How Importers May Hedge Demand Uncertainty

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Abstract

This paper examines how firms deal with demand uncertainty when importing intermediate goods takes time, and orders have to be placed before the realization of demand is known. We consider two strategies to hedge this uncertainty: building up inventory of imported goods, and relying on more expensive domestic supplies to cover peak demand. Which strategy is optimal depends on the price of imported relative to domestic goods, and on the degree of demand uncertainty. We also show that there are relative import prices and degrees of demand uncertainty for which the firm chooses not to hedge uncertainty and may thus stock out. The optimal hedging strategy implies a non-monotonic relationship between firm-level output volatility and the relative import price.

JEL classification: F12, L81.

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1 Introduction

It is by now well documented that the time required for goods to be shipped between countries and to clear customs represents a significant barrier to international trade (Hummels and Schaur, 2013). This barrier is especially big for firms exposed to demand uncertainty, since by the time goods arrive at their destination, the quantity shipped may no longer be optimal. Consider, for instance, a firm that sources an input from abroad. Given the delay associated with international trade, the firm has to order the input before knowing the realization of demand. If demand turns out to be bigger than expected, the firm risks a stockout, which may mean idling production capacity and/or foregoing profitable sales opportunities. In case of lower-than-expected demand, the firm may accumulate stocks of unused goods that may be costly to store or subject to depreciation or spoilage.

Several recent papers have explored strategies a firm engaged in international trade may use to hedge demand uncertainty. Evans and Harrigan (2005), for instance, show that retailers tend to source items subject to uncertain demand from nearby countries, even if production costs there are higher than in alternative, more distant locations. Hummels and Schaur (2010) explain that firms may resort to expensive but fast air shipments to cover peak demand, while relying on slow but inexpensive ocean shipping for baseline demand. Novy and Taylor (2014) argue that to protect against demand uncertainty firms tend to keep much larger inventories of traded goods on hand than of domestically sourced goods.

In this paper we pursue a more general approach in which both dual sourcing as in Hummels and Schaur (2010), and inventory buildup as in Novy and Taylor (2014) may arise as special cases. We explore the firm’s optimal strategy choice in a model in which the firm may source a homogeneous input good abroad or domestically. Import prices are

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1 Ocean shipping times between various ports around the world can be downloaded from several webpages, such as, https://www.searates.com/de/reference/portdistance or https://www.championfreight.co.nz/times.pdf. The World Bank provides information on the time required for border and documentary compliance when goods are exported: http://www.doingbusiness.org/data/exploretopics/trading-across-borders.

2 Strategies of this sort have been extensively analyzed in the operations management literature, where they are known as dual sourcing strategies. See Boute and Van Mieghem (2015) for a recent article and literature review.

3 See also Alessandria et al. (2010a, 2010b) on the need of firms to hold greater inventories of traded than of domestic goods.
lower than domestic prices, but imports have to be ordered before demand is known, and they take one period to arrive; imports are hence the cheap, but inflexible source. Orders for domestic goods can be placed after demand has been observed; domestic suppliers are hence the flexible, but expensive source.

When determining how much to produce, the firm makes two decisions in every period. First, it decides whether to source inputs from abroad and, if it does, how much to import knowing that these inputs take time to be delivered. Second, it decides whether to source inputs domestically and, if so, how much to buy knowing that these purchases can be used immediately. These decisions obviously depend on the price difference between imported and domestic inputs, and they depend on the degree of demand uncertainty on the output market. Not surprisingly, a firm facing no uncertainty only chooses the cheaper source. The degree of uncertainty on the output market is thus essential for choosing to simultaneously order from both the foreign and the domestic source. Our model is simple enough to show that, as the imported input becomes cheaper relative to the domestic input, for instance due to a reduction in trade costs, the firm optimally sources a greater share of inputs from abroad for any given level of demand uncertainty, and builds up an inventory of imported inputs. Once the import price is sufficiently low, the firm finds it optimal to phase out domestic sourcing altogether, and hedge demand uncertainty entirely through inventory build-up.

Why is this analysis important, and what does our approach allow us to demonstrate that could not have been shown simply by considering each hedging strategy in isolation? First, we can show that the response of imports and of inventory of imported goods to a mean-preserving increase in demand uncertainty depends non-monotonically on the price differential between imported and domestic inputs. Specifically, an increase in demand uncertainty turns out to reduce imports and inventory when the price differential is sufficiently small, but to increase imports and inventory when the price differential is sufficiently big. This is because the firm relies more on domestic sourcing to hedge demand uncertainty when the price differential is small, but more on inventory build-up when the price differential is big. By allowing only one way to hedge demand uncertainty, one may falsely conclude that an increase in demand uncertainty either reduces imports and inventory (because the firm raises the share of domestically sourced inputs), or raises

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4Alternatively both could be foreign with the difference coming from the mode of transportation (ocean shipping vs air shipping) as in Hummels and Schaur (2010) for exports, or one source being more distant (China) than the other (Mexico) as in Evans and Harrigan (2005).
imports and inventory as the firm builds up its safety stock.

Second, we are able to show that the firm’s ability to adjust current output to demand conditions, and thus firm-level output volatility, also depends non-monotonically on the price differential between imported and domestic inputs and thus on the volume of imports. If the price differential is small, the firm relies on domestic sourcing when demand turns out to be high. Given that these inputs arrive immediately, the firm is able to adjust output to demand shocks. If the price differential is big, the firm hedges against demand shocks by building up its inventory of imported inputs. But with sufficient inventory on hand, the firm is also able to adjust current output to demand conditions. For price differentials in an intermediate range, the ability of firms to adjust output is limited. On the one hand, the firm may find it too expensive to respond to a positive demand shock by sourcing domestically. On the other hand, imported inputs are not cheap enough to build up sufficient inventory to cover such demand shocks. Instead it may turn out to be optimal for the firm to stock out, in which case output is completely independent of the demand realization. Again, if we allow for only one way to hedge demand uncertainty, we may wrongly conclude that there exists a monotonic relationship between firm-level output volatility and the price differential, respectively the volume of imports.

The articles most closely related to ours are Hummels and Schaur (2010) and Novy and Taylor (2014). Hummels and Schaur look at exports by air and by ocean shipping (see also Aizenman, 2004). The firm in their model has to decide how best to hedge demand uncertainty by determining how much to ship via ocean versus air transport, and simulation results from the model are used to motivate an empirical study of the link between the increased use of air shipping and the fall in the relative price of air shipping. Their model, however, does not allow for inventory build-up, where the firm carries over unsold goods into future periods. Thus greater demand uncertainty for a good automatically leads to a greater share of air shipments. Moreover, for a given degree of demand uncertainty, a negative contemporaneous demand shock tends to reduce air shipments more than ocean shipments. In Novy and Taylor’s paper an input is either imported or sourced domestically but not both. High-price domestic inputs arrive without delay, but low-price imports arrive only with a time lag; hence the necessity in the case of imported inputs to hedge demand uncertainty by building up inventory. A greater degree of demand uncertainty in this model thus leads to a greater build-up of inventory, and a contemporaneous negative demand shock hits imported inputs more than domestic inputs,
because firms first run down their inventories of imported inputs before re-ordering.

As already indicated above, our model can be viewed as encompassing both of these strategies as special cases: dual sourcing from Hummels and Schaur (2010), and inventory build-up from Novy and Taylor (2014). In particular, in our model, the choice between sourcing an input domestically or from abroad is endogenous, as is the choice of how much inventory to carry over into next period. In this way we are able to show that which strategy a firm uses to hedge demand uncertainty depends on the price differential between domestic and imported inputs. This implies that in our paper, the response to changes in demand uncertainty and to contemporaneous demand shocks also depends on this price differential.

Another novel aspect of our paper is that we examine what the use of different hedging strategies implies for firm-level output volatility. How output (or employment) volatility differs between trading and non-trading firms has recently received considerable attention in the literature. Interest in this issue comes from the question whether trading firms exhibit a different output or employment volatility than non-trading firms. Suppose, for instance, they bring more volatility. Then, increased trade might harm risk-averse households by making their consumption patterns more volatile leading to lower welfare gains from trade than predicted by standard trade models. The literature identifies two potential reasons why trading firms may exhibit a different volatility of output or employment than non-trading firms: they may be exposed to foreign supply or demand shocks that are imperfectly correlated with domestic shocks (e.g., Vannoorenberghe, 2012), or they may react differently to domestic shocks than non-trading firms, or both (Buch et al., 2009; Kurz et al., 2017). Our paper is linked to the latter strands of the literature, since it deals with the ability of importers to hedge demand volatility and thus with their ability to react to domestic shocks. Our contribution is to propose a microeconomic mechanism linking imports to firm-level output volatility, whereas much of the existing literature is either descriptive (e.g., Kurz and Senses, 2016) or relies on macroeconomic modelling approaches (e.g., Kurz et al., 2017).

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5See, for example, Kurz et al. (2017), Kurz and Senses (2016), Benz et al. (2016), Vannoorenberghe (2012), Buch et al. (2009). Industry-level studies are provided, for instance, by di Giovanni and Levchenko (2009).

6See, for instance, Newbery and Stiglitz (1984) for a general treatment of how trade may reduce social welfare in risky economies without insurance.

7See also Sundaram (2018) for a study examining the causal relationship between trade liberalization and firm-level output volatility.
volatility of importers may change non-monotonically with the price differential between imported and domestic goods. In particular, we show that an increase in this price differential, which goes hand in hand with an increase in international trade, may reduce output volatility for low levels of the price differential and thus trade, but increase output volatility for sufficiently high levels of the price differential and trade. This suggests that empirical studies that assume a monotonic relationship between firm-level volatility and trade may be misspecified.

The remainder of the paper is organized as follows. In the next section, we illustrate the hedging strategies used by firms, in particular dual sourcing, using data for an anonymous US steel wholesaler who relies both on importing and domestic sourcing for certain steel products. We then propose a simple model consistent with this example in Section 3 and derive the firm’s optimal inventory. In Section 4 we analyze the optimal hedging strategy, and we show how inventory responds to changes in the price differential between imported and domestic inputs and to changes in demand uncertainty. There we also investigate what the optimal hedging strategy implies for firm-level output volatility. Section 5 concludes by revisiting the example of the US steel wholesaler and confirming that for two specific products, for which inventory data are available, the wholesaler’s observed inventory investment strategies are consistent with our model’s predictions. In the Appendix, we collect proofs of our results.

2 An Example of Dual Sourcing

Dual sourcing is illustrated by the case of an (anonymous) US steel wholesaler sourcing products domestically and abroad. Among the steel products bought by this wholesaler, we concentrate on the Hot Rolled Coil (HRC) product class. This class of products has a total of 3,918 purchase transactions between July 1997 and November 2006 of which 1,952 (49%) are domestic, 1,274 (32%) are foreign, and the remaining 692 purchases...
(19%) have unidentified origins.\footnote{The name of the source countries is not known. We only observe whether a product is sourced abroad or not. For each transaction, there is a date, product code, unit price, weight, and total amount paid.} The HRC purchases are divided into 41 sub-products depending on the thickness (the gauge) and the width (varying between 48 and 98 inches) of the products.\footnote{Two of these sub-products have less than 10 purchases and the top 28 sub-products represent about 90\% of the 3,918 transactions; the most purchased sub-product (HRC25048) represents 8\% of the HRC purchases.} Every sub-product has both domestic and foreign purchases.

Figure 1 shows the daily frequency of domestic and foreign purchases over the 1997-2006 sample period.\footnote{The transactions are missing during the period July-December 2004, the gap around 16,500 in Figure 1.} It shows that dual sourcing is systematically taking place over the period although the relative frequency of domestic and foreign purchases is not constant over time. Figure 2 shows the average domestic and foreign prices as well as the share of domestic and foreign purchases (by weight) over the sample period. Notice that it is not always the case that the domestic price is higher than the foreign price. This is especially the case in 2000-03, which includes the period between March 20, 2002 and December 4, 2003 when the United States imposed Section 201-tariffs on 272 different 10-digit HS steel products.\footnote{The temporary tariffs ranged from 8 to 30\% depending on the steel products (compared to an average of 6\% for all steel products.)} Still, irrespective of which price is higher, it is still the case that both sources
are used (as Figure 2 illustrates) even if, as expected, the source with the lower price commands the higher share. It is also interesting to note that the shift in market shares in favor of domestic sourcing does not occur when the domestic price becomes lower than the foreign price, but rather when the differential between the domestic and the foreign price becomes low even if the foreign price is still lower, suggesting that the wholesaler is willing to pay a premium on domestic purchases.

Table 1 shows that, on average, foreign prices are 13% lower than domestic prices with very similar coefficients of variation indicating that the volatility of the domestic and foreign prices is similar. Despite higher prices, domestic purchases represent 33% of the total weight ordered. A critical difference between domestic and foreign purchases is the average size of the orders with domestic orders amounting to about a third of the average size of foreign orders. Associated with these different average orders is the fact that domestic orders are more frequent than foreign orders. Computing the difference in terms of the number of days between domestic orders (and separately between foreign orders) for each sub-product (unweighted averages), the mean of the distribution is about 16 days between two consecutive domestic orders and about 28 days in between two consecutive foreign orders. Moreover, the corresponding coefficient of variation is lower for the foreign-order distribution (.49) than for the domestic-order one (.562).

<table>
<thead>
<tr>
<th>Origin</th>
<th>Number purchases</th>
<th>Average price</th>
<th>CV price</th>
<th>Average Weight</th>
<th>CV Weight</th>
<th>Total share by weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic</td>
<td>1,952</td>
<td>20.57</td>
<td>.362</td>
<td>88,312</td>
<td>.954</td>
<td>33%</td>
</tr>
<tr>
<td>Foreign</td>
<td>1,274</td>
<td>17.83</td>
<td>.359</td>
<td>223,232</td>
<td>1.135</td>
<td>55%</td>
</tr>
<tr>
<td>Unknown</td>
<td>692</td>
<td>18.53</td>
<td>.429</td>
<td>91,644</td>
<td>2.25</td>
<td>12%</td>
</tr>
<tr>
<td>Total</td>
<td>3,918</td>
<td>19.39</td>
<td>.381</td>
<td>132,478</td>
<td>1.42</td>
<td>100%</td>
</tr>
</tbody>
</table>

The example of this class of products shows that domestic purchases are frequent, small and not regular, while foreign purchases are large, less frequent and show a somewhat greater regularity. An explanation is that the foreign source is the primary and – on tariff of 0 to 1% prior to the policy change) imposed on all but a handful of foreign sources (185 products received a 30% tariff, 60 received a 15% tariff, 15 received a 13% tariff and 7 received an 8% tariff); see Bown (2013), Hufbauer and Goodrich (2003), Read (2005).
Figure 2: Purchase Price and Sourcing Shares by Weight
average – cheaper source, but foreign purchases take time to arrive. Domestic orders are delivered immediately but are more expensive on average, and are thus not used all the time. In the following section, we propose a simple model consistent with this explanation.

3 The Model

Consider a firm facing the inverse linear demand \( p_t = a + \epsilon_t - bq_t \) for its product in period \( t \), where \( \epsilon_t \) is a random shock uniformly distributed according to the c.d.f. \( F(\epsilon_t) = (\epsilon_t + \Delta)/2\Delta \), \( q_t \) denotes output in period \( t \), and \( \Delta < a \) holds such that the random shock is small relative to the size of the market. For each unit of output, the firm needs one unit of a homogeneous input that it can obtain from two possible sources: a domestic source or a foreign source. Domestic sourcing is immediate in the sense that inputs can be ordered and delivered after the demand in that period has been revealed; hence we may think of domestic orders as involving just-in-time delivery. The domestic order in period \( t \) is associated with the domestic unit cost \( w_t \) and quantity \( y_t \). Foreign sourcing is inflexible, because an order has to be placed before the realization of demand is known, and it takes time for that order to be delivered. In particular, an input purchase made (and paid) in period \( t - 1 \), at the foreign unit cost \( v_{t-1} \) and involving a quantity denoted by \( m_{t-1} \), can only be used in production in period \( t \) or later. We interpret the unit input cost as including the unit price as well as transport and other transaction costs involved in purchasing the input. In the case of the foreign unit cost, these other costs will typically include import tariffs, customs clearing costs, insurance, etc. We may therefore think of a decrease in the foreign unit cost as reflecting a decrease in trade costs. Since domestic and foreign inputs used in a given period are not bought during the same period, we take into account the discount factor, \( \delta < 1 \), so that, at time \( t \), the appropriate comparison of the two input costs is \( w_t \) and \( v_{t-1}/\delta \). We consider the case where the foreign, inflexible source is cheaper than the domestic, flexible source (i.e., \( w_t \geq v_{t-1}/\delta \)).

The trade-off between the two sources is clear: the foreign source is relatively cheap but it forces a firm to commit to it before demand is known and it can only be used in production next period, whereas the domestic source is relatively expensive but ‘immediate’ in the sense that an order can be placed and received once the current demand is known. In any period, the firm may end up not using the entire quantity of inputs that it purchased. We denote the volume of these unsold units in period \( t \) by \( z_t^0 \) and they become
part of the available inputs to be used in $t + 1$. We refer to $z^0_t$ as the excess inventory (or inventory build-up). It turns out to be analytically convenient to assume that the firm attaches an exogenous and constant value $\rho_t > 0$ to each of these units. We make the further assumption that $\rho_t < v_{t-1}/\delta \leq w_t$. Thus, while the firm places a positive value on unsold units, this value is not so high that it would voluntarily accumulate excess inventory. We let $z_t$ denote total inventory in period $t$ and define it as the volume of inputs available for use at the beginning of the period. Thus total inventory is equal to $z_t = m_{t-1} + z^0_{t-1}$, that is, the sum of imports purchased in period $t - 1$ and arriving at the beginning of period $t$, and the excess inventory inherited from period $t - 1$. We assume that $z^0_{t-1}$ is known when the firm chooses $m_{t-1}$. The cost of storing inventory between periods is normalized to zero.

It must be clear from above that, at the beginning of a period, the cost of the foreign inputs is sunk whereas the cost of domestic inputs is avoidable. Thus, a firm always prefers using its entire available inventory before buying from the domestic source so that, in any period, it faces three possibilities: (i) it uses less than its inventory $z_t$ and does not order from the domestic source; (ii) it uses its entire inventory but does not order from the domestic source; or (iii) it uses its entire inventory and buys, as well as uses, $y_t$ from the domestic source.

To make this more precise, consider a firm’s profit denoted by $\pi_t(q_t)$:

$$\pi_t(q_t) = \begin{cases} (a + \epsilon_t - bq_t)q_t + \rho_t(z_t - q_t) & \text{if } q_t \leq z_t, \\ (a + \epsilon_t - bq_t)q_t - w_t(q_t - z_t) & \text{if } q_t > z_t. \end{cases}$$  \hfill (1)

The optimal output in $t$ is therefore given by:

$$q^*_t(\epsilon_t) = \begin{cases} \frac{a + \epsilon_t - \rho_t}{2b} & \text{if } \frac{a + \epsilon_t - \rho_t}{2b} \leq z_t, \\ z_t & \text{if } \frac{a + \epsilon_t - \rho_t}{2b} < z_t < \frac{a + \epsilon_t - \rho_t}{2b}, \\ \frac{a + \epsilon_t - w_t}{2b} & \text{if } \frac{a + \epsilon_t - w_t}{2b} \geq z_t. \end{cases} \hfill (2)$$

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\[14\] We discuss the case where $\rho_t$ is endogenously determined in Appendix A.5.

\[15\] Our model could easily accommodate the case where $\rho_t > v_{t-1}/\delta$, but this is not the focus of our paper.

\[16\] This implies in particular that, when a foreign order is placed, the firm knows the realization of the demand of the current period.
Eq. (2) shows that demand, and thus optimal output, must be high enough relative to available inventory $z_t$ before the firm buys from the domestic source, simply because the marginal revenue has to exceed the unit cost of the domestic source. If it does not, the firm uses at most its available inventory depending on the comparison between the marginal revenue of using one unit now and the value of holding on to it, $\rho_t$. Not surprisingly the lower $\rho_t$, the greater is the incentive to use this unit today. Hence a lower $\rho_t$ and a higher domestic unit cost $w_t$ imply a wider range of inventory levels over which the firm decides at time $t$ to use up its entire available inventory, i.e., to stock out, without purchasing any domestic inputs.

From (2), we can derive the critical demand realizations where the firm is indifferent between using all inventory or not, $\varepsilon(z_t)$, and where it is indifferent between buying an additional unit from the domestic source or not, $\bar{\varepsilon}(z_t)$:

$$
\varepsilon(z_t) = 2bz_t - a + \rho_t, \quad \bar{\varepsilon}(z_t) = 2bz_t - a + w_t,
$$

with $\varepsilon(z_t) < \bar{\varepsilon}(z_t)$ from our earlier assumptions. Thus (2) can be rewritten as:

$$
q_t^* (\varepsilon_t) = \begin{cases} 
\frac{a + \varepsilon_t - \rho_t}{2b} & \text{if } \varepsilon_t \leq \varepsilon(z_t), \\
 z_t & \text{if } \varepsilon(z_t) < \varepsilon_t < \bar{\varepsilon}(z_t), \\
\frac{a + \varepsilon_t - w_t}{2b} & \text{if } \varepsilon_t \geq \bar{\varepsilon}(z_t). 
\end{cases}
$$

Eq. (4) is useful for three reasons. First it makes clear that the firm’s sourcing strategy depends on demand realizations. Demand can be: (i) low enough so that the firm does not use its entire inventory and therefore accumulates excess inventory, $z_t - q_t^* (\varepsilon_t)$, that it may use next period; (ii) in an intermediate range such that it uses up its entire foreign source inventory, $q_t^* (\varepsilon_t) = z_t$, but does not order from the domestic source; or (iii) high enough that it uses inputs from both foreign and domestic sources. In the latter case, the purchase of domestic inputs is equal to:

$$
y_t^* (\varepsilon_t) = \frac{a + \varepsilon_t - w_t}{2b} - z_t.
$$

Second, (4) makes clear that, in order for a firm to effectively face these three options, the realizations of demand must be feasible given the support $[-\Delta, \Delta]$. In particular, (4) is consistent with $[-\Delta, \Delta]$ provided that $-\Delta < \varepsilon(z_t) < \bar{\varepsilon}(z_t) < \Delta$ which, using (3), requires
that \(\tau(z_t) - \xi(z_t) = w_t - \rho_t < 2\Delta\). Hence, given \(w_t\) and \(\rho_t\), (4) requires a relatively high degree of demand uncertainty.

Third, it makes it easy to characterize graphically the possible cases that may arise. Figure 3 where \([\xi(z_t), \tau(z_t)]\) is fully contained in \([-\Delta, \Delta]\), illustrates the possible realizations of demand consistent with (4). If the demand uncertainty is low as in Figure 4 however, and thus if \([-\Delta, \Delta]\) is fully contained in \([\xi(z_t), \tau(z_t)]\), then the firm’s optimal output can only be \(q^*_t(\epsilon_t) = z_t\) irrespective of the realization of demand.

We can now proceed to the determination of optimal inventory \(z_t^*\). Notice that the cases illustrated by Figures 3 and 4 are just two possible outcomes among several that we have to examine. Going through all of these cases is obviously repetitive. So we show only one case here and refer the reader to Appendix A.1 for a derivation of the optimal inventory in the other cases. We focus on the case in which the firm’s expected marginal revenue in period \(t\) is consistent with the optimal output \(q^*_t\) as provided by (4) and thus for realizations of demand consistent with Figure 3. The expected marginal revenue in period \(t\), \(E[MR_t]\), from a unit of input sourced from abroad in period \(t - 1\) is equal to:

\[
\delta E[MR_t] = \delta \left[ \int_{-\Delta}^{\xi(z_t)} \rho_t \frac{d\epsilon_t}{2\Delta} + \int_{\xi(z_t)}^{\tau(z_t)} (a + \epsilon_t - 2bz_t) \frac{d\epsilon_t}{2\Delta} + \int_{\tau(z_t)}^{\Delta} w_t \frac{d\epsilon_t}{2\Delta} \right].
\]  

(6)

The marginal revenue is equal to \(\rho_t\) for low demand realizations \((-\Delta \leq \epsilon_t \leq \xi(z_t))\) and thus when the firm holds on to units for the next period. It is equal to \(a + \epsilon_t - 2bz_t\) when the entire inventory \(z_t\) is used, and is equal to \(w_t\) when the demand realizations are
sufficiently high \((\tau(z_t) \leq \epsilon_t \leq \Delta)\) that purchasing from the domestic source is required. Using (3) to evaluate (6), equating the outcome to the foreign price \(v_{t-1}\), and solving for \(z_t\), we obtain as optimal inventory:

\[
z_{t,123}^* = \frac{2a - (w_t + \rho_t)}{4b} + \frac{\Delta(w_t + \rho_t)}{2b(w_t - \rho_t)} - \frac{\Delta v_{t-1}/\delta}{b(w_t - \rho_t)}.
\] (7)

Here \(z_{t,123}^*\) denotes the optimal inventory in regime (123), in which all three ranges of demand realizations are feasible: building up inventory due to a low demand realization (labeled range 1), using up total inventory but without any domestic sourcing (labeled range 2), and using up total inventory and sourcing additional inputs domestically (labeled range 3). In what follows, the subscript will denote the relevant regime, i.e., the feasible range(s).

Other regimes are possible, all involving a subset of the three ranges of demand realizations. Thus, \(z_{t,123}^*\) (valid for \(\xi(z_{t,123}^*) < -\Delta < \tau(z_{t,123}^*) < \Delta\)) refers to the optimal inventory which is always fully used but with some demand realizations requiring domestic sourcing; \(z_{t,12}\) (valid for \(-\Delta < \xi(z_{t,12}) < \Delta \leq \tau(z_{t,12})\)) is the optimal inventory when domestic sourcing never takes place and where the available inventory might not be entirely used. Finally, \(z_{t,2}\) (valid for \(\xi(z_{t,2}) < -\Delta\) and \(\tau(z_{t,2}) > \Delta\)) is the optimal inventory when it is always entirely used and no domestic sourcing takes place (illustrated by Figure 4).

Lemma 1 summarizes results derived in Appendix A.1.

**Lemma 1.**

1. In addition to \(z_{t,123}^*\), the feasible inventory levels are:

\[
\begin{align*}
    z_{t,2}^* &= \frac{a - v_{t-1}/\delta}{2b}; \\
    z_{t,12}^* &= \frac{a + \Delta - \rho_t - 2\sqrt{\Delta(v_{t-1}/\delta - \rho_t)}}{2b}; \\
    z_{t,23}^* &= \frac{a - w_t - \Delta + 2\sqrt{\Delta(w_t - v_{t-1}/\delta)}}{2b}.
\end{align*}
\]

2. The conditions under which \(z_{t,2}^*\) and \(z_{t,123}^*\) hold are mutually exclusive.

3. Two inventory levels are not feasible: \(z_{t,1}^*\) and \(z_{t,3}^*\).

**Proof.** See Appendix A.1.

Two comments are in order. First, two cases never arise: \(z_{t,1}^*\) when inventories always exceed needs (requiring \(\xi(z_{t,1}^*) > \Delta\)), and \(z_{t,3}^*\) involving systematic domestic sourcing.
irrespective of demand realizations (requiring \( \tau(z_{t,3}^*) < -\Delta \)). The former would imply a permanent inventory build-up, which is not an equilibrium strategy in our model: a firm never systematically chooses to order so much from abroad that it would build up inventory for any demand realization. The latter would imply that a firm always sources at least some inputs at home for any demand realization, even if \( v_{t-1}/\delta < w_t \). Second, the conditions under which \( z_{t,123}^* \) and \( z_{t,2}^* \) hold are mutually exclusive because they do not depend on the foreign price but only on the comparison of the degree of uncertainty \((2\Delta)\) with \((w_t - \rho_t)\), with \( z_{t,123}^* \) requiring \( 2\Delta > w_t - \rho_t \), and \( z_{t,2}^* \) requiring \( 2\Delta < w_t - \rho_t \).

4 The Optimal Hedging Strategy

We are now ready to determine how a change in the foreign unit cost, \( v_{t-1}/\delta \), as might result from a change in trade costs, and a change in the level of uncertainty affect the firm’s optimal hedging strategy and thus its imports. It turns out to be analytically convenient to focus on determining how changes in the foreign unit cost and uncertainty affect \( z_t^* \) and thus the optimal ‘stock’ of inventory rather than the ‘flow’ of imports, \( m_{t-1}^* \), as is typically done in trade analysis. This is legitimate because the two concepts are closely connected. In particular, the optimal import quantity in period \( t-1 \), \( m_{t-1}^* \), is equal to the difference between the optimal inventory the firm wants to hold at the beginning of period \( t \), \( z_t^* \), and the excess inventory accumulated in \( t-1 \), \( z_{t-1}^0 \). Hence \( m_{t-1}^* = z_t^* - z_{t-1}^0 \). Given that \( z_{t-1}^0 \) is known when the firm chooses how much to order from abroad, determining \( m_{t-1}^* \) is equivalent to choosing \( z_t^* \). The advantage of picking inventory as our unit of analysis rather than imports is that it avoids having to track unsold units inherited from earlier periods without modifying the results. Its only drawback is that it may be less amenable to empirical analysis, as the inventory of imported goods is typically less easily observable than imports.

Consider the effect of a decrease in the foreign unit cost. We observe that the optimal inventory levels in the different regimes summarized in Lemma 4 all rise monotonically as the foreign cost decreases (i.e., \( \partial z_t^*/\partial(v_{t-1}/\delta) < 0 \)). However, we have to take into account the decision to order from abroad. If orders were placed before \( z_{t-1}^0 \) is known, then \( m_{t-1}^* = z_t^* - E(z_{t-1}^0) \), where \( E(z_{t-1}^0) \) is the expected excess inventory.

\(^{17}\)In Appendix A.1 we show that domestic sourcing becomes the primary and only source (i.e. \( z_{t,3}^* = 0 \)) if \( v_{t-1}/\delta > w_t \).

\(^{18}\)If orders were placed before \( z_{t-1}^0 \) is known, then \( m_{t-1}^* = z_t^* - E(z_{t-1}^0) \), where \( E(z_{t-1}^0) \) is the expected excess inventory.
account that, as the foreign unit cost decreases, the inventory regime will switch. To see this, observe that 
\( \bar{\epsilon}(z_t) - \epsilon(z_t) = w_t - \rho_t \) is independent of \( v_{t-1}/\delta \), but \( \bar{\epsilon}(z_t) \) and \( \epsilon(z_t) \) do depend on \( v_{t-1}/\delta \). In particular, given (3),

\[
\frac{\partial \epsilon(z_t)}{\partial (v_{t-1}/\delta)} = \frac{\partial \bar{\epsilon}(z_t)}{\partial (v_{t-1}/\delta)} = 2b \frac{\partial z_t}{\partial (v_{t-1}/\delta)} < 0,
\]

(8)

so that graphically a reduction in the foreign unit cost shifts the interval \( \bar{\epsilon}(z_t) - \epsilon(z_t) = w_t - \rho_t \) from left to right relative to a fixed range of length \( 2\Delta \) centered around zero. This implies two separate inventory regime paths: one conditional on \( \bar{\epsilon}(z_t) - \epsilon(z_t) = w_t - \rho_t < 2\Delta \) (see Figure 5) and the other conditional on \( \bar{\epsilon}(z_t) - \epsilon(z_t) = w_t - \rho_t > 2\Delta \) (see Figure 6).

In particular, if \( 2\Delta > \bar{\epsilon}(z_t) - \epsilon(z_t) = w_t - \rho_t \), there is a unique path from regime (3) to regime (23) to regime (12) with 
\( z^*_{t,3} = 0 \leq z^*_{t,23} \leq z^*_{t,123} \leq z^*_{t,12} \). If \( 2\Delta < \bar{\epsilon}(z_t) - \epsilon(z_t) = w_t - \rho_t \), there is a unique path from regime (3) to regime (23) to regime (2) to regime (12) with 
\( z^*_{t,3} = 0 \leq z^*_{t,23} \leq z^*_{t,2} \leq z^*_{t,12} \).

**Figure 5: Trade liberalization path with high demand uncertainty**

Consider the first path and assume that the foreign price is sufficiently high that \( v_{t-1}/\delta \geq w_t \). Clearly, a firm sources domestically only and there is no foreign sourcing irrespective of the realization of demand. This case corresponds to \( z^*_{t,3} = 0 \). When lower foreign prices bring \( w_t > v_{t-1}/\delta \), the first possible bound to intersect the fixed range \( 2\Delta \) is \( \bar{\epsilon}(z_t) \). This implies that \( z^*_{t,23} \) is the first possible optimal inventory level consistent with foreign sourcing. Since in this regime foreign sourcing is not much cheaper than domestic
sourcing, the firm relies on domestic sourcing to buffer large positive demand shocks. In the case of low demand shocks, the firm chooses to stock out rather than to be left with unsold inventory at the end of the period. Lower foreign unit costs lead the lower bound of $\bar{\epsilon}(z_t) - \underline{\epsilon}(z_t)$ to intersect the fixed range $2\Delta$ so that $z_{t,123}$ becomes the optimal inventory level. Now, in addition to purchasing domestically when demand turns out to be very high, the firm acquires so much inventory that it is stuck with unsold inventory should demand turn out to be very low. Still lower foreign unit costs imply that $\bar{\epsilon}(z_t)$ exceeds $\Delta$ which leads to $z_{t,12}$ as the only possible optimal inventory level. In this case, the foreign unit cost is so low compared to the domestic cost that the firm never sources domestically. It stocks out if demand turns out to be high and accumulates inventory when demand happens to be low. The second possible path is the same as the first one except that, since $2\Delta < \bar{\epsilon}(z_t) - \underline{\epsilon}(z_t) = w_t - \rho_t$, the optimal inventory level $z_{t,123}^*$ never arises but is replaced by $z_{t,2}^*$ instead, which is the optimal inventory when the firm prefers to stock out rather than to source domestically or to carry unsold units into the subsequent period.

We prove in Appendix A.2 that, at the foreign-unit-cost levels at which these regime switches occur, inventory is continuous in the foreign unit cost. Hence we may state the following result:

**Proposition 1.** A decrease in the foreign unit cost, $v_{t-1}/\delta$, leads to a monotonic and continuous increase in inventory.
This is not a surprising result, since it says that a fall in the foreign unit cost has the expected outcome of increasing the volume of imports\footnote{We show in Appendix \textit{A.5} that this result also holds if $\rho_t$ is endogenously determined.}. But it is interesting that, as the foreign unit cost decreases, domestic sourcing continues to play a role. The firm simply relies less on it and more on building up inventory of imported goods in order to hedge uncertainty.

We can learn more about the optimal hedging strategy by investigating directly the link between demand uncertainty and the optimal choice of inventory, respectively imports. We can do this in two ways, namely by looking at the effect of contemporaneous demand shocks, and by examining the effect of a mean-preserving increase in demand uncertainty. Consider how a contemporaneous demand shock in period $t - 1$ affects imports $m^*_{t-1}$. This shock has no effect on the optimal level of inventory $z^*_t$ the firm wants to have at the start of period $t$; rather the effect on imports comes from changes in $z^0_{t-1}$. Notice, in this respect that positive and negative contemporaneous demand shocks tend to have asymmetric effects on imports. A negative demand shock implies that the firm builds up inventory, i.e., $z^0_{t-1} > 0$, and thus necessarily reduces imports. By contrast, a sufficiently strong positive demand shock implies that the firm either stocks out or relies on domestic sourcing to buffer the unexpectedly high demand. With no build-up of inventory, i.e., $z^0_{t-1} = 0$, such a shock has no effect on imports.

Next, consider a mean-preserving increase in demand uncertainty. In principle, the firm has two ways of hedging this uncertainty, namely building up sufficient inventory, or relying on domestic sourcing to meet high demand realizations. Which option is best for the firm obviously depends on the foreign unit cost. If it is sufficiently low, the firm will react to greater demand uncertainty by raising its inventory level and accepting a greater expected inventory accumulation. Thus, more uncertainty enhances imports. If the foreign unit cost is sufficiently high, the firm will respond to more uncertainty by doing the opposite, namely by reducing the level of inventory and thus imports. This is because the firm finds it optimal to rely more on the immediacy of domestic sourcing.

A mean-preserving spread can be modeled as an increase in $\Delta$. We show in Appendix \textit{A.3} that such a mean-preserving marginal change in $\Delta$ has the following effects on the firm’s
optimal inventory level:

\[
\frac{\partial z_t^*}{\partial \Delta} = \begin{cases} 
< 0 & \text{if } z_t^* = z_{t,23}^*, \\
> 0 & \text{if } z_t^* = z_{t,123}^*, \\
= 0 & \text{if } z_t^* = z_{t,2}^*, \\
> 0 & \text{if } z_t^* = z_{t,12}^*.
\end{cases}
\] (9)

Since \(z_{t,23}^* \leq z_{t,123}^* \leq z_{t,12}^*\) for \(2\Delta > w_t - \rho_t\), (and \(z_{t,23}^* \leq z_{t,2}^* \leq z_{t,12}^*\) for \(2\Delta < w_t - \rho_t\)), (9) implies the following:

**Proposition 2.** A mean-preserving increase in demand uncertainty raises the optimal inventory, and thus imports, if the foreign unit cost is sufficiently low, and lowers optimal inventory and imports, if the foreign unit cost is sufficiently high.

**Proof.** See Appendix A.3. \(\square\)

The main insight here is that the response of imports to an increase in the level of uncertainty depends non-monotonically on the level of the foreign unit cost. This follows directly from the fact that the firm’s optimal hedging strategy is endogenously determined and depends in particular on the foreign unit cost level. It is not very surprising that inventories and imports rise with uncertainty when the foreign unit cost is sufficiently low; after all, the foreign unit cost is lower than the domestic unit cost and units that would be left unsold with low demand realizations are still valuable. The more surprising part is that the firm may prefer responding to an increase in demand uncertainty by reducing imports even if the foreign unit cost is lower than the domestic cost, as indeed happens when the cost difference is small. In this case, the firm puts a greater premium on immediacy, which implies that it relies more on expensive domestic sourcing.

Knowing how the firm best responds to demand uncertainty, we can now examine the implications for firm-level output volatility. This volatility can be measured by the variance of output, which is given by:

\[
\text{Var}(q_t^*) = \int_{-\Delta}^{\Delta} (q_t^*(\epsilon_t) - \hat{q}_t)^2 \frac{d\epsilon_t}{2\Delta},
\]

where \(\hat{q}_t = \int_{-\Delta}^{\Delta} q_t^*(\epsilon_t)d\epsilon_t/2\Delta\) is the expected output in period \(t\).

To start, as compared to the volatility of output that would arise when the firm sources domestically only, it must be clear that, in the present model, foreign sourcing brings a
lower firm-level output volatility as long as there is a positive probability of stockout. More interesting is how firm-level output volatility depends on the level of the foreign unit cost. We can show that the variance of output reacts non-monotonically to changes in the foreign unit cost. In particular, we obtain the following result:

**Proposition 3.** A reduction in the foreign unit cost raises the variance of output, if the foreign unit cost is sufficiently low, but reduces the variance of output, if the foreign unit cost is sufficiently high.

*Proof.* See Appendix A.4.

Because a reduction in the foreign unit cost increases the firm’s foreign sourcing and thus its optimal inventory, this result is proved in Appendix A.4 by studying how output volatility changes with a marginal increase in $z_t$. Specifically, we prove that

$$
\frac{\partial \text{Var}(q_t^*)}{\partial z_t} = \begin{cases} 
< 0 & \text{in regime (23)} \\
> 0 & \text{in regime (123)} \\
= 0 & \text{in regime (2)} \\
> 0 & \text{in regime (12)}
\end{cases}
$$

How the variance of output changes with $z_t$ is illustrated by Figure 7 for the case of low demand uncertainty, and by Figure 8 for the case of high demand uncertainty.

To understand this result, consider first the case of low demand uncertainty illustrated by Figure 7. In regime (2), indicated below the graph, the firm prefers to stock out rather than to source domestically or to build up inventory and risk carrying unsold units into the subsequent period. Since output is always constrained by the available inventory in this regime, the firm is unable to adjust output to the observed demand realization, and hence $\text{Var}(q_t^*) = 0$. This corresponds to the flat section of the variance shown in the figure. Any increase in $z_t$, while allowing the firm to raise output, has no effect on the variance, and thus $\frac{\partial \text{Var}(q_t^*)}{\partial z_t} = 0$.

Consider now regime (23) where the firm engages in domestic sourcing for high demand realizations. In this regime, the firm hedges demand uncertainty by sourcing inputs domestically. The greater is $z_t$, and hence the less the firm relies on domestic sourcing in

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20 The figures are consistent with the parameters $a = 10$, $b = 1$, $\rho = 1$, $w = 4$, and $\Delta = 1$ (respectively, $\Delta = 4$).
Figure 7: Variance of output - low demand uncertainty

case of high demand realizations, the smaller is the firm’s scope for adjusting output to
the observed demand realization. This means that \( \text{Var}(q^*_t) \) decreases when \( z_t \) increases.

Suppose that we are in regime (12), where there is no domestic sourcing and the firm
hedges demand uncertainty through inventory build-up. It is this inventory build-up that
allows the firm to increase output in response to a positive demand shock. The greater is
\( z_t \), the better the firm is able to adjust output to the realization of demand, and hence
the greater is variance of output. In this regime, therefore, \( \partial \text{Var}(q^*_t)/\partial z_t > 0 \).

Next, consider the case of high demand uncertainty illustrated by Figure 8. Starting in
autarky, and thus at \( z_t = 0 \), an increase in \( z_t \) first reduces \( \text{Var}(q^*_t) \); this occurs in regime
(23). As \( z_t \) rises further, we move into regime (123), where \( \text{Var}(q^*_t) \) first decreases but then
increases. Finally, in regime (12), \( \text{Var}(q^*_t) \) increases with \( z_t \) until we reach the point where
\( z_t \) is so big that output can freely adjust to any realization of demand.

5 Conclusions

In this paper we show that when a firm can source inputs both abroad and domestici-
cally, it chooses domestic sourcing to hedge demand uncertainty if the price differential
between domestic and foreign sourcing is small. This strategy is chosen because, even if
the domestic price is higher than the foreign one, the immediacy of domestic sourcing makes this strategy particularly valuable to adjust output in response to high demand. If the price differential is big, however, then the firm uses foreign sourcing and it hedges domestic demand uncertainty by buying a large enough quantity of imports to satisfy a high demand even if it implies building-up inventory should demand turn out to be low. When the price differential is in an intermediate range, the firm may choose not to hedge demand uncertainty at all. In essence, the immediate domestic sourcing is too expensive to be used when the demand is high and, even if foreign sourcing is the primary source, buying a large volume to satisfy a high demand is not worth it either as it might end up as unsold inventory with too low a future value when the demand is low. In this case, the firm is content with buying a limited quantity making sure that it can use it entirely irrespective of the realization of demand (at least for low enough levels of demand uncertainty). Ordering less even if it means stocking out can thus also be an optimal strategy.

We can turn to the case of the steel wholesaler introduced above to illustrate this implication of our theory. Hall and Rust (2000) report that product stockout (including near stockout) is not a rare event. Indeed, two steel products for which we have inventory data (see Figure 9 and Figure 10) both exhibit several stockouts (defined arbitrarily as
Stockouts occurred seven times over the sample period for the first product and three times for the second one despite the availability of two supply sources. In both cases, purchase orders are smaller on average during stockout periods than during other periods. Thus an important implication of our theory, namely that the firm may optimally choose to stock out by keeping orders small is indeed confirmed by these two steel products.

A key insight of the paper is that the optimal hedging strategy is endogenous and depends, in particular, on the differential between foreign and domestic unit costs. This has several implications. The first one is that a reduction in trade costs, in the form of a greater cost differential, has the expected outcome of increasing the level of imports and optimal inventories. But there is more to the hedging strategy than this. When the cost differential

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21Products PL100 and PL075 are steel plates of dimension 96,000x240,000" which differ only by their thickness, respectively 1 and 3/4 inch.

22For product PL100, six of the seven stockout episodes occurred in 2000-01 (February/March, August, September, October, November 2000 and May 2001). During the period September 1999 to August 2001 encompassing these episodes, the average purchase-order was 56% smaller than during the 18-month period prior to September 1999 and 27% smaller than during the 18-month period after September 2001. For product PL075, using two of the episodes and defining the stockout period as a two-month period such that the date at which the stockout occurs (Jan 9, 2002, and June 10/11, 2003) corresponds to the middle of the period, the average purchase-order is 30% lower during the first stockout period (respectively 51% lower during the second one) as compared to the average purchase-order over the entire sample.
between domestic and foreign sourcing is low, the immediacy of domestic sourcing implies that the firm prefers buying domestically despite the fact that the domestic cost is higher than the foreign one. In other words, the level of imports is lower than without that hedging strategy. At the other extreme, a large differential between domestic and foreign unit costs, as would exist near free trade, boosts imports by convincing the firm to import more than it would otherwise, simply because it is not a problem for the firm to accumulate inventory in case of low demand. Thus by lowering imports when imports are relatively expensive and by raising them when they are cheap, the hedging strategies make imports more sensitive to trade costs than they would otherwise be.

Here too, we can take the example of product PL100. Considering three distinct sub-periods during which domestic and foreign purchases take place, it is the case that overall foreign sourcing dominates when the foreign price is lower than the domestic one as during the two first sub-periods. Importantly, when the foreign price is the same as the domestic price as during the third sub-period (July 1, 2000 - Jan. 31, 2001), the total

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23 Total domestic sourcing per period represents 13% of total foreign sourcing during the first sub-period (March 1, 1998- Aug 31, 1998) when the price differential is 3.1% in favor of foreign sourcing, and 28% of total foreign sourcing during the second sub-period (Dec 1, 1999 - May 31, 2000) when the foreign price is 10% lower than the domestic price. Obviously there is more than just the price differential to determine the relative importance of each source; for instance, the price levels and the price volatility.
domestic sourcing is nearly three times as large as the total foreign sourcing. Thus, the two sources are still used but the relative share of each one is very sensitive to the price differential. The corresponding links with inventory levels are also interesting since, during the first two sub-periods, the average inventory levels are significantly higher than during the third sub-period when the two prices are the same. Thus, for this product, there is evidence that inventories are on average bigger when foreign sourcing is relatively more important.

The second implication is that these hedging strategies allow the firm to adapt its behavior with respect to changes in demand uncertainty. In particular, a mean-preserving increase in demand uncertainty works against trade liberalization when the price differential is low, and goes in the same direction as trade liberalization when the price differential is high. Again, when the price differential is low, more demand uncertainty is not met by running the risk of accumulating inventories but by using domestic sourcing. Thus at the margin, more demand uncertainty decreases the level of foreign sourcing by increasing the value of immediacy. The exact opposite occurs when the price differential is high: more demand uncertainty increases imports because in this way the firm can meet a higher demand even if the firm could end up with more excess inventory. The firm chooses the option to import more because it knows the opportunity cost of unsold inventory is low anyway.

The firm’s goal is thus not to eliminate or to minimize its output volatility. Indeed the firm’s output volatility is higher when it hedges demand uncertainty than when it does not. What an importer does when hedging is to adopt a strategy that takes into account the specific environment in which the demand uncertainty occurs; a strategy with which the sales associated with the upside of the demand uncertainty can be met while minimizing the cost associated with the downside. The cost differential between domestic and foreign inputs is this specific environment that requires an importer to adopt a different strategy when this differential is low (immediate domestic sourcing) and when it is high (foreign sourcing with possible inventories). By adjusting its strategy the firm’s output volatility is non-monotonic with respect to this cost differential, a feature that has not been identified by earlier analyses.

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24 Average inventories are respectively 1,940,689 pounds during the first sub-period; 909,159 during the second one, and 541,428 during the third one when the average domestic and foreign prices are the same.
Appendix

A.1 Proof of Lemma 1

Whenever it exists, the optimal inventory is found by finding $z_t$ such that $\delta E[MR_t(z)] = v_{t-1}$ (i.e., such that the discounted expected marginal revenue is equal to the marginal cost of a foreign-sourced unit). Consider $z_{t,2}^*$ corresponding to the case where sales are always equal to inventory and thus where there is no inventory build-up and no domestic sourcing. The discounted expected marginal revenue is:

$$\delta E[MR_t] = \delta \int_{-\Delta}^{\Delta} (a + \epsilon_t - 2bz_t) \frac{d\epsilon_t}{2\Delta}.$$

Setting it equal to the unit cost of foreign inputs, $v_{t-1}$, yields the optimal inventory:

$$z_{t,2}^* = \frac{a - v_{t-1}/\delta}{2b}.$$  \hspace{1cm} (A.1)

This case requires $\xi(z_{t,2}^*) < -\Delta$ and $\overline{\epsilon}(z_{t,2}^*) > \Delta$ which can be rewritten as:

$$\Delta < \min\{v_{t-1}/\delta - \rho_t, w_t - v_{t-1}/\delta\},$$

which in turn implies $2\Delta < w_t - \rho_t$. The optimal foreign inventories $z_{t,123}^*$ and $z_{t,2}^*$ are mutually exclusive.

Inventory level $z_{t,12}^*$ corresponds to the case where the firm never sources domestically. The discounted expected marginal revenue is given by:

$$\delta E[MR_t] = \delta \left( \int_{-\Delta}^{\Delta} (a + \epsilon_t - 2bz_t(\epsilon)) \frac{d\epsilon_t}{2\Delta} + \int_{z(t)}^{\Delta} (a + \epsilon_t - 2bz_t) \frac{d\epsilon_t}{2\Delta} \right),$$

leading to the optimal inventory:

$$z_{t,12}^* = \frac{a + \Delta - \rho_t - 2\sqrt{\Delta(v_{t-1}/\delta - \rho_t)}}{2b}.$$  \hspace{1cm} (A.2)

This case requires $\xi(z_{t,12}^*) > -\Delta$ and $\overline{\epsilon}(z_{t,12}^*) > \Delta$ which can be rewritten as:

$$w_t - \rho_t > 2\sqrt{\Delta(v_{t-1}/\delta - \rho_t)}, \quad \Delta > v_{t-1}/\delta - \rho_t,$$

which in turn implies $w_t + \rho_t > 2v_{t-1}/\delta$. 

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\( z_{t,23}^* \) corresponding to the case where the wholesaler never wants to carry over inventory but could source domestically. The discounted expected marginal revenue is:

\[
\delta E [MR_t] = \delta \left( \int_{-\Delta}^{\Delta} (a + \epsilon_t - 2bz_t) \frac{d\epsilon_t}{2\Delta} + \int_{\epsilon(z_t)}^{\Delta} w_t \, d\epsilon_t \right),
\]

leading to the optimal foreign inventory:

\[
z_{t,23}^* = \frac{a - w_t - \Delta + 2\sqrt{\Delta(w_t - v_{t-1}/\delta)}}{2b}. \tag{A.3}
\]

Since this case requires \( \epsilon(z_{t,23}) < \Delta \) and \( \epsilon(z_{t,23}) < -\Delta \), which, given \( z_{t,23}^* \), can be written as:

\[
\Delta > (w_t - v_{t-1}/\delta), \quad w_t - \rho_t > 2\sqrt{\Delta(w_t - v_{t-1}/\delta)},
\]

which in turn implies that \( w_t - \rho_t > 2(w_t - v_{t-1}/\delta) \).

Given our assumption that \( w_t \geq v_{t-1}/\delta \), there is no interior solution for \( z_{t,3}^* \), which corresponds to the case where the firm always engages in domestic sourcing (\( \epsilon(z_t) > \Delta \)). However, not surprisingly, if we allow \( w_t < v_{t-1}/\delta \), then the firm never sources abroad so that \( z_{t,3}^* = 0 \). Case 1 where sales would always be smaller than inventory (\( \epsilon(z_t) > \Delta \)) can be excluded since we assume \( \rho_t < v_{t-1}/\delta \). We would have \( \delta E[MR_t(\cdot)] = \delta \int_{-\Delta}^{\Delta} \rho_t \frac{d\epsilon_t}{2\Delta} = \delta \rho_t < v_{t-1} \) by assumption. But \( \epsilon(z_t) > \Delta \) implies \( -a + \rho_t > \delta \) which contradicts the assumption.

### A.2 Proof of Proposition

We want to show that the level of inventory is continuous at the foreign-unit-cost levels where regime switches occur along both inventory paths. Consider first the inventory path with high demand uncertainty, where \( 2\Delta > w_t - \rho_t \). The switch from regime (23) to regime (123) occurs when \( \epsilon(z_{t,23}) = \epsilon(z_{t,123}) = -\Delta \). The level of foreign unit cost at which \( z_{t,123}^* = z_{t,23}^* \) is given by \( \frac{v_{t-1}}{\delta} = w_t - \frac{(w_t - \rho_t)^2}{4\Delta} \). At this level, we indeed find that:

\[
\epsilon(z_{t,23}) = 2bz_{t,23}^* - a + \rho_t = a - w_t - \Delta + 2\sqrt{\Delta \frac{(w_t - \rho_t)^2}{4\Delta}} - a + \rho_t = -\Delta,
\]

and

\[
\epsilon(z_{t,123}) = 2bz_{t,123}^* - a + \rho_t = \frac{2a - (w_t + \rho_t)}{2} + \frac{\Delta(w_t + \rho_t)}{2(w_t - \rho_t)} \frac{2\Delta \left( w_t - \frac{(w_t - \rho_t)^2}{4\Delta} \right)}{(w_t - \rho_t)} - a + \rho_t = -\Delta.
\]
The switch from regime (123) to regime (12) occurs for $\epsilon(z_{t,123}^*) = \bar{\epsilon}(z_{t,12}^*) = \Delta$. We find that $z_{t,123}^* = z_{t,12}^*$ if $\frac{v_{t-1}}{\delta} = \rho_t + \frac{(w_t - \rho_t)^2}{4\Delta}$. At this level of foreign unit cost, we can indeed confirm that:

$$\epsilon(z_{t,123}^*) = 2bz_{t,123}^* - a + w_t = \Delta = \bar{\epsilon}(z_{t,12}^*) = 2bz_{t,12}^* - a + w_t.$$ 

Next, consider the inventory path for low demand uncertainty, where $2\Delta < w_t - \rho_t$. The switch from regime (23) to regime (2) occurs for $\epsilon(z_{t,23}^*) = \bar{\epsilon}(z_{t,2}^*) = \Delta$. We obtain $z_{t,23}^* = z_{t,2}^*$ for $\frac{v_{t-1}}{\delta} = w_t - \Delta$. At this level of foreign unit cost we can verify that $\epsilon(z_{t,23}^*) = \bar{\epsilon}(z_{t,2}^*) = \Delta$ holds. The regime switch from (2) to (12) occurs at $\epsilon(z_{t,2}^*) = \epsilon(z_{t,12}^*) = -\Delta$. At the foreign unit cost level at which $z_{t,2}^* = z_{t,12}^*$, namely $\frac{v_{t-1}}{\delta} = \Delta + \rho_t$, we can verify that indeed $\epsilon(z_{t,2}^*) = \epsilon(z_{t,12}^*) = -\Delta$.

A.3 Proof of Proposition 2

Given (7), (A.3) and (A.2), we have:

$$\frac{\partial z_{t,23}^*}{\partial \Delta} = \frac{1}{2b} \left( -1 + \sqrt{(w_t - v_{t-1}/\delta)/\Delta} \right) < 0,$$

provided $\Delta > (w_t - v_{t-1}/\delta)$, which is precisely one condition for $z_{t,23}^*$ to be valid (see point 3 in Appendix A.1).

Similarly,

$$\frac{\partial z_{t,12}^*}{\partial \Delta} = \frac{1}{2b} \left( 1 - \sqrt{(v_{t-1}/\delta - \rho_t)/\Delta} \right) > 0,$$

provided $\Delta > v_{t-1}/\delta - \rho_t$. This is also precisely one of the conditions for $z_{t,12}^*$ to be valid (see point 2 in Appendix A.1).

Finally, $\partial z_{t,2}^*/\partial \Delta = 0$ and

$$\frac{\partial z_{t,123}^*}{\partial \Delta} = \frac{1}{2b(w_t - \rho_t)}(w_t + \rho_t - 2v_{t-1}/\delta)$$

is positive or negative depending on the values of $w_t$, $v_{t-1}/\delta$ and $\rho_t$ with the constraint that $w_t > v_{t-1}/\delta > \rho_t$ as assumed.

A.4 Proof of Proposition 3

Because a reduction in the foreign unit cost increases the firm’s optimal inventory, we can study how output volatility changes with a marginal increase in $z_t$. In particular, we want
to show:

\[
\frac{\partial \text{Var}(q_t^*)}{\partial z_t} = \begin{cases} 
< 0 & \text{in regime (23)} \\
> 0 & \text{in regime (123)} \\
= 0 & \text{in regime (2)} \\
> 0 & \text{in regime (12)}
\end{cases}.
\]

Consider regime (123). From \(2\), we have:

\[
\frac{\partial q_t^*(\epsilon_t)}{\partial z_t} = \begin{cases} 
0 & \text{if } \frac{a+\epsilon_t-\rho_t}{2b} \leq z_t, \\
1 & \text{if } \frac{a+\epsilon_t-w_t}{2b} < z_t < \frac{a+\epsilon_t-\rho_t}{2b}, \\
0 & \text{if } \frac{a+\epsilon_t-w_t}{2b} \geq z_t.
\end{cases}
\]

Since the expected output is

\[
\hat{q}_{t,123} = \int_{-\Delta}^{\tau(z_t)} \frac{a - \rho_t + \epsilon_t}{2b} \frac{d \epsilon_t}{2\Delta} + \int_{\tau(z_t)}^{\tau(z_t)} \frac{z_t}{2\Delta} \frac{d \epsilon_t}{2\Delta} + \int_{\tau(z_t)}^{\tau(z_t)} \frac{a - w_t + \epsilon_t}{2b} \frac{d \epsilon_t}{2\Delta}
\]

\[
= \frac{(2a - (\rho_t + w_t))(2\Delta - (w_t - \rho_t))}{8b\Delta} + \frac{z_t(w_t - \rho_t)}{2\Delta},
\]

then,

\[
\frac{\partial \hat{q}_{t,123}}{\partial z_t} = \frac{w_t - \rho_t}{2\Delta} > 0.
\]

It follows that:

\[
\frac{\partial q_t^*(\epsilon_t)}{\partial z_t} - \frac{\partial \hat{q}_{t,123}}{\partial z_t} = \begin{cases} 
-\frac{w_t - \rho_t}{2\Delta} & \text{if } \frac{a+\epsilon_t-\rho_t}{2b} \leq z_t, \\
1 - \frac{w_t - \rho_t}{2\Delta} & \text{if } \frac{a+\epsilon_t-w_t}{2b} < z_t < \frac{a+\epsilon_t-\rho_t}{2b}, \\
-\frac{w_t - \rho_t}{2\Delta} & \text{if } \frac{a+\epsilon_t-w_t}{2b} \geq z_t.
\end{cases}
\]

The impact of an decrease in the foreign unit cost on the variance of output is therefore
equal to:

\[ \frac{\partial \text{Var}(q_t)}{\partial z_t} = 2 \int_{-\Delta}^{\Delta} \left( \frac{\partial q_t^*(\epsilon_t)}{\partial z_t} - \frac{\partial \hat{q}_{t,123}}{\partial z_t} \right) \left( q_t^*(\epsilon_t) - \hat{q}_{t,123} \right) \frac{d\epsilon_t}{2\Delta} + \frac{\partial \epsilon_t(z_t)}{\partial z_t} (q_t^*(\epsilon(z_t)) - \hat{q}_{t,123})^2 \frac{1}{2\Delta} \]

\[ + \frac{\partial \epsilon_t(z_t)}{\partial z_t} (q_t^*(\epsilon(z_t)) - \hat{q}_{t,123})^2 \frac{1}{2\Delta} \]

\[ + \frac{\partial \epsilon_t(z_t)}{\partial z_t} (q_t^*(\epsilon(z_t)) - \hat{q}_{t,123})^2 \frac{1}{2\Delta} \]

\[ = -2 \frac{\partial z_t - \rho_t}{\Delta} \int_{-\Delta}^{\Delta} (q_t^*(\epsilon_t) - \hat{q}_{t,123}) \frac{d\epsilon_t}{2\Delta} + 2 \int_{-\Delta}^{\Delta} (q_t^*(\epsilon_t) - \hat{q}_{t,123}) \frac{d\epsilon_t}{2\Delta} \]

\[ = \left( \frac{\partial z_t - \rho_t}{\Delta} \right) (z_t - \hat{q}_{t,123}). \]

Note that the second and the third line are both equal to zero and so is the first term on the fourth line. Comparing \( z_t^* \) and \( \hat{q}_{t,123} \) shows that both \( z_t - \hat{q}_{t,123} > 0 \) and \( z_t - \hat{q}_{t,123} < 0 \) are possible.

In particular:

\[ z_t - \hat{q}_{t,123} = \frac{1}{8b\Delta} \left( 2a(w_t - \rho_t) - (w_t^2 - \rho_t^2) + 2\Delta(w_t + \rho_t) + 4b z_t(w_t - \rho_t) - 4\Delta(a - 2bz_t) \right). \]

Hence we have:

\[ \text{sign}(z_t - \hat{q}_{t,123}) = \text{sign} \left[ 2a(w_t - \rho_t) - (w_t^2 - \rho_t^2) + 4b z_t(w_t - \rho_t) + 2\Delta(w_t + \rho_t) - 4\Delta(a - 2bz_t) \right] \]

\[ = \text{sign} \left[ 2a(w_t - \rho_t) - (w_t^2 - \rho_t^2) + 4b z_t(w_t - \rho_t) + 2\Delta(w_t + \rho_t) - 4\Delta(a - 2bz_t) \right] \]

\[ = \text{sign} \left( (w_t - \rho_t - 2\Delta) [2a - (w_t + \rho_t)] + 4b z_t [(w_t - \rho_t) + 2\Delta] \right), \]

where the first term is negative due to \( w_t - \rho_t - 2\Delta < 0 \) and the second term is positive.

Consider now the other three regimes starting with regime (2) where \( \epsilon(z_t) < -\Delta < \Delta < \bar{z}(z_t) \). It is straightforward to find that \( \partial q_t^*(\epsilon_t)/\partial z_t \) is 1 irrespective of \( \epsilon_t \) consistent with that regime so that \( \hat{q}_{t,2} = \int_{-\Delta}^{\Delta} z_t d\epsilon_t/2\Delta = z_t \). It follows that \( \partial \hat{q}_{t,2}/\partial z_t = 1 \) and \( \partial q_t^*(\epsilon_t)/\partial z_t - \partial \hat{q}_{t,2}/\partial z_t = 0 \) so that \( \partial \text{Var}(q_t)/\partial z_t = 0 \).

The last two regimes have two of the three possible ranges of demand. In the regime (12), valid for \(-\Delta < \epsilon(z_t) < \Delta < \bar{z}(z_t) \),

\[ \frac{\partial q_t^*(\epsilon_t)}{\partial z_t} = \begin{cases} 0 & \text{if } \epsilon_t \leq \epsilon(z_t), \\ 1 & \text{if } \epsilon(z_t) < \epsilon \leq \Delta < \bar{z}(z_t). \end{cases} \]

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The expected output is:

\[
\hat{q}_{t,12} = \int_{-\Delta}^{\Delta} \left( a - \rho_t + \epsilon_t \right) \frac{d\epsilon_t}{2\Delta} + \int_{\epsilon(z_t)}^{\Delta} \frac{z_t}{2\Delta} \frac{d\epsilon_t}{2\Delta} = \frac{-(a - \rho_t - \Delta)^2}{8b\Delta} + \frac{z_t}{2\Delta} (a + \Delta - \rho_t - b\epsilon_t),
\]

so that

\[
\frac{\partial \hat{q}_{t,12}}{\partial z_t} = \frac{(a + \Delta - \rho_t - 2b\epsilon_t)}{2\Delta} = \frac{\Delta - \epsilon(z_t)}{2\Delta} > 0.
\]  

(A.4)

Note that \(\frac{\partial \hat{q}_{t,12}}{\partial z_t} < 1\) since \(\epsilon(z_t) > -\Delta\). It follows that:

\[
\frac{\partial q_t^* (\epsilon_t)}{\partial z_t} - \frac{\partial \hat{q}_{t,12}}{\partial z_t} = \begin{cases} 
-\frac{\Delta - \epsilon(z_t)}{2\Delta} & \text{if } \epsilon_t \leq \epsilon(z_t), \\
1 - \frac{\Delta - \epsilon(z_t)}{2\Delta} & \text{if } \epsilon(z_t) < \epsilon_t \leq \Delta < \tau(z_t).
\end{cases}
\]

The impact of a decrease in the foreign unit cost on the variance of output is therefore equal to (where we left out the two terms in \((\partial \epsilon(z_t)/\partial z_t)(q_t^*(\epsilon(z_t)) - \hat{q}_{t,12})^2\) because they sum to zero):

\[
\frac{\partial \text{Var}(q_t)}{\partial z_t} = 2 \int_{-\Delta}^{\Delta} \left( \frac{\partial q_t^* (\epsilon_t)}{\partial z_t} - \frac{\partial \hat{q}_{t,12}}{\partial z_t} \right) \frac{(q_t^* (\epsilon_t) - \hat{q}_{t,12}) d\epsilon_t}{2\Delta} = 0
\]

\[
= \frac{(z_t - \hat{q}_{t,12})}{\Delta} [\Delta - \epsilon(z_t)]
\]

\[
= \frac{[\Delta - \epsilon(z_t)]}{\Delta} \left( z_t + \frac{(a - \rho_t - \Delta)^2}{8b\Delta} - \frac{z_t}{2\Delta} (a + \Delta - \rho_t - b\epsilon_t) \right)
\]

\[
= \frac{[\Delta - \epsilon(z_t)]}{\Delta} \frac{1}{8b\Delta} (\Delta - a + \rho_t + 2b\epsilon_t)^2 > 0.
\]

In the regime (23), valid for \(\epsilon(z_t) < -\Delta < \tau(z_t) < \Delta\),

\[
\frac{\partial q_t^* (\epsilon_t)}{\partial z_t} = \begin{cases} 
1 & \text{if } -\Delta \leq \epsilon_t \leq \tau(z_t), \\
0 & \text{if } \epsilon_t > \tau(z_t).
\end{cases}
\]
The expected output is:

\[
\hat{q}_{t,23} = \int_{-\Delta}^{\tau(z_t)} z_t \frac{d\epsilon_t}{2\Delta} + \int_{\tau(z_t)}^{\Delta} \left( \frac{a - w_t + \epsilon_t}{2b} \right) \frac{d\epsilon_t}{2\Delta} = \frac{(2b\hat{z}_t - a)^2 + 4b\hat{z}_t(w_t + \Delta) + (\Delta - w_t)(2a - w_t + \Delta)}{8b\Delta},
\]

so that:

\[
\frac{\partial \hat{q}_{t,23}}{\partial z_t} = \frac{(2b\hat{z}_t - a + w_t + \Delta)}{2\Delta} = \frac{\tau(z_t) + \Delta}{2\Delta} > 0. \tag{A.5}
\]

Note that \(\frac{\partial \hat{q}_{t,23}}{\partial z_t} < 1\) since \(\tau(z_t) < \Delta\). It follows that:

\[
\frac{\partial q_t^*(\epsilon_t)}{\partial z_t} - \frac{\partial \hat{q}_{t,23}}{\partial z_t} = \begin{cases} 
1 - \frac{\tau(z_t) + \Delta}{2\Delta} & \text{if } -\Delta \leq \epsilon_t \leq \tau(z_t), \\
-\frac{\tau(z_t) + \Delta}{2\Delta} & \text{if } \epsilon_t > \tau(z_t).
\end{cases}
\]

The impact of a decrease in the foreign unit cost on the variance of output is therefore equal to (where we left out the two terms in \(\frac{\partial \tau(z_t)}{\partial z_t}(q_t^*(\tau(z_t)) - \hat{q}_{t,23})^2\) because they sum to zero):

\[
\frac{\partial \text{Var}(q_t)}{\partial z_t} = 2 \int_{-\Delta}^{\Delta} \left( \frac{\partial q_t^*(\epsilon_t)}{\partial z_t} - \frac{\partial \hat{q}_{t,23}}{\partial z_t} \right) \left( q_t^*(\epsilon_t) - \hat{q}_{t,23} \right) \frac{d\epsilon_t}{2\Delta}
\]

\[
= -2 \frac{\tau(z_t) + \Delta}{2\Delta} \int_{-\Delta}^{\Delta} \left( q_t^*(\epsilon_t) - \hat{q}_{t,23} \right) \frac{d\epsilon_t}{2\Delta} + 2 \int_{-\Delta}^{\tau(z_t)} (\hat{q}_{t,23}) \frac{d\epsilon_t}{2\Delta} = 0
\]

\[
= \frac{[\tau(z_t) + \Delta]}{\Delta} \left[ z_t - \frac{(2b\hat{z}_t - a)^2 + 4b\hat{z}_t(w_t + \Delta) + (\Delta - w_t)(2a - w_t + \Delta)}{8b\Delta} \right]
\]

\[
= - \frac{[\tau(z_t) + \Delta]}{8b\Delta^2} (a - w_t + \Delta - 2b\hat{z}_t)^2 < 0.
\]

### A.5 Endogenous ρ

Suppose that \(\rho_t\) is endogenously determined by the condition \(\rho_t = \delta E(MR_{t+1})\). We want to show that, for all feasible \(z_t^*\), a reduction in the foreign unit cost raises \(z_t^*\), even if \(\rho_t\) is endogenous. The potential problem to look out for is that a change in \(\nu_{t-1}/\delta\) may affect \(z_t^*\) not only directly but also indirectly through \(\rho_t\). This is the case, if the expectation of future foreign unit costs enters \(E(MR_{t+1})\). We also check that the endogenous value of \(\rho_t\) is consistent with the assumptions of the model.

We start with the two feasible \(z_t^*\) for which the foreign unit cost does not enter
\( E(MR_{t+1}) \), namely \( z^*_{t,2} \) and \( z^*_{t,23} \). Since both are independent of \( \rho_t \), there is no indirect effect of \( v_{t-1}/\delta \) on either \( z^*_{t,2} \) or \( z^*_{t,23} \). Evaluating \( E(MR(z^*_{t,2})) \) leads to \( E(MR(z^*_{t,2})) = v_{t-1}/\delta \) and likewise \( E(MR(z^*_{t,23})) = v_{t-1}/\delta \). Hence \( \rho_t = \delta E(MR(z^*_{t+1,2})) \), respectively \( \rho_t = \delta E(MR(z^*_{t+1,23})) \), implies \( \rho_t = v_t \), where \( v_t \) is the expected foreign unit cost of purchases ordered in period \( t \) (and used in period \( t + 1 \)). Since \( v_{t-1}/\delta > \rho_t \), then \( v_{t-1} > \delta \rho_t = \delta v_t \) which holds for any \( v_{t-1} \geq \delta v_t \) since \( \delta < 1 \).

Consider regime (12). Like above, \( \rho_t = \delta E(MR(z^*_{t+1,12})) \) implies \( \rho_t = v_t \). A reduction in the foreign unit cost now has two effects on \( z^*_{t,12} \), namely a direct effect of raising \( z^*_{t,12} \) whenever \( v_{t-1} \) decreases, and a further indirect effect going in the same direction by lowering \( \rho_t \) through a lower future expected price \( v_t \).

In regime (123), like in regime (12), a reduction in the foreign unit cost has both a direct and an indirect effect on \( z^*_{t,123} \). Using \( \rho_t = v_t \) to evaluate the combination of these two effects, we obtain

\[
\text{Sign} \left( \frac{dz^*_{t,123}}{d\rho_t} \right) = -\text{Sign} \left\{ 4w_t \Delta (1 - \delta) + \delta (w_t - \rho_t)^2 \right\} < 0.
\]

Hence a reduction in the foreign unit cost unambiguously raises \( z^*_{t,123} \).

References


