The Fundamental Relations of Syntax and Logico-conceptual Structure

(a working set of ideas)

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1 Introduction.

My intention here is to create a set of terms for minimal syntax and their conceptual counterparts. We will start with a prime, which cannot be defined, and build up the definitions from these primes. The following box contains the corresponding units of conceptual structure and their correspondent units of syntactic structure.

2 Worlds

I use the term *world* here as an abstract set of all possible things, ideas, entities and so forth. A world may be a set that can contain "subworlds" or a hyperworld may contain one or more worlds.

1.1 Empty World

It is conceivable that there is a world that is entirely empty. Such a world is entirely uninteresting except for set theory, which I will discuss below. An empty world is equivalent to an empty set. The numbering theory proposed by Russell (19--) uses the empty world or set as the basic of numbers. Zero (ø) is the standard notation for the empty set.

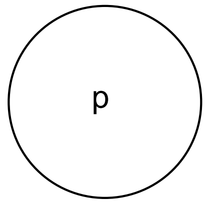
1.2 First world

Next, I imagine a world with one 'thing'. It is immaterial what this 'thing' is. The thing has no properties or features. If it did, this simple world would no longer be so simple.[1]

[1] . It is possible in our world to call it a predicator. I will do so later. Whatever it is called is for the benefit of the reader

However, there is nothing I can say about the thing. Since this world contains one thing, no other things exist that modify the thing in Figure 1 below. I will follow the Gödel (19--) system of representing variables as ‘p, q, r, ...’. ‘p’ is a primitive sign, which means that it cannot be analyzed into further components:

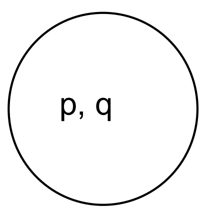
(1)



The circle is just an arbitrary representation of the world. We could represent the thing in any way for our own benefit.

1.3 Second world.

Let us imagine a world with two ‘things’ or predicators. This is a slightly more interesting world. We may identify each predicator as A and B, but such labels do not exist in this world, as a label would be another thing. Labels are for our benefit. In this world two predicates, ‘p and ‘q’ exist independent of each other, The world is also rather uninteresting--what could be so interesting about two predicators that have no relation what so ever? That they share the same world is not very interesting since it is already known.



I will call this world ‘World 2.’ In world 2 there are only two things. It is impossible to compare them because a comparison would require at least a third thing (compare). World 2 is almost as uninteresting as World 1.

In symbolic terms, we can rewrite (5) as

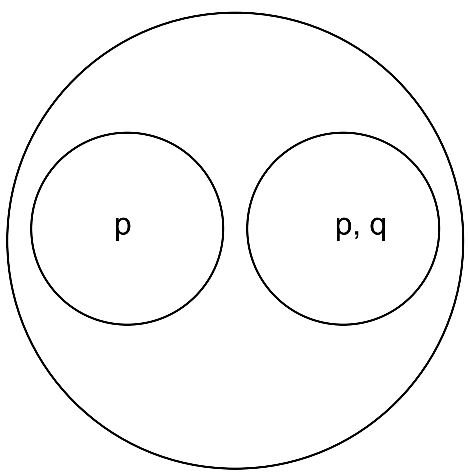
(2) {p, q},

where ‘{’ and ‘}’ are used to enclose a set; the members of this set are separated by a comma. The two sets are unordered. A set appears to be a third concept in this world. Perhaps, it follows axiomatically that ‘p’ and ‘q’ automatically form a set and that there is no distinction between ‘p’ and ‘q’ and {p, q}. The differences in writing are notational variants and that they are not a property of any world per se, except in the world of notational writing, which need not concern us here.

3 World 3

In the next world I want to establish a relationship between ‘p’ and ‘q’. The concept of a relation leads to a new thing. There must be a world of three things. The third thing is ‘relation.’ As there may be different kinds of relations from our perspective, I will have to one step further to capture the kind of relation that I am after here.

First, Suppose that World 3 is a world that may contain two (or more?) subworlds. The first subworld, Subworld 1, contains just ‘p.’ The second subworld contains ‘p’ and ‘q’. However, there is no subworld that contains just ‘q’:



‘P’ can exist without ‘q’, but ‘q’ cannot exist without ‘p’ in World 3. The relation between ‘p’ and ‘q’ is a thing that I will call a dependency ().

World 3 now contains two subworlds, ‘p’ and ‘p, q’. Perhaps now, it would be more appropriate to replace ‘world’ with a set. One set contains p, and the second set contains ‘p, q’:

(3) a. {p} = P

{p, q} = PQ

{P, PQ} = W3

‘p’ is a member of the set P, called subworld 1. ‘p’ and ‘q’ are members of the set {PQ} called subworld 2. These two sets, P and PQ, are subsets of the larger set W called World 3.

(4) a. p $\in$ {p}

q $\in$ {p, q} ’

{{p}. {p. q}} $\in$ W3

1.4 Argument

It is possible now to consider ‘q’ an argument of ‘p’. It is an argument of ‘p’ because if ‘p’ doesn’t exist, then ’q’ cannot either in World 3. However, this appears to be incorrect. If P takes no argument, {p, {}}, it is not the same thing as P that takes an argument {p, {q}). The two Ps are distinct — one that takes an argument and one that does not.

Suppose instead that ‘q’ is possible as a stand-alone and ‘q’ is related to ‘p’:

(5) {{q}. {p, q}}.

There are two things (players, participants)— ‘q’ and ‘p’. ‘q’ is still ‘q’ whether it stands alone or whether it is related to {p}. This relation is an argument relation, or ‘q’ is an argument of ‘p’ or it is not. This is common in natural languages and universal semantics. We talk about ‘book’ as an object (thing) or we can talk about ‘book’ as related to the thing ‘read’:

(6) a. The book is on the table. (thing)

John read a book. (thing as argument of 'read')

Technically, *book* in (8a) is an argument of the preposition *on*. Note that one cannot say as a complete sentence:

(7) a. *On the table.

*The book is on.

Something must be on the table, and the book must be on something.

Perhaps a better example is the following:

(8) a. A book is something that one reads.

That is a book.

These two examples may be as close to a stand-alone example that we will find in natural languages. Noun phrase may occur in expressions such *Eek*! *A snake*! However, what it means is:.

(9) There is a snake!

The expletive *there* denotes existence in (11), and the argument of it is the noun phrase that one is asserting the existence of.

The bottom line is that if 'p' is not complete without 'q', then 'q' is an argument of 'p':

(10) Argument

X is an argument of Y iff the function of Y is incomplete without X.

In the symbolization used by Reichenbach (1947) and others, we could write (?) as (13) where the set P is a function and the set Q , an argument P, is enclosed in parentheses:

(11) P(Q),

In terms of set theory, there would be but one set such that ‘p’ is its only member; it is called a singleton in set theory:

(12) {p}.

The set P corresponds to ‘World I’: P = W1. The set PQ corresponds to W2: PQ = W2.

It is interesting to look at the truth table for (1):

p
T

There is one member ‘p.’ Even though truth is not a member of this world, it must be true from our perspective. Given a set {x} in W1, ‘x’ is always true. Another thing is required before it will be possible to distinguish between true and not true

1.5 Negation

Negation is a unary operator: it takes only one argument:

(13) a. not the right page

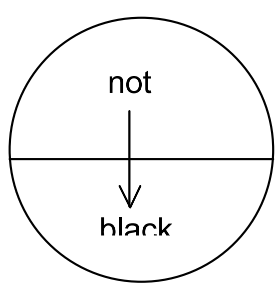
not in a life time

not black

not go

In an alternate version of World 3, there would be a thing, negation, and the relation between the thing and negation:

(14)



1.6 Polarization

In place of negation, I will introduce the operator Polarized (Pol). One of the most common operators is Pol. It is prevalent in natural languages and logic. Pol takes on argument. Pol is a binary feature set. Pol contains two features [±Neg]:

(15) Pol = {[+Neg, -Neg]}

The feature [-Neg] means 'it is the case'. In English [-Neg] is normally unmarked phonetically, but it may have a phonetic form [sow] as *so* in the following dialogue of two children quarrelling:[2]

(16) a. I did not.

You did so.

Did not.

Did so.

Did not.

Did so. (and so forth)

In some sense, [-Neg] is implicitly true in W1, since falseness or negation is a thing excluded in W1 if 'p' is a thing or member of the set W1. In English [+Neg] often takes the form of *not*:

(17) a. John did not go.

Mary could not **not** have seen that. [3]

The operator *not* in English means roughly "it is not the case that." However this interpretation still contains the word *not*. In (6b) the first *not* is modifying 'Mary could'. The second *not* is modifying 'have seen that':

(18) It is not the case that Mary could **not** have seen that.

[Neg] is a primitive since it cannot be defined in terms of the other predicate. Note that *not* does not modify nouns but clauses. Thus, it cannot be the next step after World 2 or World 3, since clauses are not a part of either world.

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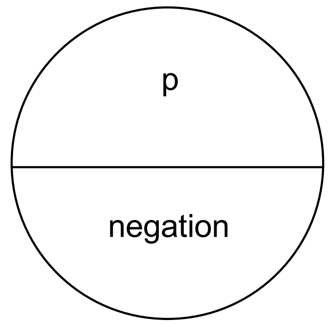
[2] This form of *so* should not be confused with *so* that functions as a pronominal in place of an omitted phrase.
[3] The second *not* is bolded to mark emphatic stress.

It is possible to consider this world, World 4, to consist of two things: ‘p’ and negation. Negation takes an argument:

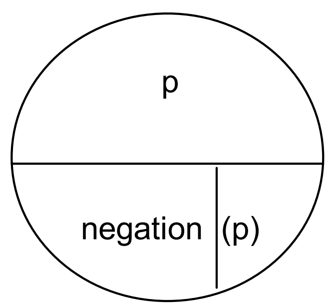
(19)



World 4 contains two things, but neither are dependant on the other as long as there are only two things.

It turns out that negation is an operator. From our perspective, negation cannot stand alone but ‘p’ can. In W5 negation cannot occur out ‘p’:

(20)

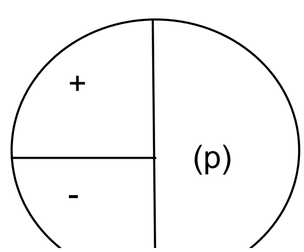


In (22) there are three things: ‘p’, ‘negation’, and ‘argument’.

Note if W5 is a complete set, then ‘negation’ as a stand alone similar to Figure (12) is not possible.

One advantage of a binary system is that it fits in well with other binary systems. In the case of simple negation, it requires one more thing than in the above system. Pol would be the new thing.

(21)



Polarization requires four things: 'p', Pol, plus and minus. However, this is not a bad thing. Once Pol and its two features are incorporated into this world, W6, they are available for all worlds using Pol. In Figure (23) the parentheses is one thing representing argument, and 'p' remains as the remaining thing.

The negative operator in logic is usually written as a tilde:

(22) $\sim 1 = \mathrm{Neg}(1) = \{\mathrm{Neg}(\{1\})$.

or with a macron:

(23) $\bar{1}$[4]

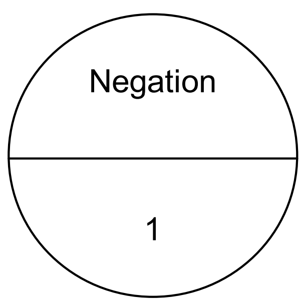
It may also be written as a negative functional, where 'f' is a negative function:

(24) f(1).

The first form in (24) is probably the most common notation.

The negative operator is a predicate with a special property denoting a relation. Suppose that there is in World 2 a predicate called a negation operator. It is possible to conceive of this world such that *one* and negation are unrelated:

(25)

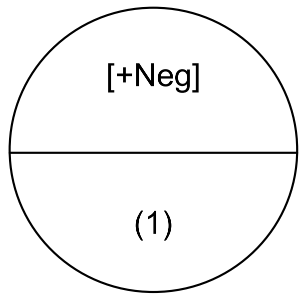


Given the predicates 'p' and 'q,' 'p' may be a notational variant of '1,' and 'q' a notation variant of negation. If there is no relation between negation and *one*, then negation is in some sense incomplete unless negation is referring the concept of negation. In figure (13), there is no relation between the negative predicate and the second predicate, the number one. If negation refers to function (as an operator), then such a world as (11) including all similar worlds is incomplete. Negation requires an argument, but this relation does not exist in World 2.

1.7 Operator (Function) and Argument

Now suppose that there is a relationship between negation and ‘1’: the negative value of the number one. *1*requires an argument, represented by parentheses:

(26)



[+Neg] implies the existence of [-Neg], and [-Neg] implies [+Neg]. For example, if it possible to have no rice, then it is possible to have rice. 1, [±Neg], and the argument constitute the world (or set) in (28). It might appear that (28) does not correctly represent the meaning of Figure (28). ‘1’ is always an argument of Pol (±Neg). It doesn’t have independent status. That is fine. Figure (28) simply means that 1 is always plus or minus. It does not occur independently of Neg.

It is interesting to point out that count nouns in English (but not Hungarian or Chinese, for example) seem always to occur as an argument of Pol, similar to ‘1’ in Figure (20). However, that is not entirely true. If a noun occurs in a compound noun, it usually is not modified for number:

(27) a. the book shelf

b. the lampshade

The shelf can be for one book or for more. We tend to think of a shade to be for one lamp, but this is not necessarily so. Quantity is simply not a factor here.

I should point out that I am referring to 1 here as a number, not as a quantifier. If it is a quantifier, then it is an operator that needs a nominal predicate or it will be incomplete:

(28) a. one book = one (book)

b. *one. book = {{one}, {book}}

In (29b) *one* and *book* occur in distinct subsets, whereas in (29a) *book* is a subset of the set *one book*. The sense of incompleteness is represented linearly with the asterisk. The asterisk may also denote a defective construction. The period is a notation to denote the lack of an argument for *one*. That is, *book* is not an argument of *one*. As a number, *one* is not an operator. Thus, the orthographic string *one* is ambiguous out of context. The period in (29b) eliminates the potential ambiguous reading of the normally written string 'one book'.

Of course, if 1 is a quantifier, (29a) can be an argument of [±Neg]:

(29) a. not one book

[+Neg] (1 (book))

(30) a. one book

[-Neg] (1 (book)).

There are two distinct forms spelled as *one*. I will temporarily identify the quantifier as 'one_q'. The following set of phrases based on (16) obtains:

(31) a. one.

one. book

*one book

*one_q

one_q book

*one_q. book

Example (17a) is not defective because the period means that no argument is given. The number *one* does not take an argument. The same holds for (17b). (17c) is defective because the lack of period here indicates that *book* is an argument of *one*, but the number cannot take an argument. (17d) is defective because the quantifier requires an argument. (17e) is not defective since *book* is the argument of *one*$_q$. (17f) is defective because the period blocks the interpretation of *book* as an argument

1.8 Unary Operators

Negation is a unary operator in that it takes only one argument. Unary operators are quite uncommon in logic. A few more can be found in mathematics: the square of a number, the square root of a number, but not addition and multiplication. In a mathematical statement, if square takes no argument, it is incomplete as is the square root:

(32) a. *square

square(2)

square(x)

*square root

square root(4)

square root(x)

The symbol 'x' in (17c) and (17f) is a variable. It stands for any number and it is an argument. Examples (17a) and (17d) are acceptable in metalinguistic contexts.

1.9 Binary Operators

Many operators are binary — they take two arguments. Examples in English include the conjunctions *and* and *or*:

(33) a. the book and the pen

b. the book or the pen

Both conjunctions cannot take only one argument:

(34) a. *John read the book and.

*John read and the book.

Formally, I will represent the argument structure as:

(35) and(x, y).

‘x’ and ‘y’ are the arguments of *and.* The same hold for *or*. The conjunction *or* has two meanings — the inclusive and the exclusive meaning of *or*. The exclusive meaning of *or* can usually be paraphrased with *ether … or*:

(36) a. John bought a Ford or a Toyota. (I can’t remember which)

b. John bought either a Ford or a Toyota. (not both of them)

The inclusive meaning means either one or both:

(37) a. John want a Ford or a Toyota

John wants a Ford, a Toyota, or both.

Sometimes the difference in meaning between the two conjunctions can only be determined contextually.

4 Lexemes

Natural languages distinguish between lexical items and grammatical items. Briefly, grammatical items are required by the grammar and they are not optional. In certain contexts, a particular grammatical item is not required. Such grammatical items I call operators here. Grammatical operators may be unary. In the following list, the operator is unmarked and its argument is enclosed square brackets:

(38) a. not [read a newspaper]

b. not [red]

c. not [in the pool]

Without argument here, the grammatical feature Tense is an operator whose argument is a verb. In terms of binary features Tense, contains [±Past]. The features make constitute the operator Tense. Tense takes one argument. If Tense takes one arguments, then its features do since that are part of Tense. :

(39) a. played

b. {[+Past]({play})}

Example (35gg) is replaced with (35a). This holds for all the verbal operators: voice, aspect, relevance, tense, subject, mood, polarity.

Verbs, nouns, adjectives, and P are lexemic. Verbs, adjectives, and P must take at least one argument. They may take up to four arguments, in the theory that I am advocating. In other theories, they may take more. It is not known if there is an upper limit.

5 Categories

The categorical or functional status of P (prepositions, postpositions) is controversial. Baker (2003) believes that they are functional, not lexical, items. His argument is based in part on the fact that they form a closed class. It is not necessarily the case that closed lists are functional words. It is a hypothesis that can be challenged. Ps assign theta roles and in this respect, they behave more like lexical items. I will put this question aside here.

6 Bibliography