Excess Demand and Rationing: Selling to an Input

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Abstract. This paper develops a model that explains the persistence of excess demand for some goods. It offers that, for some goods, consumers care about who else is consuming the good. As such, their willingness to pay depends on their beliefs about the other consumers. We demonstrate that screening mechanisms that impose costs in negative correlation to an individual’s (positive) externality can increase profits while appearing to generate excess demand. We feel that such a model is appropriate in that casual observation seems to indicate that it does well in predicting which goods would use such a screening mechanism and which would not.

Keywords: equilibrium excess demand, pricing, distributional waits, scalping

JEL Classification:

1. Introduction

There are some goods for which excess demand seems to be the norm. These goods include concert and sporting tickets, as well some games and toys1, especially when they are first introduced. The fact that excess demand is so persistent for these goods suggests that it may in fact be optimal for the producer. The literature has produced several models that lead to the creation of excess demand being part of a price setter’s profit maximization behavior. Many of these models entail demand uncertainty, either on behalf of the price setter2, or on behalf of the consumer3. Other models tell a story of price discrimination between low and high value customers4.

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1See, for example Wolpin’s (1989) analysis of the excess demand for Nintendo’s Super Mario Brothers.


This paper offers another explanation, in which customers care about who else is purchasing the good and rationing can act as a screening mechanism.

We consider an environment in which the utility an individual derives from the consumption of a good depends on the attributes of the other consumers. For example, the enjoyment of a concert, movie or sporting event can be influenced by the behavior of others in attendance. For popular music concerts and most sporting events, being part of an audience that cheers lustily can greatly enhance the experience. Such behavior could detract from the enjoyment of an opera, however. Many children’s toys are designed to be played with by more than one child at a time. If the toy is such that a skill at playing the game is developed over time, such as with video games, then the child’s enjoyment of the toy would be increased if his or her friends also had the toy and developed their skill at a similar pace. In such an environment, an individual’s willingness to pay depends on who else in consuming the good. As such, individuals are not just potential customers, they are also inputs.

If a monopolist had perfect knowledge about each person’s willingness to pay and their quality as an input to other people’s enjoyment of the good, it could face a tradeoff between selling to a person willing to pay a lot for the good but did not do much for the enjoyment of other people, and one that was not willing to pay very much but contributed greatly to other people’s experiences. In such a case, it could certainly be possible that the monopolist would prefer to sell to the latter, or the one with the lower willingness to pay. In the absence of such information, however, the monopolist would like to find a mechanism that screened potential customers according to these characteristics in order to achieve maximal profits. The use of such a screening mechanism would lead to the appearance of excess demand. That is, there would be people who did not purchase the good that were willing to pay more than the monetary price but were not willing to do whatever the screening mechanism required. One of the contributions of this paper is to explicitly model the rationing process so that the market is in equilibrium once all costs are taken into consideration.
The screening mechanism we consider in this paper entails imposing costs on the customer that are inversely proportional to that individual’s quality as an input. We consider such a mechanism because we feel that examples can be found. The purchase of tickets for concerts, or even some movies such as the Star Wars films, often requires lining up for lengthy periods of time. For example, Star Wars: Episode I - The Phantom Menace opened on May 19th, 1999 worldwide. In the United States and Canada, ticket lineups started more than a month in advance\(^5\). While the opportunity cost of time may have varied across the people in line, lining up was as much a social event as it was a means to get tickets\(^6\). Bands often release tickets through their fan clubs and sites first, and radio stations frequently hold contests for tickets and goods (such as the new Nintendo game system) in which people must demonstrate how much of a fan they are (i.e. their quality as an input). It seems that it is a regular occurrence that goods and tickets are often made available to a subset of the population before becoming widely available. This model predicts that this group of people should be “desirable” in the sense that others would be willing to pay more knowing that this group has already purchased.

Formally, we introduce a screening mechanism by incorporating ideas of distributional waits (Bucovetsky, 1984) and waiting time auctions (Holt and Sherman, 1982). More precisely, we consider the monopolist to be selling the commodity under a two part pricing mechanism, one part of which is a regular (monetary) price, the other part of which is a non-monetary price, which can be thought of as any cost a potential customer has to jump through in order to obtain a ticket. Whatever the precise component, the important part is that it introduces a utility cost to customers (which is separate and potentially differently distributed than the monetary cost across customers). In our model, any mechanism that imposes fewer costs on customers that


\(^6\)See http://www.liningup.net for photos and descriptions of the events held during the lineups for Star Wars Episodes 1 - 3.
have higher qualities as inputs would be effective for selecting the most desired customers. We call such a mechanism a “line-up” even if it does not entail actually waiting in a line. By asking customers to wait in line in order to purchase a ticket, the promoter can ensure that low-quality customers, who are willing to pay a higher monetary price but are less willing to stand in line, do not attend. This leads to a better concert experience (a higher level of the consumption externality) which increases everyone’s willingness to pay. We begin by showing that the monopolist maximizes profits with a line-up when there exists a negative correlation between quality as an input and willingness to pay if the amount high-quality (but low income) customers are willing to pay for the best concert experience is greater than the amount the low-quality (but high income) customers are willing to pay for the worst concert experience.

We then consider whether correlation between a consumer’s willingness to pay and her desired attribute is necessary for line-ups. We address this point by considering a version of the model where both attributes of consumers are (jointly) uniformly distributed. In this setting it can be shown that some line-up will be used if the value of the externality to a consumer is large enough, where this critical value (positively) depends on the capacity constraint.

This model generates some testable predictions about the use of screening. In the model, buyers of tickets have two private attributes: one is their willingness/ability to pay (which is a function of who else is consuming), the other is a private attribute which contributes to a (positive) consumption externality. If an agent’s quality as an input is positively correlated with their willingness/ability to pay, then we show that the monopolist will simply clear the market by setting a high enough price. In other words, for events such as the opera, where viewing members of “high society” may be part of the enjoyment, we should not expect to observe persistent excess demand. However, if the desired attribute is either negatively or not correlated with willingness/ability to pay, then the promoter may choose to screen for the desired

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7 A January 27, 2007 search of the New York Craigslist site (http://www.craigslist.com) found 397 posts from people either looking for or selling Knicks tickets. A random sampling of these posts found that sellers always mentioned the face value and mentioned they would take the best offer.
attribute using the non-monetary price. If it is true that tickets to sporting events typically either go to true fans or corporations (who would be low-quality as an input, but have high willingness to pay), then we should expect to see line-ups and promotions to fan clubs as common occurrences, which they seem to be.

The idea that people may care about who else consumes a good is not wholly new, given the fairly broad literature going back to Veblen’s (1899) observation that one consumer’s demands may well depend on those of other consumers. A common name for this effect is that of “social externalities,” as in Becker (1991), although Becker considers the valuation a consumer places on a commodity to depend on the level of consumption by others as opposed to the attributes of those others. Becker shows how the dependence of one consumer’s demand on aggregate quantity demand can lead to a positive relationship between price and aggregate quantity demanded. Together with a capacity constraint in this upward sloping region, the profit maximizing firm may then be able to increase price without lowering quantity demanded and (if constrained) without lowering sales.

DeSerpa and Faith (1996) construct the analogous result for a “mob good”. In their setup customers are heterogeneous in an attribute (they call it “noise”) which increases the willingness to pay of all customers, but which is inversely related to customer’s base willingness to pay. The monopolist then has an incentive to reduce price in order to attract higher noise customers. With a capacity constraint and line-up customers are chosen randomly to obtain a ticket, and the resulting expected level of noise exceeds that of the situation where the price is market clearing. This makes no assumption as to the correlation between an individual’s quality as an input and their willingness to pay and is much more explicit about the rationing mechanism. Both of these models have customers who, in equilibrium, are rationed. That is, customers who are willing to pay more than the monetary price but do not obtain a ticket. This raises the usual question of how the demand behavior of those customers ought to be modified in order to be consistent. After all, normal demand curves are

Another search found 82 posts for opera tickets. A random sampling of these posts found that sellers mentioned face value and indicated a selling price below.
derived under the assumption (by the consumer) that any quantity she demands can be obtained. In this paper we present a model which is consistent.

In our model, the marginal consumer is just indifferent between attempting to purchase a ticket and not doing so in equilibrium. In both Becker (1991) and DeSerpa and Faith (1996) it is not clear if the marginal customer actually would line up, since no explicit allowance is made for the fact that some customers do not get served. While in the versions presented here the marginal customer is also served, that is not necessary. The model can easily be extended to accommodate some customers not being served. Put differently, in the model presented below the market clears \textit{ex ante} if price and line-up costs are taken into account. Nevertheless this model can address excess demand as it is described in, for example, Becker (1991): Once tickets are allocated via the monopolist’s mechanism, opportunities for re-trading exist. More precisely, at the posted face value of a ticket (but without having to incur line-up costs) there exists excess demand, i.e., many more customers are willing to pay the face value than there are seats. The presence of scalpers is only indicative of this fact, not actual unserved customers at the initial selling stage.

An interesting implication for the resale of tickets arises when promoters use line-ups as a screening mechanism. If customers are allowed to resell their tickets at any price, then promoters are unable to use line-ups as a screening mechanism. High income customers with high line-up costs (but with low quality) are willing to pay more for a ticket in the absence of a line-up. So, once low income customers with low line-up costs have purchased a ticket, they would like to sell their ticket to a high income customer. However, in this framework there is a good reason for anti-scalping legislation. The seller and buyer are also in a second supplier-demander relationship where the buyer of the ticket supplies the input. If the buyer is allowed to resell, then that is tantamount to input substitution. The quality of the input will be affected and thus the buyer of the input suffers economic damages if the supplier

\footnote{In the model as presented, customers in the line-up can be viewed as all being served at the same time. If a waiting line auction as in Holt and Sherman (1982) is used instead, customers are served first-come-first-served, and thus have to line up earlier in order to ensure tickets. The only modification from a standard auction is the fact that loosing customers also pay a price.}
of the input is allowed to substitute (resell). As a result, the concert experience is diminished for all. Resale, however, would be foreseeable, and so the promoter would not be able to use line-ups as a screening mechanism and so would sell only to the high income/low quality customers at a lower price. Anti-scalping laws are thus efficiency enhancing/preserving. If resale is legal only at the posted price, then the ticket holders will not want to resell, and so the input quality is preserved. Courty (2003) also considers the implications of resale. He finds that monopolists cannot do better by allowing resale, which is also true in our model. Our model has implications for efficiency, however, which are not present in Courty.

The outline of this paper is as follows. In the next section we present the general model and demonstrate that equilibria with positive line-ups exist. Section 3 will consider a version with perfect (positive or negative) correlation between the variables, in which case more precise results can be obtained. Section 4 considers the effect of anti-scalping laws. The conclusion follows.

2. The Model

There is a continuum of potential customers and a monopolist. While this model applies broadly, we shall use terminology applicable to the sale of concert tickets. As such, we shall refer to the monopolist as a concert promoter and to the sale of concert tickets. Customers will similarly be referred to as fans and concert attendees. Each customer is characterized by two values, \((v_i, q_i)\), normalized to lie in \([0, 1]^2\). The value of these is private information to the customer. The interpretation of \(v_i\) is the valuation of that customer for a ticket to a concert of minimal quality. The interpretation of \(q_i\) is that customer’s ‘quality’ as a concert goer. This measures the amount of (positive) externality that the given customer contributes to the concert experience if he attends.\(^9\) Suppose that there is a unit mass of consumers who are distributed on the unit square \([0, 1]^2\) according to some probability density \(f(v, q)\).

\(^9\)In other words, \(v_i\) is a (private) consumption value for the customer, while \(q_i\) is a (private) production value of the customer to the promoter.
Let $f_V(v)$ and $f_Q(q)$ denote the marginal distributions and let $f_{V|Q}(v|q)$ and $f_{Q|V}(q|v)$ denote the conditional distributions.

Aside from the idiosyncratic private valuation of the concert, given by $v_i$, there also exists a common value component, made up from the contributions of all those attending through their $q_i$. Denote this common value component, or concert experience, by $e$. It is assumed that the individual contributions to the concert experience are aggregated in such a way that $e$ depends only on the average quality of those in attendance and that all consumers have the same valuation of the concert experience. Specifically, it is assumed that there exists a continuous and differentiable function $e(q)$, where $q$ denotes the average $q_i$ of the attendees. Note that since there is a continuum of customers, a single individual’s contribution to the concert experience is zero. This formulation also has the advantage that it does not build in a preference for either large or small events.

Concert tickets are sold via a two part pricing system: one part is a monetary price, denoted $p$, which corresponds to the face value of the ticket. The second part is a non-monetary component, denoted by $\ell$, which can be thought of as the line length or any other special procedures a fan has to follow in order to qualify for purchasing a ticket. Other such features may include the cost of having to make plans far in advance of the concert date, or having to spend time searching for the exact moment of the start of online ticket sales. Each potential customer has a money-equivalent cost for $\ell$. The cost of lining up may depend on various personal characteristics, such as one’s wage (i.e. the opportunity cost of time). One characteristic which we wish to consider is the customer’s quality component $q_i$. In particular, we wish to consider the possibility of an inverse relationship between $q_i$ and the cost of lining up. This may arise from a greater sense of anticipation while physically lining up, or from easier access to information about online ticket sales. The cost of lining up is thus denoted by $C(q, \ell)$, where $C_\ell(\cdot) > 0$ and $C_q(\cdot) < 0$. 

3. Profit Maximization

The promoter maximizes profits by choosing \((p, \ell)\):

\[
\max_{p, \ell} (p - c) N(p, \ell) - F
\]

where \(c\) is the (constant) marginal cost of an additional ticket sale. Such costs may include the printing of the ticket and the marginal cost of renting a larger venue. \(F\) denotes the fixed costs. The first order conditions for this problem are

\[
(p - c) \frac{\partial N (\cdot)}{\partial p} + N (\cdot) = 0
\]

\[
(p - c) \frac{\partial N (\cdot)}{\partial \ell} \leq 0
\]

where the derivative with respect to \(\ell\) holds with equality if the optimal level of \(\ell\) is positive. Given that \(p > c\) for positive profits, a necessary condition for the promoter to use a lineup is \(\frac{\partial N (\cdot)}{\partial \ell} > 0\) for some \(\ell\). That is, aggregate demand must be increasing in the lineup length at least somewhere.\(^{10}\) In order to analyze aggregate demand, the following section considers the behavior of consumers in response to changes in ticket prices and lineup lengths.

4. Consumer Behavior

Total customer utility from the purchase of a ticket for a concert with externality level \(e\) at price \(p\) and line length \(\ell\) is given by

\[
V_i = v_i + e(q) - p - C(q_i, \ell).
\]

The reservation utility level is normalized to 0 for all consumers. A given consumer assuming a concert experience of \(e(q)\) will therefore purchase a ticket at \((p, \ell)\) if

\[
V_i(p, \ell, q) = v_i + e(q) - p - C(q_i, \ell) \geq 0.
\]

Hence, we get a relationship between \(v\) and \(q\) with consumers who have higher \(v\) and/or \(q\) buying, and those with lower values not buying. This defines a line in \([0, 1]^2\)

\(^{10}\)A sufficient condition for the second order conditions to hold is \(N_{pp} (\cdot) < 0, N_{\ell\ell} (\cdot) > 0.\)
Figure 1. Different Purchasing regions

\[ v = p - e(q) + C(q, \ell). \]

Note that \( \frac{dv}{dq} = C_q(q, \ell) \leq 0 \), and that the region of buying customers is convex if \( C_{qq}(q, \ell) \geq 0 \). Some examples are given in Figure 1. In Figure 1 the lines denote marginal customers for different values of \( p, q, \) and \( \ell \). All customers above a given line wish to purchase a ticket at these values, all those below do not.\(^{11}\) Also note that a change in either \( p \) or \( e(q) \) causes a (parallel) shift of the line of marginal consumers (up if either \( p \) increases or \( e(q) \) decreases.) However, a change in \( \ell \) shifts the curves up but also changes the slope. The slope becomes steeper if \( C_{\ell q}(\cdot) < 0 \) and flatter if \( C_{\ell q}(\cdot) > 0 \).

\(^{11}\)This follows from the fact that \( \partial V(\cdot)/\partial v_i = 1 > 0 \) and \( \partial V(\cdot)/\partial q_i = -C_q(\cdot) > 0 \) by assumption.
The number of customers and their average quality are found by simultaneously solving the following two equations:

\begin{align}
N &= \int_0^1 \int_{p-e(\bar{q})+C(q, \ell)}^1 f(v, q) \, dv \, dq \\
\bar{q} &= \frac{1}{N} \int_0^1 \int_{p-e(\bar{q})+C(q, \ell)}^1 q f(v, q) \, dv \, dq
\end{align}

Since these equations are continuous maps from \([0, 1]^2\) to \([0, 1]^2\), a fixed point occurs and there exists a solution. It is assumed that the corresponding Jacobian matrix is positive semi-definite so that this solution is unique for every pair \((p, \ell)\). Denote the determinant of the Jacobian by \(|J|\). Of particular interest is how \(N\) and \(\bar{q}\) change as \(p\) and \(\ell\) change. The partial derivatives of \(N\) and \(\bar{q}\) with respect to \(p\) are as follows:

\begin{align*}
\frac{\partial N}{\partial p} &= -\frac{\int_0^1 f(v, q) \, dq}{|J|} < 0 \\
\frac{\partial \bar{q}}{\partial p} &= \frac{\int_0^1 q f(v, q) \, dq - \frac{1}{N} \int_0^1 f(v, q) \, dq \int_0^1 q f(v, q) \, dv \, dq}{N |J|} \\
&= \frac{E[v|v = \bar{v}] - \bar{q}}{N |J|}
\end{align*}

where \(\bar{v} = p - e(\bar{q}) + C(q, \ell)\). Note that an increase in \(p\) always decreases attendance while the effect on average quality is ambiguous. In particular, the average quality of attendees will increase (decrease) if the marginal attendee is of greater (lesser) quality than the average. It should be further noted that if \(v\) and \(q\) are independent so that \(f(v, q) = f_v(v) f_q(q)\), then an increase in \(p\) has no effect on average quality or concert experience.

The effect of \(\ell\) on \(N\) and \(\bar{q}\) is more complex and depends on \(C_{q\ell}(\cdot)\) as well as the correlation between \(v\) and \(q\). Specifically, the partial derivatives are

\begin{align*}
\frac{\partial N}{\partial \ell} &= -\frac{1}{|J|} \left[ \int_0^1 C_{\ell}() f(v, q) \, dq - \frac{e'(\bar{q})}{N} \int_0^1 q f(v, q) \, dq \int_0^1 C_{\ell}(\cdot) f(v, q) \, dq \\
&\quad + \frac{e'(\bar{q})}{N} \int_0^1 f(v, q) \, dq \int_0^1 q C_{\ell}(\cdot) f(v, q) \, dq \right] \\
\frac{\partial \bar{q}}{\partial \ell} &= -\frac{1}{N |J|} \left[ \int_0^1 q C_{\ell}(\cdot) f(v, q) \, dq - \bar{q} \int_0^1 C_{\ell}(\cdot) f(v, q) \, dq \right]
\end{align*}
If lineup costs are independent of input quality, then \( \frac{\partial N}{\partial \ell} = - \frac{C_{\ell}(1)}{|J|} \int_0^1 f(v, q) \, dq < 0 \) and the promotor would never use lineups when maximizing profits. This leads us to the following lemma:

**Lemma 1.** A necessary condition for a promotor to use lineups in order to maximize profits is that there exists correlation between an agent’s quality as an input and his/her willingness to line up. Formally, a necessary condition for the promotor to choose \( \ell > 0 \) is \( C_{\ell q} < 0 \).

However, contrary to the impression left by previous work, it is not necessary that there exist correlation between an individual’s quality as an input and his/her valuation of the concert:

**Proposition 1.** Correlation between an individual’s quality as an input, \( q \), and his/her willingness to pay, \( v \) is not necessary for a promotor to use a lineup to maximize profits.

This proposition is proved via an example. Suppose \( v \) and \( q \) are independent and distributed uniformly on the unit square. Then \( f(v, q) = f_Q(q) = f_V(v) = f_{Q|V}(q|v) = f_{V|Q}(v, q) = 1 \). Let the consumer’s utility from the concert experience be \( e(\overline{q}) = \alpha \overline{q} \), \( \alpha \leq 2 \) and the cost function be \( C(q, \ell) = (1 - q) \ell \) so that \( V_i = v_i - p + \alpha \overline{q} - (1 - q)\ell \). If the promotor does not use a line, then the average quality will be \( \frac{1}{2} \) for any price he might set and he maximizes profits by choosing \( p = \frac{1}{2} + \frac{\alpha}{4} \).

Maximal profits without a line are therefore \( \left(2(1-\alpha) + \alpha\right)^2 - F \).

Note that if the promotor does use a line, then \( v = p - \alpha \overline{q} + (1 - q)\ell \) defines a linear relationship between \( q \) and \( v \), where \( \frac{\partial v}{\partial q} = -\ell \). Suppose that the promotor chooses the line length and price such that consumers in the area depicted in Figure 2 attend the concert. Note that the slope of the line in the figure is \(-2\), so that the line length must be 2. The area of this region is \( \frac{1}{4} \). Since the average quality of consumers in this region is \( \frac{5}{6} \), the promotor must choose a price of \( \frac{5\alpha}{6} \) in order for this to be the equilibrium attendance.
Figure 2. When the promoter chooses $\ell = 2$ and $p = \frac{5\alpha}{24}$, consumers in the region in the upper right corner above the line attend the concert.

The promoter’s profits in this scenario are therefore $\frac{5\alpha}{24}$. These profits are greater than without a line when

$$\frac{5\alpha}{24} > \left(\frac{2(1 - c) + \alpha}{4}\right)^2$$

$$\alpha \in \left(\frac{6c - 1 - \sqrt{24c - 35}}{3}, \frac{6c - 1 + \sqrt{24c - 35}}{3}\right)$$

For example, if $c = 2$, then the promoter makes greater profits from this line and price pair when $\alpha \in (2.4, 4.8)$, approximately. If $\alpha = 4$, then the promoter makes profits of $\frac{1}{4}$ without a lineup and profits of $\frac{1}{3}$ with this particular lineup.

5. Conclusion

This paper has added to the literature on social externalities and mob goods in two dimensions. One is that it presents a closed model in which line-ups are consistent with equilibrium behavior. The other is that it corrects a possible misconception left by previous work, namely that a negative correlation between a consumer’s quality and her willingness to pay for the good is necessary. What is necessary for line-ups to occur in equilibrium (and possible rationing), is that the consumer’s quality and utility cost of line-ups be correlated. It is not necessary, however, that the correlation extend to the willingness to pay.

Given that a positive social externality is supplied by a high quality consumer (however that may be defined) the profit maximizing firm has an incentive to screen
for just such consumers for the sole reason that it increases profits. It coincidentally also increases social welfare. Scalping, that is, the resale of tickets without the utility cost of waiting, would jeopardize both. The firm loses profits because only high willingness to pay (but low quality) consumers could be attracted, but social welfare also declines, since the level of the externality is reduced. Anti-scalping legislation therefore is not necessarily an attempt to increase private profits, but can be viewed as a welfare enhancing measure, a point first observed by DeSerpa and Faith (1996). It is interesting to note that it is not resale per se, but resale at a higher than posted price which is the problem, since at the posted price no ticket holder would wish to sell. Hence the observed policy of allowing resale at prices up to the face value appears to be welfare maximizing.

References


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