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LAW AND CAUSALITY

A. UNIVERSALS OF LAW AND OF FACT (1928)

1 The difference according to Johnson¹ is that universals of law apply over a wider range than do universals of fact, i.e. over a wider range than everything, which is impossible (range of x in $(x). \phi x \supset \psi x$ is everything).

2 The difference according to many is that when all A are B is a universal of fact it is short for this A is B , that A is B , . . . ; this is not true; the universal may in the first instance be discovered in this sense, but as soon as it is told to someone else it ceases to have this sense, because the hearer does not know how many, or what, A s there are, but merely that all that there are are B . But this does not mean that to the hearer the universal is one of law.

3 The difference according to Braithwaite² is that universals of law are believed on grounds which are not demonstrative. This will not do because

- (a) some universals of law are not believed at all
- (b) some universals of fact are believed on non-demonstrative grounds
- (c) some universals of law are believed on grounds which in his sense are demonstrative.

Any one of these contentions is enough to upset his definition. Let us take them in turn.

4 (a) We many of us think many characteristics of

¹ W. E. Johnson, *Logic, Part III* (1924), Chapter 1.

² R. B. Braithwaite, "The Idea of Necessary Connexion", *Mind* 36 (1927), pp. 467-77; 37 (1928), pp. 62-72.

offspring are caused by (unknown) characteristics of the chromosomes of the uniting cells; but on Braithwaite's view to think this is to think that we know what characteristics of the chromosomes they are; it is not enough to say

$(\exists \phi): \phi(\text{chromosomes}) \supset_{\text{always}} \psi(\text{offspring});$

if we say the universal is causal we mean

$(\exists \phi): \phi(\text{chromosomes}) \supset_{\text{always}} \psi(\text{offspring})$ is believed,

which is patently false until ϕ has been discovered. It might be replied that what we want is not 'is believed' but either 'will be believed', 'would be believed', or 'could be believed'.

5 Of these amended versions 'will' clearly will not do; the causes of hereditary characteristics are not altered, if a new barbarian invasion checks the progress of science; 'would' would be circular as it means that certain circumstances would cause it to be believed; 'could' be believed would either mean this too or else something radically different to be considered later.

6 (b) What is clearly only a universal of fact, e.g. 'everyone there was asleep', may easily not be believed on demonstrative grounds: e.g. it may be believed on testimony; or because I said something which anyone who was awake would probably have answered.

7 (c) This point is not so clear as the others, owing to an ambiguity as to what 'universal of law' is supposed to include; if it means a universal whose subject term does not mention any particular portion of space-time, it would be better to make this clearly part of the definition. Otherwise take 'Whenever this balloon was filled with hydrogen and let go, it rose'; this, or something like it only more complicated, is surely a universal of law and yet might be believed as a result of observing all its instances.

8 In order to get nearer a correct solution let us classify universals a little more precisely; as we have the following

classes

- (1) the ultimate laws of nature
- (2) derivative laws of nature, i.e. general propositions deducible from the ultimate laws
- (3) what are called laws in a loose sense; i.e. general propositions deducible from the ultimate laws together with various facts of existence assumed to be known by everyone, e.g. bodies fall
- (4) universals of fact; but these cannot be sharply distinguished from (3); on a determinist view all of them could be deducible from the ultimate laws together with enough facts of existence.

9 This table of classes might perhaps suggest the following solution; the fundamental distinction is between (1) and (2) on the one hand, and (3) and (4) on the other, and it is that universals in classes (1) and (2) mention no particular portion of space-time whereas those in (3) and (4) do (hence the need for facts of existence to deduce them). Between (1) and (2), and between (3) and (4), the distinction is a vague one, in the first case of artificial arrangement, in the second of amount of fact required for their deduction.

10 This solution would not, however, do; because there are universals belonging to (3) and (4), which mention no particular portion of space-time but still do not follow from the ultimate laws; thus all Conservative prime ministers of England between 1903 and 1928 have names beginning with B; and so probably all Conservative prime ministers of a country with 40,000,000-50,000,000 inhabitants, whose capital is called 'London' and has 7,000,000 inhabitants . . . at a time when that country has between 2-27 years previously lost a queen who has ruled for 64 years . . . have their names beginning with B. If we put in enough detail we shall (unless the world repeats itself endlessly with just a few details

different each time) get a true generalization which mentions no particular portion of space-time but this would not be a law of nature.

11 What is it then that is true of universals of classes (1) and (2) and not of those classes (3) and (4)? We have seen that it is not their spatio-temporal indifference, not that they are believed; nor we may remark is it any combination of these characters, for the fact that they are believed or might be believed is quite irrelevant; anything whatever can be believed on authority or testimony. Also the difference would still persist if we knew everything.

12 This last point gives us the clue; even if we knew everything, we should still want to systematize our knowledge as a deductive system, and the general axioms in that system would be the fundamental laws of nature. The choice of axioms is bound to some extent to be arbitrary, but what is less likely to be arbitrary if any simplicity is to be preserved is a body of fundamental generalizations, some to be taken as axioms and others deduced. Some other true generalizations will then only be able to be deduced from these by the help of particular facts of existence. These fundamental generalizations will then be our universals of classes (1) and (2), the axioms forming class (1).

13 As it is, we do not know everything; but what we do know we tend to organize as a deductive system and call its axioms laws, and we consider how that system would go if we knew a little more and call the further axioms or deductions there would then be, laws (we think there would be ones of a certain kind but don't know exactly what). We also think how all truth could be organized as a deductive system and call its axioms ultimate laws.

14 The property of a universal that it *would* be an axiom in a deductive system covering everything is not really hypothetical; the concealed if is only a *spurious* one; what

is asserted is simply something about the whole world, namely that the true general propositions are of such forms that they form a system of the required sort with the given proposition in the required place; it is the facts that form the system in virtue of internal relations, not people's beliefs in them in virtue of spatio-temporal ones. Of course the system is required to be as simple as possible; but this is another vague formal property, not a causal one, or if causal shorn of its causality; see § 16.

15 It will be objected that when we use the notion of a law as in a statement of causal implication, we do not say anything about a grand deductive system. The answer is that we do do this so soon as we pass beyond the mere material or formal implication. But that the important part of statements of causal implication is always just the material or formal implication which has no reference to system. It is only the philosopher or systematizer or emotionalist who is interested in the rest. All the practical man wants to know is that all people who take arsenic die, not that this is a causal implication, for a universal of fact is *within its scope* just as good a guide to conduct as a universal of law.

16 A danger always to be thought of, is that belief being a causal fact we must not involve it in the analysis of cause. The above theory avoids that; see § 14. An alternative way of avoiding it is to say that the belief, if any, that occurs in the analysis of cause is belief shorn of its causality, i.e. with the causal implications reduced to material ones.

17 The laws involved in *causal implications* are classes (1), (2) above. Not class (3); in the cases in which we should naturally appeal to a universal of class (3), we can by extending our r make $pr \supset q$ an instance of one of class (2) instead; and it is the possibility of doing this within the implied limitations on r which in effect distinguishes class (3) from class (4).

B. GENERAL PROPOSITIONS AND CAUSALITY (1929)

Let us consider the meaning of general propositions in a clearly defined given world. (In particular in the common sense material world.) This includes the ordinary problem of causality.

As everyone except us¹ has always said these propositions are of two kinds. First *conjunctions*: e.g. 'Everyone in Cambridge voted'; the variable here is, of course, not people in Cambridge, but a limited region of space varying according to the definiteness of the speaker's idea of 'Cambridge', which is 'this town' or 'the town in England called Cambridge' or whatever it may be.

Old-fashioned logicians were right in saying that these are conjunctions, wrong in their analysis of what conjunctions they are. But right again in radically distinguishing them from the other kind which we may call *variable hypotheticals*: e.g. Arsenic is poisonous: All men are mortal.

Why are these not conjunctions?

Let us put it this way first: What have they in common with conjunctions, and in what do they differ from them? Roughly we can say that when we look at them subjectively they differ altogether, but when we look at them objectively, i.e. at the conditions of their truth and falsity, they appear to be the same.

(x) . ϕx differs from a conjunction because

(a) It cannot be written out as one.

(b) Its constitution as a conjunction is never used; we never use it in class-thinking except in its application to a finite class, i.e. we use only the applicative rule.

¹ [I.e. Ramsey and Braithwaite; see A above.]

(c) [This is the same as (b) in another way.] It always goes beyond what we know or want; cf. Mill on 'All men are mortal' and 'The Duke of Wellington is mortal'. It expresses an inference we are at any time prepared to make, not a belief of the primary sort.

A belief of the primary sort is a map of neighbouring space by which we steer. It remains such a map however much we complicate it or fill in details. But if we professedly extend it to infinity, it is no longer a map; we cannot take it in or steer by it. Our journey is over before we need its remoter parts.

(d) The relevant degree of certainty is the certainty of the particular case, or of a finite set of particular cases; not of an infinite number which we never use, and of which we couldn't be certain at all.

$(x) \cdot \phi x$ resembles a conjunction

(a) In that it contains all lesser, i.e. here all finite, conjunctions, and appears as a sort of infinite product.

(b) When we ask what would make it true, we inevitably answer that it is true if and only if every x has ϕ ; i.e. when we regard it as a proposition capable of the two cases truth and falsity, we are forced to make it a conjunction, and to have a theory of conjunctions which we cannot express for lack of symbolic power.

[But what we can't say we can't say, and we can't whistle it either.]

If then it is not a conjunction, it is not a proposition at all; and then the question arises in what way can it be right or wrong.

Now in the case of a proposition right and wrong, i.e. true or false, occur doubly. They occur to the man who makes the proposition whenever he makes a truth-function of it, i.e. argues disjunctively about the cases of its truth and falsity.

Now this we never do with these variable hypotheticals except in mathematics in which it is now recognized as fallacious. We may seem to do so whenever we discuss the different theories obtainable by combining different proposed laws of nature. But here, if P is such a law, we do not consider the alternatives P , i.e. $(x) \cdot \phi x$, and \bar{P} , i.e. $(x) \cdot \overline{\phi x}$, but we consider either having P or not having P (where not having it as a law in no way implies the law's falsity, i.e. $(\exists x) \cdot \overline{\phi x}$) or else having $P = (x) \cdot \partial \phi x$ or having $Q = (x) \cdot \overline{\phi x}$.

The other way in which right and wrong occur in connection with propositions is to an onlooker who says that the man's belief in the proposition is right or wrong. This, of course, turns simply on what the onlooker thinks himself and results from identity or difference between his view and what he takes to be that of the man he is criticizing. If A thinks p and thinks also that B thinks p , he says B thinks truly; if he thinks p and thinks that B thinks \bar{p} , he says B thinks falsely. But criticism may not always be of this simple type; it is also possible when B thinks p , and A thinks neither p nor \bar{p} , but regards the question as unsettled. He may deem B a fool for thinking p , without himself thinking \bar{p} . This happens almost always with hypotheticals. If B says 'If I eat this mince pie I shall have a stomach-ache', and A says 'No, you won't', he is not really contradicting B 's proposition—at least if this is taken as a material implication. Nor is he contradicting a supposed assertion of B 's that the evidence proves that so-and-so. B may make no such assertion, in fact cannot always reasonably even if he is in the right. For he may be in the right without having *proof* on his side.

In fact agreement and disagreement is possible in regard to any aspect of a man's view and need not take the simple form of ' p ', ' \bar{p} '.

Many sentences express cognitive attitudes without being propositions; and the difference between saying yes or no

to them is not the difference between saying yes or no to a proposition. This is even true of the ordinary hypothetical [as can be seen from the above example, it asserts something for the case when its protasis is true: we apply the Law of Excluded Middle not to the whole thing but to the consequence only]; and much more of the variable hypothetical.

In order therefore to understand the variable hypothetical and its rightness or wrongness we must consider the different possible attitudes to it; if we know what these are and involve we can proceed easily to explain the meaning of saying that such an attitude is right or wrong, for this is simply having such an attitude oneself and thinking that one's neighbour has the same or a different one.

What then are the possible attitudes to the question—Are all men mortal?

- (1) To believe it with more or less conviction.
- (2) Not to have considered it.
- (3) Not to believe it because it is unproven.
- (4) *Not* to believe it because convinced that a certain type of man, who *might* exist, would be immortal.
- (5) To disbelieve it as convinced that a particular man is immortal.

We have to analyse these attitudes; obviously in the first instance the analysis must be in terms of beliefs in singular propositions, and such an analysis will suffice for our present purpose.

To believe that all men are mortal—what is it? Partly to say so, partly to believe in regard to any x that turns up that if he is a man he is mortal. The general belief consists in

- (a) A general enunciation,
- (b) A habit of singular belief.

These are, of course, connected, the habit resulting from

the enunciation according to a psychological law which makes the meaning of 'all'.

We thus explain

- (1) In terms of the notion of a '*habit*';
- (2) Offers no problem;
- (3) May seem to give a problem if we ask—What is it that the thinker considers? But there is really no problem: it is not considering whether a thing is so or not, nor again considering whether or not to do something, but a kind of intermediary. The idea of the general statement rises, the evidence is considered and it falls again.

In (4) and (5) it falls more decisively for the reasons given: namely, in (4) we have another general statement which combined with the proposed one would give a conclusion we are disinclined for (itself a third general statement, namely 'All men are not of that type'); and in (5) we have a singular statement flatly contradicting the one proposed.

Variable hypotheticals or causal laws form the system with which the speaker meets the future; they are not, therefore, subjective in the sense that if you and I enunciate different ones we are each saying something about ourselves which pass by one another like 'I went to Grantchester', 'I didn't.' For if we meet the future with different systems we disagree even if the actual future agrees with both so long as it *might* (logically) agree with one but not with the other, i.e. so long as we don't believe the same things. (Cf. If A is certain, B doubtful, they can still dispute.)

Variable hypotheticals are not judgments but rules for judging 'If I meet a ϕ , I shall regard it as a ψ '. This cannot be *negated* but it can be *disagreed* with by one who does not adopt it.

These attitudes seem therefore to involve no puzzling idea except that of habit; clearly any proposition about a habit

is general, and hence the criticism of a man's general judgments is itself a general judgment. But since all belief involves habit, so does the criticism of any judgment whatever, and I do not see anything objectionable in this. There is a feeling of circularity about it, but I think it is illusory. Anyway we shall recur to it below.

This account of causal laws has a certain resemblance to Braithwaite's, and we must compare them closely to see whether it escapes the objections to which his is liable. He said that a universal of law was one believed on grounds not demonstrative, and I said¹ that that would not do for three separate reasons:—

(a) Some universals of law are not believed at all, e.g. unknown causal laws.

(b) Some universals of fact are believed on grounds not demonstrative.

(c) Some (derivative and localized) universals of law are believed on demonstrative grounds.

I, therefore, put up a different theory by which causal laws were consequences of those propositions which we should take as axioms if we knew everything and organized it as simply as possible in a deductive system.

What is said above means, of course, a complete rejection of this view (for it is impossible to know everything and organize it in a deductive system) and a return to something nearer Braithwaite's. A causal generalization is not, as I then thought, one which is simple, but one we trust (cf. the ages at death of poets' cooks). We may trust it because it is simple, but that is another matter. When I say this I must not be misunderstood; variable hypotheticals are not distinguished from conjunctions by the fact that we believe

¹ [In A above.]

them, they are much more radically different. But the evidence of a variable hypothetical being (often at least) a conjunction, such a conjunction is distinguished from others in that we trust it to guide us in a new instance, i.e. derive from it a variable hypothetical.

This explains how Braithwaite came to say that laws were those which were believed; but, put as he puts it, it is of course wrong, being open to the objections made above. Braithwaite's problem was to explain the meaning of 'P is a law of nature'. Our solution is that to say this, is to assert P after the manner of a variable hypothetical. [Or of course we may extend law of nature to any conjunction following from one in the above sense.] But this solution is incomplete because it does not at all explain what we mean when we speak of an unknown law of nature, or one described but not stated, e.g. the law that characteristics of people depend in some way on chromosomes (but no one knows how), or, he has discovered a law governing the extension of springs (but I don't know what law), where in the second instance I say he believes a variable hypothetical, and further imply that it is true, but since I do not know what it is I cannot myself adopt his attitude towards it.

Thus in each of these cases we seem to be treating the unknown law as a true proposition in the way our theory says is impossible.

The same difficulty also occurs in the finitist theory of mathematics, when we speak of an unknown true mathematical proposition. In this clearer field the solution should be easier and then extensible to the other.

An unknown truth in the theory of numbers cannot be interpreted as an (unknown) proposition true of all numbers, but as one proved or provable. Provable in turn means provable in any number of steps, and on finitist principles the number must in some way be limited, e.g. to the humanly

possible. 'So-and-so has discovered a new theorem' means therefore that he has constructed a proof of a certain limited size.

When we turn to an unknown causal law, what is there to correspond to the process of proof on which the above solution turns? Clearly only the process of collecting evidence for the causal law, and to say that there is such a law, though we don't know it, must mean that there are such singular facts in some limited sphere (a disjunction) as would lead us, did we know them, to assert a variable hypothetical. But this is not enough, for there must not merely be facts leading to the generalization, but this when made must not mislead us. (Or we could not call it a true causal law.) It must therefore be also asserted to hold within a certain limited region taken to be the scope of our possible experience.

There was nothing corresponding to this in the mathematical case, for a mathematical generalization must if proved hold in any particular case, but an empirical generalization cannot be proved; and for there to be evidence leading to it and for it to hold in other cases also are separate facts.

To this account there are two possible objections on the score of circularity. We are trying to explain the meaning of asserting the existence of an unknown causal law, and our explanation may be said to be in terms of the assertion of such laws, and that in two different ways. We say it means that there are facts which *would lead* us to assert a *variable hypothetical*; and here it may be urged that this means that they would lead us in virtue of one possibly unknown causal law to form a habit which would be constituted by another unknown causal law.

To this we answer, first, that the causal law in virtue of which the facts would lead us to the generalization must not be any unknown law, e.g. one by which knowledge of the facts would first drive us mad and so to the mad generalization, but the

known laws expressing our methods of inductive reasoning; and, secondly, that the unknown variable hypothetical must here be taken to mean an unknown statement (whose syntax will of course be known but not its terms or their meanings), which would, of course, lead to a habit in virtue of a known psychological law.

What we have said is, I think, a sufficient outline of the answers to the relevant problems of analysis, but it is apt to leave us muddled and unsatisfied as to what seems the main question—a question not of psychological analysis but of metaphysics which is 'Is causation a reality or a fiction; and, if a fiction, is it useful or misleading, arbitrary or indispensable?'

We can begin by asking whether these variable hypotheticals play an essential part in our thought; we might, for instance, think that they could simply be eliminated and replaced by the primary propositions which serve as evidence for them. This is, I think, the view of Mill, who argued that instead of saying 'All men die, therefore the Duke of Wellington will', we could say 'Such-and-such men have died,¹ therefore the Duke will'. This view can be supported by observing that the ultimate purpose of thought is to guide our action, and that on any occasion our action depends only on beliefs or degrees of belief in singular propositions. And since it would be possible to organize our singular beliefs without using variable intermediaries, we are tempted to conclude that they are purely superfluous.

But this would, I think, be wrong; apart from their value in simplifying our thought, they form an essential part of our mind. That we think explicitly in general terms is at the root

¹ We may be inclined to say that the evidence is not merely that *A, B, C* have died, but that *A, B, C* have died, and none so far as we know not died; i.e. 'all we know about have died'. But the extra is not part of the evidence, but a description of it, saying 'and this is all the evidence'.

of all praise and blame and much discussion. We cannot blame a man except by considering what would have happened if he had acted otherwise, and this kind of unfulfilled conditional cannot be interpreted as a material implication, but depends essentially on variable hypotheticals. Let us consider this more closely.

When we deliberate about a possible action, we ask ourselves what will happen if we do this or that. If we give a definite answer of the form 'If I do p , q will result,' this can properly be regarded as a material implication or disjunction 'Either not- p or q .' But it differs, of course, from any ordinary disjunction in that one of its members is not something of which we are trying to discover the truth, but something within our power to make true or false.¹ If we go on to 'And if q , then r ', we get more material implications of a more ordinary kind.

Besides definite answers 'If p , q will result', we often get ones 'If p , q might result' or ' q would probably result'. Here the degree of probability is clearly not a degree of belief in 'Not- p or q ', but a degree of belief in q given p , which it is evidently possible to have without a definite degree of belief in p , p not being an intellectual problem. And our conduct is largely determined by these degrees of hypothetical belief.

Now suppose a man is in such a situation. For instance, suppose that he has a cake and decides not to eat it, because he thinks it will upset him, and suppose that we consider his conduct and decide that he is mistaken. Now the belief on which the man acts is that if he eats the cake he will be ill, taken according to our above account as a material implication. We cannot contradict this proposition either before or after the event, for it is true provided the man doesn't eat the

¹ It is possible to take one's future voluntary action as an intellectual problem: 'Shall I be able to keep it up?' But only by dissociating one's future self.

cake, and before the event we have no reason to think he will eat it, and after the event we know he hasn't. Since he thinks nothing false, why do we dispute with him or condemn him?

Before the event we do differ from him in a quite clear way: it is not that he believes p , we \bar{p} ; but he has a different degree of belief in q given p from ours; and we can obviously try to convert him to our view.¹ But after the event we both know that he did not eat the cake and that he was not ill; the difference between us is that he thinks that if he had eaten it he would have been ill, whereas we think he would not. But this is *prima facie* not a difference of degrees of belief in any proposition, for we both agree as to all the facts.

The meaning of these assertions about unfulfilled conditions, and the fact that whether the conditions are fulfilled or not makes no difference to the difference between us, the common basis, as we may say, of the dispute lies in the fact that we think in general terms. We each of us have variable hypotheticals (or, in the case of uncertainty, chances) which we apply to any such problem; and the difference between us is a difference in regard to these. We have degrees of expectation, vague or clear, as to the outcome of any state of affairs whenever or wherever it may occur. Where there is apt to be ambiguity is in the definition of the state of affairs; for instance, in considering what would have happened if a man had acted differently, we are apt to introduce any fact we know, whether he did or could know it, e.g. the actual position of all the cards at bridge as opposed to their probabilities of position from his point of view. But what is clear is that

¹ If two people are arguing 'If p will q ?' and are both in doubt as to p , they are adding p hypothetically to their stock of knowledge and arguing on that basis about q ; so that in a sense 'If p , q ' and 'If p , \bar{q} ' are contradictories. We can say they are fixing their degrees of belief in q given p . If p turns out false, these degrees of belief are rendered *void*. If either party believes \bar{p} for certain, the question ceases to mean anything to him except as a question about what follows from certain laws or hypotheses.

our expectations are general ; when the sort is clearly defined we expect with the same probabilities in any case of the sort. If not, and we expected differently in every real case, expectation in an imaginary case could have no meaning.

All this applies, of course, equally well to the consequences of any hypothetical event and not only to human actions. I have chosen to expound it with reference to the latter because I think they are of quite peculiar importance in explaining the special position possessed by causal laws, which are an important but not the only type of variable hypotheticals. In order to deal with this question let us begin with hypotheticals in general.

'If p , then q ' can in no sense be true unless the material implication $p \supset q$ is true ; but it generally means that $p \supset q$ is not only true but deducible or discoverable in some particular way not explicitly stated.¹ This is always evident when 'If p then q ' or 'Because p , q ' (*because* is merely a variant on *if*, when p is known to be true) is thought worth stating even when it is already known either that p is false or that q is true. In general we can say with Mill that 'If p then q ' means that q is inferrible from p , that is, of course, from p together with certain facts and laws not stated but in some way indicated by the context. This means that $p \supset q$ follows from these facts and laws, which if true is in no way a hypothetical fact ; so that, in spite of the sound of *inferrible*, Mill's

¹ 'If p , then q ' may also mean $pr \supset q$ where r is not a fact or law, or not only composed of facts or laws, but also composed of propositions in a secondary system. E.g., from a solipsistic standpoint, 'If I open my eyes I shall see red.' The hypotheticals in Mill's theory of the external world are of this nature, and cannot be used to define the external world. All that could be used are *laws* from which, combined with my past experience, it might follow that if I open my eyes I shall see red. But this could not cover *conjectures* as to the external world, unless we think sufficient knowledge of law would enable us to make all such conjectures certain. I do conjecture something ; this can only be hypothetical if the hypothesis can refer to a secondary system.

Mill's view must be replaced by saying that the external world is a secondary system, and that any proposition about it commits one to no more judgments than a denial of all courses of experience inconsistent with it.

explanation is not circular as Bradley thought. Of course that $p \supset q$ follows from the facts is not a proposition of logic but a description of the facts : 'They are such as to involve $p \supset q$.' Corresponding to the kind of laws or facts intended we get various subtle syntactical variations. For instance, 'If he was there, he must have voted for it (for it was passed unanimously), but if he had been there, he would have voted against it (such being his nature).' [In this, law = variable hypothetical.]

One class of cases is particularly important, namely those in which, as we say, our 'if' gives us not only a *ratio cognoscendi* but also a *ratio essendi*. In this case which is e.g. the normal one when we say 'If p had happened, q would have happened', $p \supset q$ must follow from a hypothetical $(x) . \phi x \supset \psi x$ and facts $r, pr \supset q$ being an instance of $\phi x \supset \psi x$ and q describing events not earlier than any of those described in pr . A variable hypothetical of this sort we call a *causal law*.

We have now to explain the peculiar importance and objectivity ascribed to causal laws ; how, for instance, the deduction of effect from cause is conceived as so radically different from that of cause from effect. (No one would say that the cause existed because of the effect.) It is, it seems, a fundamental fact that the future is due to the present, or, more mildly, is affected by the present, but the past is not. What does this mean ? It is not clear and, if we try to make it clear, it turns into nonsense or a definition : 'We speak of *ratio essendi* when the protasis is earlier than the apodasis Df.' We feel that this is wrong ; we think there is some difference between before and after at which we are getting ; but what can it be ? There are differences between the laws deriving effect from cause and those deriving cause from effect ; but can they really be what we mean ? No ; for they are found *a posteriori*, but what we mean is *a priori*. [The Second Law of Thermodynamics is *a posteriori* ; what is peculiar is that it

seems to result merely from absence of law (i.e. chance), but there might be a law of shuffling.]

What then do we believe about the future that we do not believe about the past; the past, we think, is settled; if this means more than that it is past, it might mean that it is settled *for us*, that nothing now could change our opinion of it, that any present event is irrelevant to the probability for us of any past event. But that is plainly untrue. What is true is this, that any possible present volition of ours is (for us) irrelevant to any past event. To another (or to ourselves in the future) it can serve as a sign of the past, but to us now what we do affects only the probability of the future.

This seems to me the root of the matter; that I cannot affect the past, is a way of saying something quite clearly true about my degrees of belief. Again from the situation when we are deliberating seems to me to arise the general difference of cause and effect. We are then engaged not on disinterested knowledge or classification (to which this difference is utterly foreign), but on tracing the different consequences of our possible actions, which we naturally do in sequence forward in time, proceeding from cause to effect not from effect to cause. We can produce A or A' which produces B or B' which etc. . . .; the probabilities of A, B are mutually dependent, but *we* come to A first from our present volition.

Other people we say can affect only the future and not the past for two reasons; first, by analogy with ourselves we know they can affect the future and not the past from their own point of view; and secondly, if we subsume their action under the general category of cause and effect, it can only be a cause of what is later than it. This means ultimately that by affecting it we can only affect indirectly (in our calculation) events later than it. In a sense my present action is an ultimate and the only ultimate contingency.

[Of course it is our own past we know we cannot affect; our own future we know we can. The branching out of influence with at most the velocity of light is known by experience.]

It is clear that the notion and use of causal laws presupposes no 'law of causation' to the effect that every event has a cause. We have some variable hypotheticals of the form 'If ϕx , then ψx ' with ψ later than ϕ , called causal laws: others of the form 'If ϕx , then probability α for ψx '; this is called a chance. We suppose chance to be ultimate if we see no hope of replacing it by law if we knew enough facts. There is no reason to suppose it is not ultimate. A law is a chance unity; of course, as is shown in my essay on chance, the chances do not give actual degrees of belief but a simpler system to which the actual ones approximate. So too we do not believe the laws for certain.

On the view that we have been explaining, causal necessity is not a fact; when we assert a causal law we are asserting not a fact, not an infinite conjunction, nor a connection of universals, but a variable hypothetical which is not strictly a proposition at all, but a formula from which we derive propositions.

The most obvious criticism of this view is that it is circular, for it seeks to explain away causality by means of a notion, namely that of a variable hypothetical, which itself involves causality. For the existence of a variable hypothetical depends on our using it as such, i.e. according to a causal law of our own nature proceeding from it to particular beliefs. We must try to make the answer to this criticism really clear, for it is certainly unsound.

One minor point may be made first: variable hypotheticals involve causality no more and no less than ordinary beliefs; for it belongs to the essence of any belief that we deduce from it, and act on it in a certain way, and this notion involves

causality just as much as does the variable hypothetical. The causal law connected with the latter is more complicated, but not essentially different. E.g. there is no hierarchy of types of causal laws, but merely growing homogeneous complication like $(x) \dots, (x)(y) \dots, (x)(y)(z) \dots$

But now for the main point. The world, or rather that part of it with which we are acquainted, exhibits as we must all agree a good deal of regularity of succession. I contend that over and above that it exhibits no feature called causal necessity, but that we make sentences called causal laws from which (i.e. having made which) we proceed to actions and propositions connected with them in a certain way, and say that a fact asserted in a proposition which is an instance of causal law is a case of causal necessity. This is a regular feature of our conduct, a part of the general regularity of things; as always there is nothing in this beyond the regularity to be called causality, but we can again make a variable hypothetical about this conduct of ours and speak of it as an instance of causality.

But may there not be something which might be called real connections of universals? I cannot deny it, for I can understand nothing by such a phrase; what we call causal laws I find to be nothing of the sort.

So too there may be an infinite totality, but what seem to be propositions about it are again variable hypotheticals and 'infinite collection' is really nonsense.

Variable hypotheticals have formal analogies to other propositions which make us take them sometimes as facts about universals, sometimes as infinite conjunctions. The analogies are misleading, difficult though they are to escape, and emotionally satisfactory as they prove to different types of mind. Both these forms of 'realism' must be rejected by the realistic spirit.

The sort of thing that makes one want to take a realistic

view of causality is this. Suppose the human race for no reason always supposed strawberries would give them stomach-ache and so never ate them; then all their beliefs, strictly so-called, e.g. that if I eat strawberries I shall have a pain, would be true; but would there not really be something wrong? Is it not a fact that if they had eaten them they wouldn't have had a pain?

No, it is not a fact; it is a consequence of my rule. What is a fact is that I have eaten them and not had a pain. If we regarded the unfulfilled conditional as a fact we should have to suppose that any such statement as 'If he had shuffled the cards, he would have dealt himself the ace' has a clear sense true or false, which is absurd. We only regard it as sense if it, or its contradictory, can be deduced from our system. Otherwise we say 'You can't say what would have happened', which sounds like a confession of ignorance, and is so indeed, because it means we can't foretell what *will* happen in a similar case, but not because 'what would have happened' is a reality of which we are ignorant.

But their system, you say, fitted all the facts known to them; if two systems both fit the facts, is not the choice capricious? We do, however, believe that the system is uniquely determined and that long enough investigation will lead us all to it. This is Peirce's notion of truth as what everyone will believe in the end; it does not apply to the truthful statement of matters of fact, but to the 'true scientific system'.

What was wrong with our friends the strawberry abstainers was that they did not experiment. Why should one experiment? To increase the weight of one's probabilities: if q is relevant to p , it is good to find out q before acting in a way involving p . But if q is known it is not worth while; they knew, so they thought, what the issue of the experiment would be and so naturally couldn't bother to do it.

The difficulty comes fundamentally from taking every sentence to be a proposition ; when it is seen by considering the position of coincidences that chances are not propositions then it should be clear that laws are not either, quite apart from other reasons.

NOTES

(1) All theories, chances and laws are constructed with a view to supplementation by discovery of further facts ; these facts are always taken as known for certain. What is to be done when we are not certain of them is left quite vague, just as is the allowance to be made for uncertainty about the theory itself.

(2) Chance and law are used in the same way in a theoretical system as in a primary system ; cause, too, if the theoretical system is temporal. Of course the theoretical system is like a variable hypothetical in being there just to be deduced from ; and a law in the theoretical system is at two removes of deduction.

(3) If the consequences of a law or theory are not clear, i.e. if there is no test whether something can or cannot be deduced from it, then it must be taken *formally* ; it is a habit not of believing ψ whenever we see ϕ , but of believing the meaning of any symbol deduced from these marks.

(4) Something should be said of the relation of this theory to Hume's. Hume said, as we do, that there was nothing but regularity, but he seemed to contradict himself by speaking of determination in the mind and a feeling of determination as giving the idea of necessity. We are accused of the same circularity unjustly : he got into a mess by taking an 'idea' of necessity and looking for an 'impression'. It is not clear to me that there is such an idea and such an impression, but there may be. When we are necessitated as a result of experience to think in a particular way, we probably do have a different feeling from when we freshly make up our mind. But we must not say we feel ourselves being necessitated, for in the mind there is only regularity : the necessity is as always a figure of speech. I think

he understood this very well, and gave his readers credit for more intelligence than they display in their literal interpretations.

(5) As opposed to a purely *descriptive* theory of science, mine may be called a *forecasting* theory. To regard a law as a summary of certain facts seems to me inadequate ; it is also an attitude of expectation for the future. The difference is clearest in regard to chances ; the facts summarized do not preclude an equal chance for a coincidence which would be summarized by and, indeed, lead to a quite different theory.