Why a pandemic recession should *boost* asset prices
(...according to standard economic theory)

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Abstract: Economic recessions are traditionally associated with asset price declines, and recoveries with asset price booms. Standard asset pricing models make sense of this: during a recession, dividends are low and the marginal value of wealth is high, causing low asset prices. Here, I develop a simple model which shows that this is not true during a recession caused by *consumption restrictions*, such as those seen during the 2020 pandemic: the restrictions drive the marginal value of wealth down, and thereby drive asset prices up, to an extent that tends to overwhelm the effect of low dividends.

JEL codes: E21, G12, I19

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Economists and market participants alike have been puzzled by how quickly, and how completely, stock prices have rebounded from their crash in the early days of the Covid-19 pandemic. Traditionally, we think of recessions as causing asset price declines, and recoveries causing booms, but by late summer of 2020 stock indices in most countries had recovered beyond their previous peak, even as forecasters agree that the economic damage inflicted by the pandemic will be deep and long-lasting [5].

What can explain this disconnect? Financial analysts have proposed three explanations [3, 6]: (a) asset markets are forward-looking, so high prices could just reflect investors' expectations of a quick end to the pandemic; (b) the kinds of big companies that are represented in the major stock indices are shielded from pandemic effects, or even stand to profit from them (e.g., Big Tech and Big Pharma); (c) asset values are being supported by central bank intervention. However, (a) is not looking likely as forecasts have now for several months ruled out a quick economic recovery. Arguments (b) and (c) are plausible, but do not explain why the asset market recovery has been so broad-based; the Russell 3000 index (which covers almost all of the US equity market) has also passed its February peak, while bond markets and housing markets are setting records for high prices and low yields [1, 4].

In this paper, I construct a simple neoclassical asset pricing model in the spirit of the Lucas "tree" model [2], and I model various restrictions plausibly caused by a pandemic shock. The model suggests an alternative explanation for high asset prices: they are *caused*

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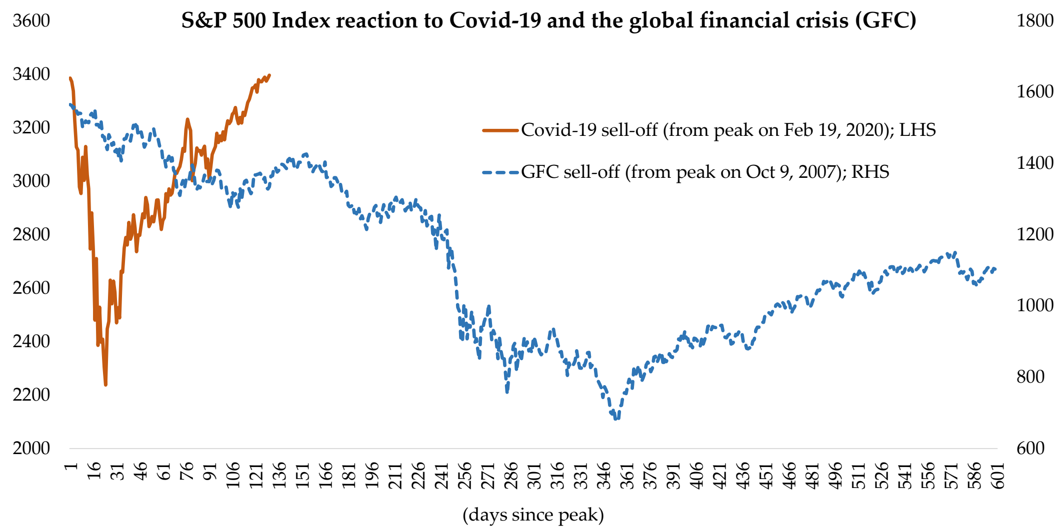


Figure 1: The "great disconnect": asset prices have fully recovered even though the global pandemic is ongoing. Data source: Yahoo Finance.

by the pandemic, not hindered by it, and more specifically they are caused by restrictions on consumption due to social distancing. (Whether distancing is voluntary or due to government mandates is not relevant to the argument.) In a typical recession, incomes fall, and households respond to shrinking budgets by reducing their consumption expenditure. This results in a rising marginal value of income, a falling desire to save, and a low valuation of financial assets. In a pandemic, on the other hand, households reduce consumption of socially-exposed goods and services in order to protect their health. Thus, it is the consumption restrictions that cause income reductions, and the result is a *falling* marginal value of income, an increasing desire to save (since additional income cannot be consumed, at least not in the way we want to most badly), and a high valuation of financial assets.

Certainly, the reduced income causes lower asset dividends as well. However, unless the pandemic is expected to last for decades, the model shows that the effect of an increased desire to save easily dominates the effect of lower dividends. If there are restrictions on production in addition to consumption, the results are weakened and may get reversed, but only if both (a) production restrictions are tighter than consumption restrictions, and (b) the supply side of the economy is highly elastic in the short run. If agents misperceive the model, they will initially underprice assets as the pandemic hits, but the increased desire to save will eventually result in high assets prices even if nobody (within the model) understands the reason for this.

The rest of this paper is organized as follows. Section 1 develops the basic model, derives results, and explains the intuition. Section 2 extends the model to cover capital losses, multiple goods, and beliefs. Section 3 discusses the limitations of the model and concludes.

1 The basic model

There are two states of the world: $s_t = \{0, 1\}$. We call $s_t = 1$ the "sick" or "pandemic" state and $s_t = 0$ the "normal" state. There is a large measure of households and firms who take market prices as given. There is a single consumption good, which is produced using labour and a capital asset *in fixed supply*. The production function is:

$$y_t = k_t^\alpha h_t^{1-\alpha}$$

The aggregate supply of capital is normalized to $K = 1$, but individual agents can buy and sell units of k at price q_t. In a period, capital yields a rental rate r_t and labor yields a wage rate w_t. Agents thus choose consumption c_t and labor supply h_t subject to the budget constraint:

$$c_t + q_t k_{t+1} = (r_t + q_t)k_t + w_t h_t$$

They seek to maximize the following standard utility function:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \frac{1}{1+\eta} h_t^{1+\eta} \right)$$

However, the twist is that consumption in a pandemic state must also satisfy the constraint $c_t \leq \hat{c}$. This can be interpreted either as a physical or legal constraint (certain activities, like going to bars or traveling internationally, are prohibited), or a part of agents' preferences whereby consuming $c_t > \hat{c}_t$ yields infinitely negative utility (people voluntarily avoid bars and air travel because of the infection risk). Either way, I assume that the constraint is slack in the normal state $s_t = 0$, and $\hat{c}$ is so low that the constraint binds in the sick state $s_t = 1$.

Household decisions thus satisfy the following Bellman equation, in Lagrangian form:

$$V(k_t, s_t) = \max_{\substack{c_t, k_{t+1} \\ \lambda_t, \mu_t}} \Bigg\{ \log(c_t) - \frac{1}{1+\eta} h_t^{1+\eta} + \beta\, \mathbb{E}_t \big\{ V(k_{t+1}, s_{t+1}) \big\}$$
$$+ \lambda_t [(r_t + q_t)k_t + w_t h_t - c_t - q_t k_{t+1}] + \mu_t s_t [\hat{c} - c_t] \Bigg\}$$

where λ_t and μ_t are the Lagrange multipliers on the budget and health constraints, respectively. (It turns out to be convenient to keep λ_t around, rather than immediately substituting it with a marginal utility term as we normally would.) Standard optimization methods tell us that the solution must satisfy the Euler equation:

$$\lambda_t\, q_t = \beta\, \mathbb{E}_t \Big\{ \lambda_{t+1} (r_{t+1} + q_{t+1}) \Big\} \tag{1}$$

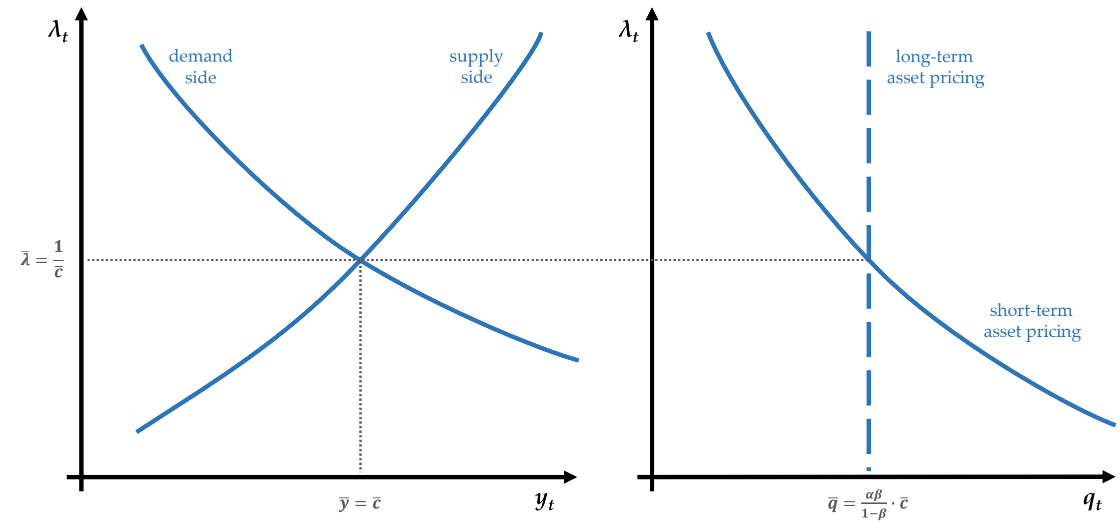


Figure 2: The "normal" state of the economy.

Notes: The "demand side" curve represents the first-order condition for consumption together with goods market clearing: $y_t = c_t = 1/\lambda_t$. The "supply side" curve represents the first-order conditions for labor supply and labor demand together with the production function: $y_t = h_t^{1-\alpha} = (1-\alpha)^{-1}\lambda_t^{(1-\alpha)/(\eta+\alpha)}$. The "short-term asset pricing" curve represents the Euler Equation with expectations about the future held fixed: $\lambda_t q_t =$ constant.

In a normal state, we have consumption equal to the inverse marginal value of wealth, but in a pandemic state, consumption is constrained: $c_t = \min\{1/\lambda_t, \hat{c}/s_t\}$. The labor supply curve is $h_t^\eta = \lambda_t w_t$, and the aggregate labor demand curve is $w_t = (1-\alpha)h_t^{-\alpha}$ (since the capital stock is fixed at 1). The rental rate on capital is $r_t = \alpha h^{1-\alpha}$, and market clearing in the goods market requires $c_t = y_t = h_t^{1-\alpha}$. An **equilibrium** is defined to be any bounded sequence of $\{c_t, h_t, r_t, w_t, \lambda_t, q_t\}_{t=0}^\infty$ satisfying these equations, for a given belief about states $\{s_t\}_{t=0}^\infty$.

Never pandemic

To simplify the analysis, I assume that the "normal" state is always believed to be a steady state; it lasts forever with no risk of a future pandemic. In that case, the optimality condition $c_t = 1/\lambda_t$ together with the market clearing equations yields the steady-state equilibrium:

$$\begin{aligned} c_t &= \bar{c} \equiv (1-\alpha)^{\frac{1-\alpha}{1+\eta}} & \qquad r_t &= \bar{r} \equiv \alpha\bar{c} \\ h_t &= \bar{h} \equiv (1-\alpha)^{\frac{1}{1+\eta}} & \qquad \lambda_t &= \bar{\lambda} \equiv \frac{1}{\bar{c}} \end{aligned} \tag{2}$$

Plugging these solutions back into Equation (1), and using $q_t = q_{t+1}$, we obtain the normal-state price of capital:

$$\bar{q} = \frac{\alpha\beta}{1-\beta}\bar{c} \tag{3}$$

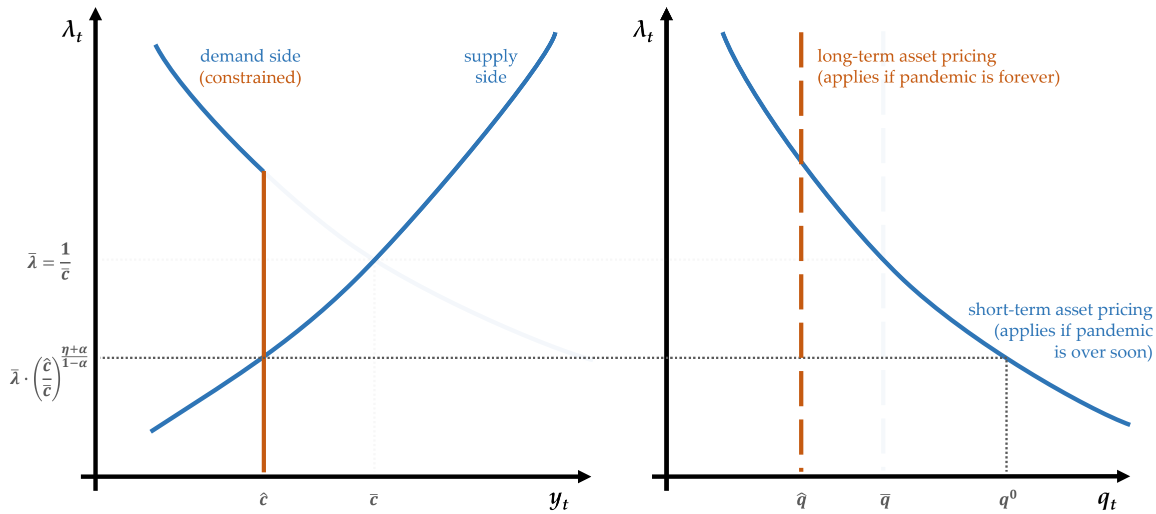


Figure 3: The "pandemic" state of the economy.

Notes: The "demand side" curve represents the first-order condition for consumption together with goods market clearing: $y_t = c_t = \min\{\hat{c}, 1/\lambda_t\}$. The "supply side" curve represents the first-order conditions for labor supply and labor demand together with the production function: $y_t = h_t^{1-\alpha} = (1-\alpha)^{-1}\lambda_t^{(1-\alpha)/(\eta+\alpha)}$. The "short-term asset pricing" curve represents the Euler Equation with expectations about the future held fixed ($\lambda_t q_t =$ constant), whereas the "long-term asset pricing" curve represents the Euler equation solved in steady state (5).

Forever pandemic

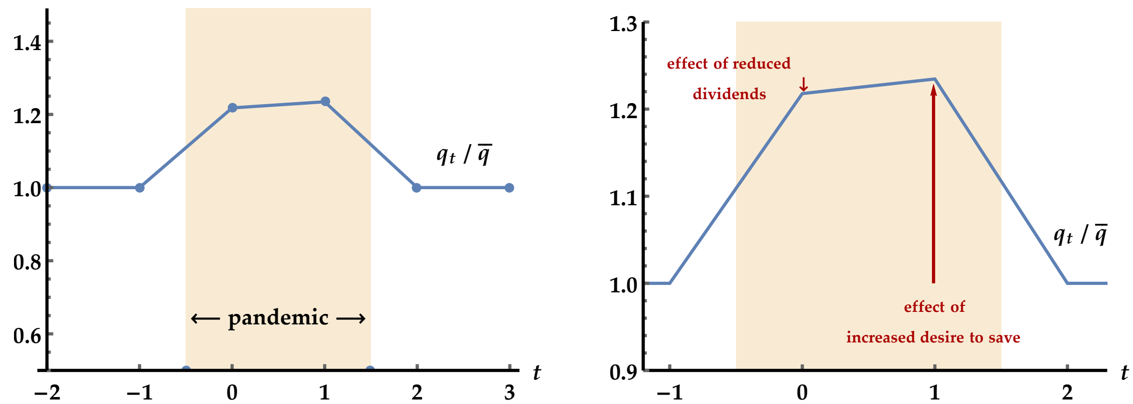
However, a pandemic *did* strike in 2020. Solving the optimality and market clearing conditions together with $c_t = \hat{c}$, in a pandemic state we have:

$$\begin{aligned} c_t &= \hat{c} & r_t &= \alpha\hat{c} \\ h_t &= (\hat{c})^{\frac{1}{1-\alpha}} & \lambda_t &= \frac{1}{1-\alpha}(\hat{c})^{\frac{\eta+\alpha}{1-\alpha}} \end{aligned} \tag{4}$$

Thus, during the ongoing pandemic all real variables are characterized by the constraint $\hat{c}$ alone. The only variable that requires knowing more than that is the price of capital, because that depends on whether agents believe the pandemic will persist or end soon.

If the pandemic is expected to persist forever, then of course we have a steady state with $q_t = q_{t+1}$. Plug this into Equation (1), evaluate at pandemic values, and we obtain the forever-pandemic asset price value which we can call $\hat{q}$:

$$\hat{q} = \frac{\alpha\beta}{1-\beta}\hat{c} \tag{5}$$



[a] Effect of consumption restrictions [b] Zooming in on [a]

Figure 4: Asset prices during a pandemic; unless the pandemic is expected to last decades, effect of lower dividends is dominated by effect of lower marginal value of wealth.

Notes: We assume the pandemic hits as a surprise in period 0, causes consumption to be restricted to 90% of its normal level, and is immediately understood to last through period 1 and end in period 2. Parameters: $\alpha = 1/3, \eta = 1, \beta = 0.95$.

Pandemic is known to last for n periods

With these tools, we can analyze what would happen if the pandemic was known to last for $n \geq 0$ more periods (excluding the current period). Iterating Equation (1), after some algebra we obtain:

$$\boxed{\frac{q^n}{\bar{q}} = (1-\beta^n)\frac{\hat{c}}{\bar{c}} + \beta^n \left(\frac{\hat{c}}{\bar{c}}\right)^{-\frac{\eta+\alpha}{1-\alpha}}} \tag{6}$$

where q^n is the price of the asset given that the pandemic is expected to persist for n more periods, and for simplicity it has been expressed in relation to the long-term normal-state asset price $\bar{q}$ (Equation 3) and consumption $\bar{c}$ (Equation 2).

Equation (6) is the main result of this paper. Notice what happens if the pandemic lasts forever ($n \to \infty$): consumption is depressed forever, hence economic activity is depressed forever, and so is the price of capital. But if the pandemic is expected to be short-lasting ($n = 0$ in particular), **the depressed economic activity results in a boost to asset prices**, since the exponent on the last term is negative. For realistic values of the discount factor β and the share of quickly-adjustable factors of production $(1-\alpha)$, the (negative) contribution of the first term is dominated by the (positive) contribution of the second term – even if the pandemic was expected to last, say, three or four years.

What is the intuition for this striking result? The key lies in the level of λ, the marginal value of wealth, during the pandemic. Generally in macroeconomics, the marginal value of wealth is inversely related to consumption – such as here, $\bar{\lambda} = 1/\bar{c}$ in the normal state – a result so basic that it has become part of the 'deep wiring' of a macroeconomist's thinking engine. However, **in a pandemic, consumption is not constrained by wealth, but by health**. Thus, the only value of an increase in wealth is that it helps the household work less, and avoid the disutility of working. But this disutility falls when the economy is constrained – indeed, Equation (4) confirms that λ is positively related to the consumption constraint.

Now, standard theory suggests that asset prices are determined by two things: expected dividends, and the value of deferring wealth into the future. It is true that dividends ($r_t = \alpha c_t$ in the simple model, perfectly correlated with aggregate consumption) are lower in the pandemic ($\hat{c}$ vs $\bar{c}$), and this channel becomes more important the longer the pandemic is expected to last. However, if the pandemic is not expected to last beyond a few years, what is much more important than dividends is the motivation to defer spending until normal activity can resume. Figure 4 illustrates this result with a numerical example.

Pandemic is expected to end at random date

We can do a similar analysis under the simplifying assumptions that we start in the pandemic, each period the pandemic ends with probability $1 - \pi$ and persists otherwise, and once the pandemic is over it never returns. We denote the asset price in this scenario by q^π (note that π is a label here, not an exponent). After some algebra, we obtain:

$$\frac{q^\pi}{\bar{q}} = \frac{(1-\beta)\pi}{1-\beta\pi} \cdot \frac{\hat{c}}{\bar{c}} + \frac{1-\pi}{1-\beta\pi} \left(\frac{\hat{c}}{\bar{c}}\right)^{-\frac{\eta+\alpha}{1-\alpha}}$$

Again, unless π is of similar magnitude to β (meaning the pandemic is expected to persist for decades), the second term dominates and the pandemic causes a boost in asset prices.

2 Bells and whistles

Certainly, the basic model from Section 1 is just that, basic. A pandemic has many effects, more than what can be captured with a simple cap on aggregate consumption. To obtain more general results, in this section I solve three extensions of the basic model – capital obsolescence, multiple goods, and incorrect beliefs by agents within the model – and discuss their implications.

2.1 Capital obsolescence

The model in Section 1 is simple and based on very standard macroeconomic principles. So, why does our intuition seem to dictate that the pandemic should decrease stock prices? One of the reasons might be the fact that social distancing restrictions – whether voluntary or not

– do not only affect the availability of consumption goods, but also the usefulness of certain kinds of capital. For example, restaurants are forced to operate at reduced capacity, and sports arenas and convention centers are kept empty. Grocery stores and airports are open, but they are being retrofitted at high costs.

Here, we can capture this channel by assuming that during the pandemic state $s_t = 1$, only a fraction of the capital stock, $\kappa < 1$, can be used. That is to say, an agent holding k_t units of capital during the pandemic is only able to rent out (and collect returns on) κk_t of them. The aggregate stock of capital remains normalized at $K = 1$ throughout this exercise.

2.1.1 No consumption restrictions

To begin with, we assume that there are no consumption restrictions. Then, consumption equals the inverse marginal value of wealth: $c_t = 1/\lambda_t$. As before, the labor supply curve is $h_t^\eta = \lambda_t w_t$, and the aggregate labor demand curve is $w_t = (1-\alpha)y_t/h_t$. The aggregate resource constraint is $c_t = y_t = \kappa^\alpha h_t^{1-\alpha}$ during the pandemic, and $c_t = y_t = h_t^{1-\alpha}$ outside of it. Either way, the rental rate on total capital is $r_t = \alpha y_t$; during the pandemic, not all capital is usable (so we need to multiply by κ), but the marginal value of *usable* capital is inversely proportional to the ratio of usable capital per unit of output (so we divide by κ). Solving, we obtain the key equations:

$$\begin{aligned} y_t = c_t = \kappa^\alpha \cdot \bar{c} \qquad &\Rightarrow \qquad r_t = \alpha \bar{c} \cdot \kappa^\alpha \\ \lambda_t = 1/c_t \qquad &\Rightarrow \qquad \lambda_t = \alpha / r_t \end{aligned}$$

We plug these results into the Euler equation (1), and notice that $\lambda_{t+1} r_{t+1}$ simplifies to a constant α. Thus, $\lambda_t q_t = \beta \mathbb{E}_t\{\alpha + \lambda_{t+1} q_{t+1}\}$, and we can simply iterate on $\lambda_t q_t$ to obtain:

$$\lambda_t q_t = \frac{\alpha\beta}{1-\beta} \qquad \Rightarrow \qquad \frac{q_t}{\bar{q}} = \frac{c_t}{\bar{c}} = \kappa^\alpha \tag{7}$$

Thus, current asset prices only depend on current consumption. In particular, this implies that during a pandemic where only a fraction κ of all capital can be used to earn returns, stock prices should be scaled down by a factor κ^α. Furthermore, it implies that the duration of the pandemic is irrelevant; and, more than that, it is irrelevant whether the pandemic-induced loss of capital is believed to be temporary or permanent!

The reason for this strong result is of course our assumption of logarithmic utility; capital obsolescence causes both an income and a price effect, and these two effects offset exactly. If the obsolete capital is gone forever, then expected future returns fall but the marginal value of saving (λ_{t+1}/λ_t) is flat. If the capital is only temporarily disabled, then expected future returns are preserved but the marginal value of saving falls by an equal amount.

To be sure, this is a special case, and we could analyze variations with more general intertemporal preferences. Nevertheless, as long as these variations do not depart too far from the logarithmic benchmark, the results are clear: (i) if the only effect of the pandemic is that some fraction of the capital stock becomes unusable, capital prices fall; (ii) the loss of usable capital is passed through to stock prices with elasticity α, the elasticity of output with respect to the relevant type of capital; (iii) it does not matter whether the disruption is temporary or permanent.

2.1.2 Capital and consumption restrictions combined

Naturally, since the 2020 pandemic has caused restrictions to both consumption and productive capacity, we should investigate the combined effect of these restrictions. To do so, I assume that $\hat{c} < \kappa^\alpha \bar{c}$; that is to say, consumption is restrained even below the level that can be achieved with the reduced capacity. In this case, during a pandemic state $s_t = 1$, output is again determined by consumption demand ($y_t = c_t = \hat{c}$), and so are returns on total capital ($r_t = \alpha y_t = \alpha \hat{c}$). After some algebra, the marginal value of wealth during the pandemic is:

$$\lambda_t \Big|_{s_t=1} = \frac{1}{1-\alpha} (\hat{c})^{\frac{\eta+\alpha}{1-\alpha}} (\kappa)^{-\frac{\alpha\eta+\alpha}{1-\alpha}}, \tag{8}$$

which is, confirming the results from the previous models, increasing in the allowed fraction of consumption $\hat{c}/\bar{c}$ and decreasing in the fraction of usable capital κ.

As before, suppose that the pandemic is known to persist for another $n \geq 0$ periods after the current one; iterating Equation (1), we obtain the following ratio of stock prices to their long-term pre-pandemic value:

$$\boxed{\frac{q^n}{\bar{q}} = (1-\beta^n)\frac{\hat{c}}{\bar{c}} + \beta^n \left(\frac{\hat{c}}{\bar{c}}\right)^{-\frac{\eta+\alpha}{1-\alpha}} (\kappa)^{\frac{\alpha\eta+\alpha}{1-\alpha}}} \tag{9}$$

Comparing this equation with the earlier result (6), the effect of the pandemic on the dividend component of the equation is exactly the same; as long as the consumption restriction is a binding constraint, dividends are proportional to aggregate consumption, no matter what happens to capital. But as explained earlier, given reasonable values for the discount factor β and unless the pandemic is expected to persist for many years, what matters for stock prices *during* the pandemic is the final value $q_0/\bar{q}$, the *last price before exit* from the pandemic. Here, the two restrictions push in opposite directions: the restriction on consumption ($\hat{c} < \bar{c}$) lowers the marginal value of wealth, while the restriction on capital ($\kappa < 1$) increases it.

In a pandemic like the one in 2020 where both restrictions operate, which one wins? In principle, this is of course a quantitative question, but even just with theory we can say a bit by comparing the elasticities. It turns out that the elasticity on $\hat{c}$ (in absolute value) exceeds the elasticity on κ by exactly η, the inverse elasticity of the labor supply. This means that

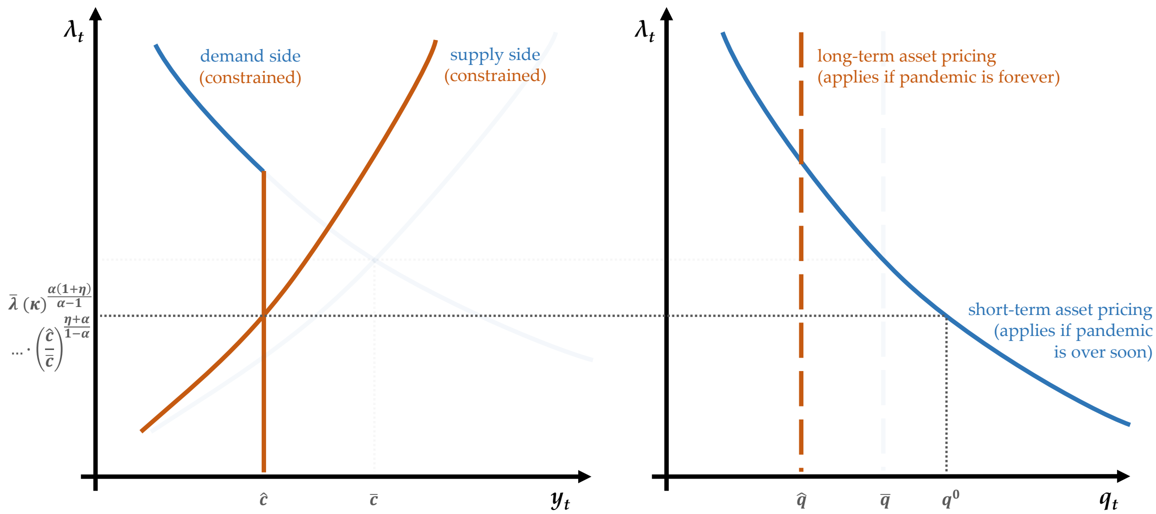


Figure 5: The “pandemic” state, with restrictions on consumption and capital use.

Notes: The “demand side” curve represents the first-order condition for consumption together with goods market clearing: $y_t = c_t = \min\{\hat{c}, 1/\lambda_t\}$. The “supply side” curve represents the first-order conditions for labor supply and labor demand together with the production function: $y_t = \kappa^\alpha h_t^{1-\alpha} = (1-\alpha)^{-1}\kappa^\alpha \lambda_t^{(1-\alpha)/(\eta+\alpha)}$. The “short-term asset pricing” curve represents the Euler Equation with future expectations held fixed ($\lambda_t q_t$ = constant), whereas the “long-term asset pricing” curve represents the Euler equation solved in steady state (5).

when the labor supply is elastic (so that $\eta \to 0$), consumption and capital restrictions are about equally strong in the magnitude of their effect on stock prices. When the labor supply is inelastic, on the other hand ($\eta \to \infty$), then the effect of consumption restrictions will dominate and even a small consumption restriction can drive asset prices arbitrarily high.

For concreteness, consider a few examples (and for simplicity, assume that the pandemic lasts for only one period in each case):

(E1) The pandemic reduces both consumption and the usable capital stock by one percent ($\hat{c}/\bar{c} = \kappa = 0.99$). In this case, since $\alpha < 1$, we have $\hat{c} < \kappa^\alpha \bar{c}$ and thus the constraint on consumption binds. The stock price q^0 increases by η percent above $\bar{q}$.

(E2) The usable capital stock falls by ten percent ($\kappa = 0.9$), but people can satisfy the health constraint if they reduce consumption by three percent ($\hat{c}/\bar{c} = 0.97$). If $\alpha \geq 0.3$, then consumption falls by 10α percent which is *more* than three percent, so the health constraint does not bind. Stock prices fall by 10α percent, the same as consumption.

(E3) The usable capital stock falls by four percent ($\kappa = 0.96$), and in order to stay healthy people must reduce consumption by two percent ($\hat{c}/\bar{c} = 0.98$). Also, suppose $\alpha = 1/3$ and $\eta = 1$. Then, stock prices stay exactly the same compared to both before and after the pandemic.

(E4) The usable capital stock falls by three percent ($\kappa = 0.97$), but in order to stay healthy people must reduce consumption by nine percent ($\hat{c}/\bar{c} = 0.91$). Also, suppose $\alpha = 1/3$ and $\eta = 1$. Then, stock prices increase by $[(9 - 3\alpha)\eta + 6\alpha]/(1 - \alpha) = 15$ percent.

These examples illustrate that when the consumption restriction $\hat{c}/\bar{c}$ and the capital usability restriction κ are similar in magnitude, the effect of the consumption restriction tends to win out and cause stock prices to go up during the pandemic. In order for stock prices to go down, we would need (a) the capital restriction to be much severe than the consumption restriction; (b) the capital elasticity in the production function, α, to be large; (c) the elasticity of short-term labor supply, $1/\eta$, to be large as well.

2.2 Two types of consumption goods

Of course, the effect of the 2020 pandemic has not been to force a reduction in all kinds of consumption spending equally. Some spending on affected goods (air travel and movie theaters) has been diverted to others (hand sanitizer and yoga mats). To capture this, I augment the model from Section 1 with two consumption goods: "social" consumption c_t^S which is subject to the health constraint $c_t^S \leq \hat{c}/s_t$, and "private" consumption c_t^P which is not constrained in this way. The utility function is changed to:

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\sigma \log c_t^S + (1 - \sigma) \log c_t^P - \frac{1}{1+\eta} h_t^{1+\eta} \right)$$

so that, ideally, households want to spend a fraction $\sigma \in (0, 1)$ of their income on social goods and the remainder on private goods. The resource constraint is:

$$c_t^S + c_t^P = y_t = k_t^{\alpha} h_t^{1-\alpha}$$

Since this is a simple extension of the basic model, I skip the Bellman equation and go straight to the solution. It turns out that output must satisfy the equation:

$$\hat{c} + (1 - \sigma)(1 - \alpha) y_t^{-\frac{\eta+\alpha}{1-\alpha}} = y_t$$

This equation clearly has a unique solution for y_t, but it cannot be solved in closed form except in a few special cases. One such special case is $\eta \to \infty$, meaning that labor supply and output are both fixed at 1, and the only problem in this economy is to allocate consumption between social and private consumption.[1] In that case, the solution for the asset price during a pandemic (which is again expected to last for $n \geq 0$ more periods) would be:

[1]Another special case is $\eta = 1 - 2\alpha$, where the exponent $(\eta + \alpha)/(1 - \alpha)$ equals 1 so the model can be solved in closed form as well. In that case, the model makes the intuitive prediction that the impact of the pandemic is mixed between S-consumption falling, output as well but less so, and P-consumption increasing. But there are no additional insights for asset prices, so I do not explore the case further.

$$\frac{q^n}{\bar{q}} = (1 - \beta^n) \cdot 1 + \beta^n \cdot \frac{1 - \hat{c}}{1 - \sigma}$$

This time, there is only a positive effect (through $\hat{\lambda} \uparrow$); because output is fixed, so are dividends. And as one would expect, the effect of a restriction on social consumption is strongest when the social sector is a big share (σ) of the economy; specifically, the elasticity of the short-pandemic stock price q^0 with respect to $\hat{c}$, evaluated near the no-restrictions steady state $\bar{c}^S = \sigma$, is:

$$\left.\frac{d \log(q_0)}{d \log(\hat{c})}\right|_{\hat{c} \to \sigma} = -\frac{\sigma}{1 - \sigma}$$

The lesson here is that when the supply side of the economy is inelastic, the impact of pandemic restrictions on asset prices can be arbitrarily large, even when the intensive margin of restrictions ($\hat{c} < \bar{c}^S$) is small; what matters is the extensive margin, or *how many* kinds of consumption are restricted, moreso than by *how much*.

2.3 Incorrect beliefs

A model proposed in August of 2020 cannot claim that investors in March of 2020, when stock prices crashed, knew that new model. Instead, it is plausible that during the early days of the pandemic when it became clear that it would cause deep and long-lasting economic damage, investors used familiar models to predict its effect on asset returns and prices.

We can obviously never be sure what exactly investors were thinking. However, within the context of the model here, we can take a pass at it by assuming that (a) reality is described by the model from Section 2.1.2, but (b) agents believe that they are living in the world of Section 2.1.1 and ignore than the effects of the health constraint $c_t \leq \hat{c}$ on asset prices.

To keep things simple, I also assume that the pandemic hits (as a complete surprise) in period 0, at which point everyone believes with certainty that the pandemic will persist through period 1 and be over in period 2. (The argument that follows will mainly focus on asset prices in periods 0 and 1, thus it does not matter much whether the pandemic is actually over in period 2.) Thus, the economy is expected to be in the "normal" steady state from period 2 on:

$$\lambda_t q_t = \bar{\lambda} \bar{q} \quad \forall t \geq 2$$

At time 0, agents believe that in period 1, the model from Section 2.1.1 will apply, hence the asset price in period 1 will be $\tilde{q}_1 = \kappa^\alpha \, \bar{q}$. Thus, in period 0, they evaluate the Euler equation:

$$\lambda_0 q_0 = \beta \mathbb{E}_t \{ \lambda_1 (r_1 + \tilde{q}_1) \}$$

They also believe that period 1 is still in the pandemic, as is period 0, thus they conclude that

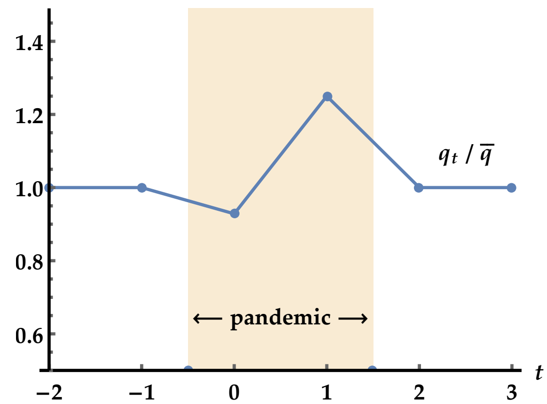


Figure 6: Asset prices during a pandemic when agents have incorrect beliefs.

Notes: We assume the pandemic hits as a surprise in period 0, causes both consumption and capital use to be restricted to 80% of their normal levels, and is immediately understood to last through period 1 and end in period 2; agents do *not* understand the effect of consumption restrictions on asset prices, but otherwise act optimally. Parameters: $\alpha = 1/3, \eta = 1, \beta = 0.95$.

$\lambda_0 = \lambda_1$ (whatever that value may turn out to be); hence, λ drops out of the Euler equation. Finally, in accordance with their model of the world, agents believe that $r_1 = \kappa^\alpha \bar{r}$ because of the ongoing restrictions on the use of capital during the pandemic. In that case, their willingness to pay for capital in period 0 is $q_0 = \kappa^\alpha \bar{q}$, the same as they believe will be the price in period 1.

However, once period 1 comes around, agents' marginal value of saving will be low, not high, due to the ongoing consumption restrictions (see Equation 8). Their willingness to pay for capital will be:

$$q_1 \;=\; \frac{\lambda_2}{\lambda_1}\,\bar{q} \;=\; \left(\frac{\hat{c}}{\bar{c}}\right)^{-\frac{\eta+\alpha}{1-\alpha}} (\kappa)^{\frac{\alpha\eta+\alpha}{1-\alpha}},$$

the 'correct' asset price as per Equation (9). Thus, the trajectory of asset prices satisfies:

$$\bar{q} \;>\; q_0 \;<\; q_1 \;\overset{?}{\gtrless}\; \bar{q} \tag{10}$$

They go through a zigzag pattern, falling at the onset of the pandemic, rising (possibly above the steady state) near its end, and returning to the old steady state once the pandemic is over. Figure 6 illustrates this result with a numerical example.

The point here is not that agents 'learn' in period 1 that they were wrong about the model of the pandemic. On the contrary, finding the 'correct' price in period 1 requires only that agents have correct beliefs about period 2 (the pandemic is over and the economy returns to steady state), and respond optimally to their own individual constraints (budget and health). They may observe the zigzag pattern for asset prices, but incorrectly attribute it to changing beliefs about the course of the pandemic.

3 Discussion

In this paper, I develop a simple variation of the standard neoclassical growth model. In the benchmark version, there are only two changes: first, a pandemic shock forces everybody to *reduce consumption* below the steady-state value, and second, capital is in fixed supply. This second assumption makes capital similar to a "Lucas tree" [2]; however, in that model, trees are the only factor of production, whereas here it turns out to be important that there is also an elastic factor of production, otherwise asset prices blow up to infinity. For the purposes of the model, I call that factor "labor", but it really represents any input into production of which the supply can be quickly adjusted.

The model implies that a pandemic causes a decrease in the marginal value of current income, which can be interpreted as an increased demand for saving, and which is translated into high asset prices. This result is not particularly dependent on how low asset dividends fall during the pandemic; it only requires that the pandemic be short (in the sense of not lasting more than a few years), and that consumption restrictions be at least as severe as restrictions on supply. It also has nothing to do with central bank intervention in asset markets; the point is that the pandemic increases the *demand for saving instruments in general*, which drives up their prices, so if a central bank swaps one kind of saving instrument (stocks and bonds) for another (money), this does little to satisfy the increased demand overall.

The model, simple as it is, does miss one big ingredient in real-world asset markets: leverage. If leverage is high, this could provide one reason for stock prices during the pandemic to stay low, or not rise as high as the model predicts. For example, a firm with a leverage ratio of 10 will see its dividends fall by 50 percent even if the aggregate economy only shrinks by 5 percent, and it is only the 5 percent that cause a higher demand for saving, not the 50. If the aggregate economy shrinks even more, the firm goes bankrupt so its asset value hits zero and never recovers, even after the pandemic. Thus, if we expect the pandemic recession to be so severe as to cause widespread *bankruptcies*, then the increased demand for saving instruments would not necessarily be enough to save the stock market from collapse.

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