_1(1, 0) - e^N_1(0, 0) \right].
\]
\[
(3.3)
\]
\footnote{While it may seem odd to cheat if it reduces the probability of winning, it may be worthwhile because of the reduction in effort costs.}
While this condition may appear intimidating, the intuition is very straightforward. On the right hand side of the inequality, the first term represents the change in probability of winning the first place prize when deviating from an equilibrium in which neither agent cheats. Note that the agent wins the tournament with probability $P(\hat{e}_1^N(1,0), e_2^N(0,0); 1,0)$ when 1 is the sole cheater, but only gets the prize when not caught, which occurs with probability $1 - \pi(0,0)$. The second term is the change in effort costs. Thus the right hand side represents the net benefit of cheating, given that agent 2 is not cheating and exerting effort $e_1^N(0,0)$. The left hand side similarly depicts the net benefits to cheating, but when agent 2 is also cheating and exerting effort $e_1^N(1,1) < e_1^N(0,0)$. We thus have that multiple equilibria exist if, at $\pi(0,0)$, the benefits to cheating increase in the other agent’s cheating decision. We have thus demonstrated the following result.

Result 1: When prizes are not re-awarded, the necessary and sufficient condition for $\pi(0,0) < \pi^N(1,1)$ is that the marginal benefit to cheating is greater when the other competitor cheats than when she does not.

Let us consider an extreme example. First, consider the hypothetical case in which agents’ efforts are constant. That is, $e_1^N(0,0) = \hat{e}_1^N(0,1) = e_1^N(1,1) = \hat{e}_1^N(1,0)$. When this happens, we have that $P(\hat{e}_1^N(1,0), e_2^N(0,0); 1,0) = 1 - P(\hat{e}_1^N(0,1), e_2^N(1,1); 0,1)$. The condition for multiple equilibria is therefore

$$\frac{1}{2} - P(\hat{e}_1^N(0,1), e_2^N(1,1); 0,1) > (1 - \pi(0,0)) \left[\frac{1}{2} - P(\hat{e}_1^N(0,1), e_2^N(1,1); 0,1)\right]$$

which must be true as long as $\pi(0,0) > 0$. The intuition for this is that the expected cost of cheating (the foregone prize) is smaller when the other agent cheats than when s/he does not. This illustrates an important externality that is created in tournaments when prizes are not re-awarded. Namely, an agent’s decision to cheat reduces the expected prize for all other agents. Since the only penalty is the loss of this expected prize, cheating on behalf of one agent thus reduces the expected sanction, which can lead to multiple equilibria. We also wish to examine a less extreme example. Let us consider a variation of the widely used tournament success function first introduced by Tullock (1980).
Figure 1: The upper curve represents the amount of monitoring required to eliminate the equilibrium in which both agents cheat and the lower curve represents the amount of monitoring required to enable an equilibrium in which neither agent cheats, as functions of $\alpha$.

Example: Let $P(e_1, e_2; \theta_1, \theta_2) = \alpha \frac{e_1 + e_2}{\theta_1 + \theta_2} + (1 - \alpha) \frac{\theta_1}{\theta_1 + \theta_2}$, where $1 \geq \alpha \geq 0$ determines the importance of cheating in winning a tournament, and where $\frac{\theta_1}{\theta_1 + \theta_2}$ is defined as $\frac{1}{2}$ when both agents do not cheat.

In this example, the relevant levels of effort can be calculated, as well as the winning probabilities. If we let $A = 1$ and $B = 0$, Figure 1 reveals that $\pi^N(1, 1)$ is greater than $\pi(0, 0)$ for all values of $\alpha$ less than 1. Obviously, when $\alpha = 1$, no enforcement is needed since cheating is useless. On the other hand, when $\alpha = 0$ cheating is the only thing that determines the winner. If agent two cheats, agent one is losing with probability one, so the probability of detection, $\pi^N(1, 1)$, has to be equal to one. If agent 2 does not cheat, agent 1’s expected payoff is $\frac{1}{2}$ when not cheating, so the probability of detection, $\pi(0, 0)$, only needs to be $\frac{1}{2}$. □

Whenever there exist multiple equilibria, the minimum level of monitoring re-

$^9$They are $e_1^N(0, 0) = \alpha \frac{A - B}{4}$, $e_1^N(1, 1) = \alpha (1 - \pi) \frac{A - B}{4}$, $\hat{e}_1^N(1, 0) = \hat{e}_2^N(0, 1) = \max \left\{ 0, \alpha [2(1 - \pi) \frac{1}{2} - 1] \frac{A - B}{4} \right\}$ and $\hat{e}_2^N(0, 1) = \hat{e}_2^N(1, 0) = \alpha (1 - \pi) \left[ \frac{2}{(1 - \pi)^2} - 1 \right] \frac{A - B}{4}$ respectively.

$^10$The probabilities of winning are $P(0, 0) = P(1, 1) = \frac{1}{2}$, $P(1, 0) = \max \left\{ 0, 1 - \frac{\alpha}{2(1 - \pi)^2} \right\}$ and $P(0, 1) = \alpha \left[ 1 - \frac{1}{2} (1 - \pi)^2 \right]$ respectively.
quired to ensure that neither agent cheats is $\pi^N(1,1) > \pi(0,0)$. If monitoring was to be anywhere between those two cutoff points, contestants may still coordinate on the equilibrium in which both cheat. If multiple equilibria are not present, choosing $\pi \geq \pi(0,0) > \pi^N(1,1)$ is sufficient to prevent cheating. It is worth mentioning at this point that this result can be generalized to a continuous cheating decision. While the issue of multiple equilibria may or may not be present when the decision to cheat is a continuous one, an increase in cheating by one of the contestants would still have the effect of reducing the expected penalty for the other. Thus cheating decisions would still be strategic complements, and a greater degree of monitoring would still be required.

We now consider the case in which the winner of the tournament is stripped of the prize if they are discovered to have cheated and the prize is then given to the other contestant, provided they were not found to have cheated as well.

4 Re-Awarding

When prizes are re-awarded, there exists an additional means for a competitor to win the first place prize. Now, an agent wins the tournament (assuming they are not found to have cheated) if the other agent is caught cheating. Each agent’s expected utility is therefore

\[ EU^R_1(\theta_1, \theta_2) = (1 - \theta_1 \pi) \left( B + [(1 - \theta_2 \pi) P(e_1, e_2, \theta_1, \theta_2) + \theta_2 \pi] \left( A - B \right) \right) - e_1, \]
\[ EU^R_2(\theta_1, \theta_2) = (1 - \theta_2 \pi) \left( A - (1 - \theta_1 \pi) P(e_1, e_2, \theta_1, \theta_2) \left( A - B \right) \right) - e_2. \]

We begin by noting that if an agent cheats, it affects the marginal return to effort for both competitors. As before, when an agent cheats there is a chance that s/he will be caught, and any effort expended essentially goes to waste. When the prize is re-awarded, the agent that is not caught automatically wins the first place prize, and so their effort also has no impact on the outcome. The following Lemma describes the relation between effort levels and compares them to the effort levels described above when the prize is not re-awarded.

**Lemma 2:** Agents exert more effort when neither cheat as compared to the case in
which only one agent cheats. Agents exert the least amount of effort when both cheat. Consequently, $e_i^R(0,0) \geq e_i^R(1,0) = e_i^R(0,1) \geq e_i^R(1,1)$, where all inequalities are strict when $\pi > 0$.

Note that when neither agent cheats, they exert the same amount of effort as when the prize is not re-awarded. We shall thus refer to this effort level as simply $e(0,0)$. More interestingly, we note that agents exert the same effort when one agent cheats and the prize is re-awarded as when the prize is not re-awarded and both agents cheat. That is $e_i^R(1,0) = e_i^R(0,1) = e_i^N(1,1)$. It should further be noted that, in addition to the direct cost of re-awarding the prize, this system also produces less effort (as compared to no re-awarding) in all equilibria in which some cheating takes place. However, in the equilibrium in which neither agent cheats, there are neither greater prize costs nor effort displacement.

Let us consider the equilibria to this game. First, we note that re-awarding leads to a difference in an agent’s payoff only when the other competitor cheats. Thus the condition for the existence of an equilibrium without cheating remains the same. That is, there exists an equilibrium with no cheating for all $\pi \geq \pi(0,0)$. However, when the other competitor cheats, there now exists an additional means to win the first place prize. Notably, we have that each agent’s expected payoff in an equilibrium in which both agents cheat is given by

$$EU_i^R(1,1) = (1 - \pi) \left[ \frac{A + B}{2} + \frac{\pi}{2} (A - B) \right] - e_i^R(1,1).$$

There exists an equilibrium in which both competitors cheat as long as neither agent prefers not to cheat. The payoff associated with this type of deviation is the solution to

$$\max_{e_1} B + \left[ (1 - \pi) P(e_1, e_2^R(1,1); 0,1) + \pi \right] (A - B) - e_1.$$

Denote the solution to this maximization problem by $\hat{e}_i^R(0,1)$. The condition for the existence of an equilibrium in which both agents cheat is therefore

$$\pi \frac{A + B}{2} \leq \left[ (1 - \pi) \left[ \frac{1}{2} - P(\hat{e}_1^R(0,1), e_2^R(1,1); 0,1) \right] - \frac{\pi^2}{2} \right] (A - B) + \left[ \hat{e}_1^R(0,1) - e_i^R(1,1) \right] (4.1)$$

Let $\pi^R(1,1)$ be the solution to the above equation when it holds with equality.
Since this paper is interested in the minimal level of monitoring required to ensure that neither agent cheats, we again seek to find conditions such that both types of equilibria exist for a given level of enforcement. Combining equations 3.1 and 4.1 gives us that

$$\pi(0, 0) < \pi^R(1, 1)$$

if and only if

$$\left(1 - \pi(0, 0)\right) \left[ \frac{1 + \pi(0, 0)}{2} - P(\hat{e}_1^R(0, 1), e^R_2(1, 1); 0, 1) - \pi(0, 0) \right] (A - B)$$

$$- [\hat{e}_1^R(1, 1) - \hat{e}_1^R(0, 1)] > \left(1 - \pi(0, 0)\right) P(\hat{e}_1^N(1, 0), e^R_2(0, 0); 1, 0) - \frac{1}{2} (A - B)$$

$$- [\hat{e}_1^N(1, 0) - e^R_1(0, 0)] . (4.2)$$

The right hand side of the equation is the marginal benefit of cheating given that the other agent is not. The left hand side is the marginal benefit of cheating when the other agent is also cheating\(^{11}\) We have thus proved the following result.

**Result 2:** When prizes are re-awarded, a necessary and sufficient condition for $$\pi(0, 0) < \pi^R(1, 1)$$ is that the benefit to cheating be greater when the other agent cheats than when s/he does not.

While this result is in many ways similar to Result 1 when there is no re-awarding, there are important differences. To compare the two results, we first refer to the two extreme examples used in the last section. Recall that if the contestant’s effort levels were constant for all cheating profiles, there existed amounts of monitoring that lead to multiple equilibria when prizes were not re-awarded. With re-awarding this is no longer true; whenever the equilibrium with no cheating exists, it is always unique. This can be seen as follows. When effort is constant, we can rewrite equation 4.2 to yield

$$-\frac{\pi(0, 0)}{2} > 0,$$

which never holds. Thus when the equilibrium in which neither agent cheats exists, it is unique. Recall that, when prizes are not re-awarded, cheating on behalf of one agent reduces the expected penalty for the other. Thus the agents’ decisions to cheat are strategic complements. When prizes are re-awarded, however, an agent’s decision to cheat creates another means for the other agent to win the first place prize. Furthermore, if the level of monitoring is sufficient to deter the an agent from

\(^{11}\)The left hand side can be obtained from a rearrangement of $$EU^R(1, 1) - EU^R(0, 1)$$. 

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cheating when the other is not, then we have that this other means of winning is sufficient to make the expected penalty increase when the other agent cheats. The other example we considered was a modification of the Tullock contest.

**Example:** Let \( P(e_1, e_2; \theta_1, \theta_2) = \alpha \frac{e_1}{e_1 + e_2} + (1 - \alpha) \frac{\theta_1}{\theta_1 + \theta_2} \), where \( \frac{\theta_1}{\theta_1 + \theta_2} \) is \( \frac{1}{2} \) when both agents do not cheat.

As when prizes are not re-awarded, effort levels\(^{12}\), and winning probabilities\(^{13}\) can be easily calculated.

It is interesting to notice that all probabilities of winning are exactly the same as for the no re-awarding case. The problem faced by a contestant when the other agent is not cheating, is exactly the same no matter if the prizes are re-awarded or not. All effort levels and probabilities of winning are naturally going to be the same. That is precisely why \( \pi(0, 0) \) is the same in both cases. The effort level chosen by a contestant when the other contestant cheats does depend on whether prizes are re-awarded, and so the probability of winning could also depend on the regime. However, since in the Tullock contest, effort levels vary proportionally when agents decide to cheat across the two enforcement regimes, the probabilities of winning the tournament are the same in both cases. Using the same numerical example where \( A = 1 \) and \( B = 0 \), it is possible to see that multiple equilibria no longer exist when prizes are re-awarded. Figure 2 represents the levels of monitoring required to eliminate both cheating and establish neither cheating as equilibria. Contrary to the case with no re-awarding, \( \pi(0, 0) \) is always above \( \pi^R(1, 1) \). When \( \alpha = 1 \), cheating does not contribute so both probabilities of detection are equal to zero. However when \( \alpha = 0 \), contestants always want to cheat without re-awarding, no matter what the other agent is doing, so \( \pi^N(1, 1) \) must be equal to one. With re-awarding, if the probability of getting caught was 1, a contestant would never want to cheat when the other does.

While the issue of multiple equilibria may be of interest by itself, it very important to deterrence. Specifically, it is important to the minimum level of enforcement that

\(^{12}\)Effort levels are given by
\[
e^R_i(0, 0) = \alpha \frac{A - B}{2}, \quad e^R_i(1, 1) = \alpha (1 - \pi)^2 \frac{A - B}{4}, \quad \hat{e}^R_i(1, 0) = \frac{\hat{e}^R_i(0, 1)}{2(1 - \pi)^2 - 1} = \frac{2(1 - \pi)^2 - 1}{4(1 - \pi)^2 - 1} \]

\(^{13}\)The winning probabilities are given by
\[
P(0, 0) = P(1, 1) = \frac{1}{2}, \quad P(1, 0) = 1 - \frac{\alpha}{2(1 - \pi)^2}, \quad P(0, 1) = \alpha \left[ 1 - \frac{(1 - \pi)^2}{2} \right]\] respectively.
guarantees no cheating. With this in mind, we can now analyze the effect that re-awarding prizes has on deterrence.

Recall that, in any equilibrium in which neither agent cheats, we have that $e_i^N(0,0) = e_i^R(0,0) = e(0,0)$. Further, we have that $\hat{e}_i^N(1,0) = \hat{e}_i^R(1,0) = \hat{e}(1,0)$. We also know that $\pi(0,0)$ is independent of the re-awarding structure. Let us now consider a comparison of $\pi^N(1,1)$ and $\pi^R(1,1)$. Recall that equation 3.2 characterizes $\pi^N(1,1)$ when it holds with equality. We have $\pi^R(1,1)$ from equation 4.1 when it holds with equality. Using these two equations, we have that $\pi^N(1,1) > \pi^R(1,1)$ if and only if

$$\left[\frac{1}{2} - P(\hat{e}_1^N(0,1), e_2^N(1,1); 0, 1)\right] (A - B) - \left[\frac{1}{2} - P(\hat{e}_1^R(0,1), e_2^R(1,1); 0, 1)\right] (A - B) > 0$$

which simply states that the marginal benefit to cheating is greater when prizes are not re-awarded than when they are.

Note that re-awarding increases the payoff to agents when they both cheat. First, it provides an extra avenue to winning the first place prize. Second, agents exert less effort when the prize is re-awarded than when it is not. However, re-awarding

![Figure 2](image.png)

Figure 2: The upper curve represents the amount of monitoring required to enable an equilibrium in which neither agent cheats and the lower curve represents the amount of monitoring required to eliminate the equilibrium in which both agents cheat, as functions of $\alpha$. 
also increases the payoff to deviating from such an equilibrium by not cheating when
the other agent does, and this increase happens through the same means. Whether
re-awarding increases or decreases the marginal benefit to cheating thus depends on
which effect dominates. Intuitively, re-awarding should have a greater impact when
not cheating (and the other agent is), because the agent does not have to discount
this way of winning by the probability that s/he is not caught as well.

More formally, let us consider the benefit to cheating when there is re-awarding.
This benefit, denoted by $\beta$, is given by

$$
\beta = \max_{e_1} (1 - \pi) [B + [(1 - \pi) P(e_1, e_2; 1, 1) + \pi] (A - B)] - e_1
\quad - \left[ \max_{e_1} B + [(1 - \pi) P(e_1, e_2; 0, 1) + \pi] (A - B) - e_1 \right]
$$

Note that these maximization problems can be rewritten as follows.

$$
\beta = \max_{e_1} (1 - \pi) \left[ B + P(e_1, e_2; 1, 1) (A - B) + \pi \left[ 1 - P(e_1, e_2; 1, 1) \right] (A - B) \right] - e_1
\quad - \left[ \max_{e_1} B + P(e_1, e_2; 0, 1) (A - B) + \pi \left[ 1 - P(e_1, e_2; 0, 1) \right] (A - B) - e_1 \right]
$$

By writing the maximization problems this way, we have isolated a term that rep-
resents the gain from winning the prize through re-awarding. In the first maximiza-
tion problem (choosing to cheat when the other agent is also cheating), this term
is $\pi \left[ 1 - P(e_1, e_2; 1, 1) \right] (A - B)$. We now consider multiplying these terms by $\theta$. If
$\theta = 0$, then we have the benefit of cheating when there is no re-awarding, and when
$\theta = 1$, we have the benefit when prizes are re-awarded. In order to see the effect that
re-awarding has on the benefit to cheating, we need only take the derivative with
respect to $\theta$. Doing so and applying the Envelope Theorem yields

$$
\frac{\partial \beta}{\partial \theta} = \pi \left[ P(\hat{e}_1(0, 1), e_2(1, 1); 0, 1) - \frac{1}{2} - P_2(\cdot) \frac{\partial e_2(1, 1)}{\partial \theta} \right] (A - B)
$$

Note that both $\pi$ and $A - B$ are positive. It is easy to show that $\frac{\partial e_2}{\partial \theta}$ is negative, as
is $P_2$. Thus a sufficient condition for this to be negative is $P(e_1, e_2; 0, 1) < \frac{1}{2}$, which
states that cheaters have a greater chance of winning the tournament. We have thus
demonstrated the following result.

**Result 3:** The necessary and sufficient condition for $\pi_N^N(1, 1) > \pi_R^R(1, 1)$ is

$$
P(\hat{e}_1(0, 1), e_2(1, 1); 0, 1) - \frac{1}{2} - P_2(\cdot) \frac{\partial e_2(1, 1)}{\partial \theta} < 0
$$
A sufficient condition is that cheaters have a greater probability of winning the tournament \((P(\hat{e}_1(0,1), e_2(1,1); 0,1) < \frac{1}{2})\).

A corollary to this result is that the tournament organizer needs to monitor (weakly) less when re-awarding prizes than when not. If the equilibrium in which neither agent cheats is unique when it exists without re-awarding, then this result demonstrates that it will also be unique with re-awarding. In this case, the tournament organizer need only monitor with \(\pi(0,0)\) for either sanction scheme. However, if multiple equilibria exist when prizes are not re-awarded, then the organizer needs to monitor with \(\pi^N(1,1)\) in order to ensure a lack of cheating. By re-awarding, either the cheating equilibrium will be eliminated so that monitoring with \(\pi(0,0)\) is sufficient to ensure no cheating, or the tournament organizer can monitor with \(\pi^R(1,1) < \pi^N(1,1)\). Thus re-awarding can save on monitoring costs.

The above result establishes a reason for tournament organizers to re-award prizes. The next section considers the effect that the prizes themselves have on the amount of monitoring required.

5 Winner Take All, Entry Fees and Other Features

Lazear and Rosen (1981) and Tullock (1980) both pointed out that effort is a function of the gap between the first-place and second-place prizes. In particular, it has been noted that winner-take-all formats induce maximal effort at the lowest cost. This stems from the first order conditions for effort, which depend on \(A-B\). Consequently, there is no value to offering generous second prizes (or third and so on) in such a simple environment. However, in reality we often observe contests which do not adopt the winner-take-all approach; the Olympics with gold, silver and bronze medals being an obvious example. Moldovanu and Sela (2001), Singh and Wittman (2000) and Szymanski and Vallentti (2005) rationalized the existence of more than one prize by the incentives that the additional prizes create when participants are heterogeneous in their probability of winning a tournament. For example, in an 8-person heat of a race, if one contestant is heavily favored to win, the others will have more incentive to exert effort if there is a second-place prize. It should be noted that neither of the
above papers suggest that if there are $N$ contestants, then there should be $N$ prizes. In this paper, there are 2 symmetric risk neutral contestants, yet there still exists a rationale for a second-place prize.

Consider an environment in which multiple equilibria are not present. Choosing a probability of detection equal to $\pi(0,0)$ is thus sufficient to prevent cheating. Suppose that $B$ were to increase and $A$ were to stay the same. In this case, the benefits to coming in first have decreased but the expected prize will have increased. Thus the cost to cheating will have increased, and the tournament organizer will be able to monitor less in order to obtain a non-cheating equilibrium. However, recall that in any equilibrium in which no cheating occurs, effort depends on $A - B$. Thus the reduction in monitoring will have come at the cost of reduced effort. Now consider an increase in $A$, holding $B$ constant. The expected prize will have again increased, but now the benefits to coming in first will also have increased. The effect on monitoring is therefore ambiguous. The following Lemma demonstrates this formally.

**Lemma 3:** When $\pi(0,0)$ is sufficient to deter cheating, an increase in $B$ will reduce the amount of monitoring required and an increase in $A$ has an ambiguous effect. That is, $\frac{\partial \pi(0,0)}{\partial B} < 0$ and $\frac{\partial \pi(0,0)}{\partial A}$ may be either positive or negative.

Kräkel (2007) finds a result similar to the above lemma, but does not fully consider the impact of prizes on monitoring costs. Both papers assume limited liability. Without this assumption, the incentive to cheat would simply be an increasing function of $A - B$. Tournament organizers would have no reason to offer a second prize $B$. Tournaments would be winner take all, and maximal fines or punishments would be imposed on cheaters and contestants would be monitored as little as possible, as in the standard Becker (1968) result. With limited liability, the expected punishment is the expected prize. Since effort depends on $A - B$, if both $A$ and $B$ were to increase, but the difference between them was unchanged, there would be no effect on effort. However, the expected prize, and therefore the expected penalty, would increase. This would enable the tournament organizer to decrease monitoring and still obtain the non-cheating equilibrium, as demonstrated in the following Result.

**Result 4:** Increasing $A$ and $B$ such that $A - B$ remains constant leads to the same effort provided by both agents (in the equilibrium with no cheating), but reduces $\pi(0,0)$.
Increasing both prizes is equivalent to increasing the sanction associated with cheating. The tournament organizer gives something that can be taken away when bad behavior is detected. Thus if monitoring is particularly costly, the organizer may find it cheaper to offer larger prizes than to increase monitoring in order to prevent agents from cheating. The same results can be derived for $\pi^N(1, 1)$ and $\pi^R(1, 1)$.

If the tournament organizer is able to charge entry fees, however, then it may actually end up to be costless for the organizer to reduce monitoring costs in this fashion. An entry fee would not influence effort choices, nor would it affect the incentives to cheat, all else equal. An entry fee, however, could be used to fund increases in both $A$ and $B$. That is, the entry fee acts as a bond that contestants post before the tournament and do not get back if they are caught cheating. If the difference between the two prizes were held the same, then efforts would be the same, each agent’s expected payoff (net of the entry fee) would be the same, but the organizer could spend less on monitoring costs\textsuperscript{14}.

6 Differential Monitoring

The analysis thus far has assumed that the tournament organizer must monitor both competitors the same. That is, there exists a single $\pi$ that represents the probability that a cheating competitor is caught. This need not be the case. For example, when prizes are not re-awarded, one could ensure the equilibrium in which neither agent cheats in the unique equilibrium by setting the level of monitoring for competitor 1 at $\pi(0, 0)$ and for competitor 2 at $\pi^N(1, 1)$. If this were done, agent 2 would not be willing to cheat no matter whether 1 were cheating or not. Agent 1 would be willing to cheat if 2 were cheating, but not if 2 is not cheating. Since 2 definitely will not cheat, 1 will also not cheat. In this manner, the organizer could achieve the same equilibrium with less total monitoring.

A more common form of differential monitoring is to check for cheating after the outcome of the tournament has been realized and to monitor the winner to a greater degree than the loser. For example, urine tests are mandatory for Olympic medal

\textsuperscript{14}We thank an anonymous referee for pointing this out.
winners but occur only with some probability for other athletes. By doing so, the organizer can exploit the fact that when multiple equilibria are present, deterring one agent from cheating helps deter the other. However, this is not the only benefit of differential monitoring. Since cheating increases one’s chances of winning, additional monitoring of the winner has a greater impact on one’s decision to cheat. As a result, a tournament organizer can reduce the total amount of monitoring required by monitoring the winner more than the loser. This is true even when multiple equilibria are not present. It should be noted that the second effect is similar to the rank-based punishment strategy proposed by Berentsen (2002).

As in the last section, we only consider the case in which the prize is not re-awarded. Let $\pi^W$ and $\pi^L$ denote the monitoring of the winner and loser of the tournament, respectively. When an agent does not cheat, his/her choice of effort is independent of any monitoring. Thus $\frac{\partial e_1^N(0, \theta_2)}{\partial \pi^W} = \frac{\partial e_1^N(0, \theta_2)}{\partial \pi^L} = 0$, with equivalent results for 2’s effort. When an agent does cheat, effort does depend on the amount of monitoring. Given agent 2’s effort, 1’s choice of effort is determined by

$$[(1 - \pi^W)A - (1 - \pi^L)B] P_{e_1}(e_1, e_2) = 1.$$ 

With differential monitoring, the two probabilities of detection have differing impacts on effort. In particular, an increase in $\pi^W$ discourages effort while an increase in $\pi^L$ encourages it. Contingent on cheating, an increase in $\pi^W$ makes it less attractive to win due to the increase in the expected punishment. On the other hand, and increase in $\pi^L$, stimulates effort because losing implies a greater chance of being punished.

Let us consider an environment in which multiple equilibria is not a concern. The minimal amount of monitoring required to deter cheating is a variation of equation 3.1:

$$\frac{A + B}{2} - e(0, 0) = (1 - \pi^L)B + P(e_1^N(1, 0), e_2^N(0, 0); 1, 0)( (1 - \pi^W)A - (1 - \pi^L)B) - \hat{e}_1^N(1, 0).$$

Using the Implicit Function and Envelope Theorems, we find that

$$\frac{\partial \pi^W}{\partial \pi^L} = -\frac{B}{A} \cdot \frac{1 - P(e_1^N(1, 0), e_2^N(1, 1); 1, 0)}{P(e_1^N(1, 0), e_2^N(1, 1); 1, 0)} < 0$$

\(^{15}\text{Note that this does not mean that medal winners are caught with probability one.}\)
When $\frac{\partial \pi^W}{\partial \pi^L} > -1$, we have that it is possible to increase the monitoring of the winner by less than the decrease of the monitoring of the loser and still maintain an equilibrium with no cheating. The necessary and sufficient condition for this is

$$\left[1 - P\left(\hat{e}_1^N(1, 0), e_2^N(1, 1); 1, 0\right)\right] B < P\left(\hat{e}_1^N(1, 0), e_2^N(1, 1); 1, 0\right) A$$

A sufficient condition is $P\left(\hat{e}_1^N(1, 0), e_2^N(1, 1); 1, 0\right) > \frac{1}{2}$, which states that cheaters have a greater chance of winning the contest, even after they reduce their effort.

## 7 Asymmetric Contests

Going back as far as Lazear and Rosen (1981), it is well understood that when contestants’ abilities differ, effort choices may not be efficient. Intuitively, if one agent has a considerably advantage in winning the tournament, there is not much incentive to provide effort for any contestant\(^{16}\). The race is already won before it starts. If this variation in ability is observable to the tournament organizer, there are many ways to deal with this problem. For example, contestants could be sorted out into skill categories, or a handicapped form of tournament could be designed such that a high ability contestant needs to show an output sufficiently higher than a lower ability contestant’s in order to win. The ability of the contestants being observable to the organizer is a strong assumption, however. Akerlof and Holden (2006) show that even if a tournament organizer could perfectly monitor contestants’ output and without cost, it may not be desirable to do so. By constructing a tournament where mistakes are made in the allocation of the prizes (with sufficient probability, but also not too frequently), it increases the chance that a low ability individual wins the tournament, which in turn increases the incentive to provide effort. Our interpretation of this result is that making mistakes can level the playing field enough to make it worthwhile competing.

The introduction of cheating makes for an interesting extension of the above results. When tournaments are symmetric, cheating displaces effort. When tourna-

\(^{16}\)In Lazear and Rosen’s model, it is assumed that the marginal return to effort is greatest when the odds of winning are even.
ments are asymmetric, however, cheating can level the playing field, thereby promoting effort. Specifically, if low ability agents were allowed to cheat more than high ability agents, by monitoring them less for example, the competition could become closer, thereby increasing the incentive to exert effort. Obviously, this would again require that ability is observable. When ability is unobservable, an argument similar to Akerlof and Holden can be made. If the tournament organizer is able to set a uniform monitoring system so that low ability individuals are the only ones how have incentive to cheat, then effort by all agents could increase.

We will now use a simple example to show how is it possible to have an equilibrium in which only low ability individuals cheat. To simplify the analysis, we consider a model without effort. While it may seem an inappropriate assumption, given that the point is to stimulate effort, it makes the intuition clear. It will be easy to see what effect effort would have if this assumption were to be relaxed after establishing the intuition. Suppose that the first agent is high ability and has a probability of winning the tournament of \( \gamma \in [0.5, 0.75] \). Further suppose that cheating increases the probability of winning by \( \theta \in [0, 0.25] \). As before, the probability of detection is \( \pi \) and the first and second place prizes are \( A \) and \( B \). We consider only the case without re-awarding. In this simple \( 2 \times 2 \) game, an equilibrium in which only agent 2 cheats exists when \( \pi \) satisfies

\[
\frac{\theta (A - B)}{B + \gamma (A - B)} < \pi < \frac{\theta (A - B)}{A - (\gamma - \theta) (A - B)}.
\]

Note that such a \( \pi \) exists as long as \( B + \gamma (A - B) > A - (\gamma - \theta) (A - B) \), or \( 2\gamma > 1 + \theta \). Intuitively, an equilibrium in which only the low ability agent cheats can exist only when cheating narrows the gap between the two agents, but doesn’t overcome 1’s natural advantage. It should further be noted that when this equilibrium exists, it is unique. The condition for only agent 1 to cheat is that \( 1 - \theta > 2\gamma \), which is never true. With regards to effort, we have that agent 1 will increase effort because of the tighter race, while the effect is ambiguous for agent 2. Agent 2 will want to decrease effort because of the tighter race, but will want to decrease effort because of the probability of getting caught. As long as agent 1 still has the greater probability of winning after the net effects of effort, it will still be the case that an equilibrium exists in which only agent 2 cheats.
8 Conclusion

This paper examines different issues that arise when a tournament organizer wants to discourage participants from undertaking activities (which we refer to as “cheating”) that increase their chances of winning, but that are undesirable. Cheating is deterred with the standard detection and punishment scheme, although a specific form of limited liability is assumed - the tournament organizer can only confiscate any earned prize. A main result centers on the potential for multiple equilibria. We demonstrate that the tournament environment creates an externality between competitors in their cheating decisions, and so for a given enforcement effort, no cheating or both agents cheating may both be equilibria. We focus on the minimum level of enforcement that would deter cheating with probability one, and then look at ways a tournament organizer can reduce the monitoring intensity and still achieve the same goal.

When a participant is caught cheating and his/her prize been stripped, the tournament organizer faces the choice of re-awarding the prize or not. Given that many prizes are costly, it might seem that the tournament organizer would be better off by keeping the confiscated prize, but in reality re-awarding is the norm. This paper demonstrates that re-awarding prizes influences that nature of the externality associated with cheating. When a participant cheats, the other participant can win just by the fact that the cheater is being caught. The resulting positive externality not only reduces the possibility for multiple equilibria, but also reduces the monitoring necessary to ensure no cheating. Consequently, a tournament organizer can trade off the cost of monitoring with the cost of re-awarding a prize.

Awarding a second-place prize, even with only two contestants, is also a way save on monitoring costs. The presence of a second-place prize increases the cost of cheating and consequently reduces the incentive do to so. Finally, the tournament organizer can reduce monitoring costs by assigning different probabilities of detection to the different agents. Specifically, if the winner is caught with a higher probability than the loser, the incentive to win by cheating is reduced.

An important feature of this paper is that it focuses on the equilibrium in which no cheating takes place. In reality, a tournament organizer might be willing to tradeoff
costs arising from cheating with prize and monitoring costs. To be properly able to analyze such a problem, one would need a formal objective function for the tournament organizer, something this paper did not undertake in order to focus on the positive aspects of enforcement. Such a question would certainly be worthy of exploration. What would be the objective function of a tournament organizer be in this context? In other models, the organizer is assume to care about output which is a function of effort and a random component. If this is the sole objective, cheating would be undesirable if it displaces effort, or in the case of sabotage, destroys the productive efforts of others. Re-awarding, offering a second-place prizes and differential monitoring all reduce efforts, but save on monitoring costs. However, is the displacement of effort the only cost to cheating? Steroid uses increases the speed of sprinters, the number of home runs hit or the top weight lifted, but can also have many negative effects. It can jeopardize contestants’ future heath, render comparisons of performance impossible (such as the comparison of Barry Bonds and Babe Ruth), or diminish interest in the competition. For example, a July 8, 2002 USA Today poll showed that 86% of baseball fans claim that compulsory testing for steroids would renew their interest in baseball. How these different costs enter the tournament organizer’s objective function would dictate the optimal levels of enforcement, effort, and cheating. We hope that this paper would stimulate research interest on these types of questions.
9 Appendix

Proof to Lemma 1: First, let us consider the two maximization problems for a given cheating profile, \((\theta_1, \theta_2)\).

\[
\max_{e_1} (1 - \theta_1 \pi) [B + P (e_1, e_2; \theta_1, \theta_2) (A - B)] - e_1,
\]
\[
\max_{e_2} (1 - \theta_2 \pi) [A - P (e_1, e_2; \theta_1, \theta_2) (A - B)] - e_2
\]

With the separability assumption, the two first order conditions are given by

\[
(1 - \theta_1 \pi) P_1 (e_1, e_2) (A - B) - 1 = 0,
\]
\[
-(1 - \theta_2 \pi) P_2 (e_1, e_2) (A - B) - 1 = 0.
\]

By symmetry, however, we have that \(P_1 (\cdot) = -P_2 (\cdot)\), and so we have that agent \(i\)'s first order condition is

\[
(1 - \theta_i \pi) P_1 (e_1, e_2) (A - B) - 1 = 0.
\]

Consider \(i\)'s choice of effort when \(\theta_1 = \theta_2 = 0\). The first order conditions reveal that \(e_1 = e_2\), and so \(e^N(0, 0)\) is given by \(P_1 (e^N(0, 0), e^N(0, 0)) (A - B) = 1\)

Now consider \(i\)'s choice of effort when \(\theta_1 = \theta_2 = 1\). Again, the first order conditions reveal that \(e_1 = e_2\), but now \(e^N(1, 1)\) is given by \((1 - \pi) P_1 (e^N(1, 1), e^N(1, 1)) (A - B) = 1\). For all \(\pi > 0\) it is the case that \(e_i^N(0, 0) > e_i^N(1, 1)\). ■

Proof to Lemma 2: With the separability assumption, the two first order conditions are given by

\[
(1 - \theta_1 \pi)(1 - \theta_2 \pi) P_1 (e_1, e_2) (A - B) = 1
\]
\[
-(1 - \theta_1 \pi)(1 - \theta_2 \pi) P_2 (e_1, e_2) (A - B) = 1
\]

Consider \(i\)'s choice of effort when \(\theta_1 = \theta_2 = 0\). By symmetry, we have that when \(e_1 = e_2\), then \(P_1 = -P_2\), and so a symmetric equilibrium exists. Thus \(e^R(0, 0)\) is given by \(P_1 (e^R(0, 0), e^R(0, 0)) (A - B) = 1\). Note that \(e^R(0, 0) = e^N(0, 0)\).

Now consider \(i\)'s choice of effort when \(\theta_1 = \theta_2 = 1\). Again, the first order conditions reveal that again \(e_1 = e_2\), but now \(e^R(1, 1)\) is given by \((1 - \pi) P_1 (e^R(1, 1), e^R(1, 1)) (A - B) = 1\). For all \(\pi > 0\), it is the case that \(e^R(0, 0) > e^R(1, 1)\).

When only one agent cheats, the FOC for both agent are given by \((1 - \pi) P_1 (e_1, e_2) (A - B) = 1\). This implies that \(e^R(1, 0) = e^R(0, 1) = e^N(1, 1)\). ■
Proof to Lemma 3: To shorten the notation in this proof, denote $P(\hat{e}(1,0), e(0,0); 1, 0)$ simply by $P(1,0)$. We first solve for $\frac{\partial \pi(0,0)}{\partial A}$. Applying the Implicit Function and Theorem to 3.1 yields

$$\frac{\partial \pi(0,0)}{\partial A} = -\left[\frac{1}{2} - (1 - \pi(0,0)) P(1,0)\right] - \frac{\partial e(0,0)}{\partial A} \left[1 + (1 - \pi(0,0)) P_2(1,0)(A-B)\right] \div B + P(1,0)(A-B)$$

Note that the denominator is positive. In the numerator, the first term $\left[\frac{1}{2} - (1 - \pi(0,0)) P(1,0)\right]$ may be either positive or negative. The second term, $\frac{\partial e(0,0)}{\partial A} > 0$, but $\left[1 + (1 - \pi(0,0)) P_2(1,0)(A-B)\right]$ may be either positive or negative. Thus the effect is ambiguous.

A similar exercise with respect to $B$, and noting that $\frac{\partial e(0,0)}{\partial B} < 0$, yields

$$\frac{\partial \pi(0,0)}{\partial B} = -\left(1 - \pi(0,0)\right) P(1,0) + \frac{1}{2} - \frac{\partial e(0,0)}{\partial B} \left[1 - (1 - \pi(0,0)) P_2(1,0)(A-B)\right] \div B + P(1,0)(A-B) < 0$$

Proof to Result 4: The fact that effort remains the same in the non-cheating equilibrium follows directly from the first order conditions. To examine the effect on $\pi(0,0)$, we note that if both $A$ and $B$ increase by the same amount, the net effect is simply $\frac{\partial \pi(0,0)}{\partial A} + \frac{\partial \pi(0,0)}{\partial B}$.

$$\frac{\partial \pi(0,0)}{\partial A} + \frac{\partial \pi(0,0)}{\partial B} = -\left[\frac{1}{2} - (1 - \pi(0,0)) P(1,0)\right] - \frac{\partial e(0,0)}{\partial A} \left[1 + (1 - \pi(0,0)) P_2(1,0)(A-B)\right] \div B + P(1,0)(A-B)$$

From the first order conditions for effort choice, it can easily be shown that $\frac{\partial e(0,0)}{\partial A} = \frac{\partial e(0,0)}{\partial B}$. This implies that

$$\frac{\partial \pi(0,0)}{\partial A} + \frac{\partial \pi(0,0)}{\partial B} = -1 + 2 \frac{\partial e(0,0)}{\partial A} (1 - \pi(0,0)) P_2(1,0)(A-B) \div B + P(1,0)(A-B) < 0.$$
10 Bibliography


