Chapter 6
Circular Motion, Orbits and Gravity

Topics:

- The kinematics of uniform circular motion
- The dynamics of uniform circular motion
- Circular orbits of satellites
- Newton’s law of gravity

Sample question:
The motorcyclist in the “Globe of Death” rides in a vertical loop upside down over the top of a spherical cage. There is a minimum speed at which he can ride this loop. How slow can he go?
Reading Quiz

1. For uniform circular motion, the acceleration
   A. is parallel to the velocity.
   B. is directed toward the center of the circle.
   C. is larger for a larger orbit at the same speed.
   D. is always due to gravity.
   E. is always negative.

Answer: B
2. When a car turns a corner on a level road, which force provides the necessary centripetal acceleration?
   A. Friction
   B. Tension
   C. Normal force
   D. Air resistance
   E. Gravity

Answer: A
Reading Quiz

• Newton’s law of gravity describes the gravitational force between
  A. the earth and the moon.
  B. a person and the earth.
  C. the earth and the sun.
  D. the sun and the planets.
  E. all of the above.

Answer: E
Looking Back: What You Already Know

From this class:

- We studied the kinematics of uniform circular motion in Chapter 3. We will review this and extend our study to include the dynamics of circular motion.
- We will make extensive use of Newton’s laws and related problem-solving techniques from Chapters 4 and 5.
- We will further develop the concept of apparent weight.

From previous classes:

- The force of gravity between all objects.
- Some ideas about the orbits of planets and satellites.
Checking Understanding

When a ball on the end of a string is swung in a vertical circle:

We know that the ball is accelerating because

A. the speed is changing.
B. the direction is changing.
C. the speed and the direction are changing.

Answer: B
Checking Understanding

When a ball on the end of a string is swung in a vertical circle:
What is the direction of the acceleration of the ball?

A. Tangent to the circle, in the direction of the ball’s motion
B. Toward the center of the circle

Answer: B
Uniform Circular Motion

\[ a = \frac{v^2}{r} = \omega^2 r \]

The instantaneous velocity \( \vec{v} \) is parallel to the circle at all points.

The instantaneous acceleration \( \vec{a} \) is directed toward the center of the circle at all points.
Examples

The disk in a hard drive in a desktop computer rotates at 7200 rpm. The disk has a diameter of 5.1 in (13 cm.) What is the angular speed of the disk?

The hard drive disk in the previous example rotates at 7200 rpm. The disk has a diameter of 5.1 in (13 cm.) What is the speed of a point 6.0 cm from the center axle? What is the acceleration of this point on the disk?
For the ball on the end of a string moving in a vertical circle:
What force is producing the centripetal acceleration of the ball?

A. gravity  
B. air resistance  
C. normal force  
D. tension in the string

Answer: D
Circular Motion Dynamics

For the ball on the end of a string moving in a vertical circle:

What is the direction of the net force on the ball?

A. tangent to the circle
B. toward the center of the circle
C. there is no net force

Answer: B
Circular Motion Dynamics

When the ball reaches the break in the circle, which path will it follow?

A.  
B.  
C.  
D.  

Answer: C
Forces in Circular Motion

\[ v = \omega r \]

\[ A = \frac{v^2}{r} = \omega^2 r \]

\[ \vec{F}_{\text{net}} = ma = \left\{ \frac{mv^2}{r}, \text{toward center of circle} \right\} \]

You tried to keep moving straight ahead.

The door provides the center-directed force that makes you move in a circle.

Center of curve
**PROBLEM-SOLVING STRATEGY 6.1**  

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**Circular dynamics problems**

**PREPARE** Begin your visual overview with a pictorial representation in which you sketch the motion, define symbols, and identify what the problem is trying to find. It is best to draw the free-body diagram with the circle viewed edge-on, the $x$-axis pointing toward the center of the circle, and the $y$-axis perpendicular to the plane of the circle.

**SOLVE** Newton’s second law for uniform circular motion, $\vec{F}_{\text{net}} = (mv^2/r$, toward center of circle), is a vector equation. Some forces act in the plane of the circle, some act perpendicular to the circle, and some may have components in both directions. In the coordinate system described above, Newton’s second law is

$$\sum F_x = \frac{mv^2}{r} \quad \text{and} \quad \sum F_y = 0$$
That is, the net force toward the center of the circle is $mv^2/r$, while the net force perpendicular to the circle is zero. The components of the forces are found directly from the free-body diagram. Depending on the problem, either

- Use the net force to determine the speed $v$, then use circular kinematics to find frequencies or angular velocities.
- Use circular kinematics to determine the speed $v$, then solve for unknown forces.

**ASSESS** Make sure your net force points toward the center of the circle. Check that your result has the correct units, is reasonable, and answers the question.
Example

A level curve on a country road has a radius of 150 m. What is the maximum speed at which this curve can be safely negotiated on a rainy day when the coefficient of friction between the tires on a car and the road is 0.40?
Example

In the hammer throw, an athlete spins a heavy mass in a circle at the end of a chain, then lets go of the chain. For male athletes, the “hammer” is a mass of 7.3 kg at the end of a 1.2 m chain.

A world-class thrower can get the hammer up to a speed of 29 m/s. If an athlete swings the mass in a horizontal circle centered on the handle he uses to hold the chain, what is the tension in the chain?
Driving over a Rise

A car of mass 1500 kg goes over a hill at a speed of 20 m/s. The shape of the hill is approximately circular, with a radius of 60 m, as in the figure at right. When the car is at the highest point of the hill,

a. What is the force of gravity on the car?

b. What is the normal force of the road on the car at this point?
Maximum Walking Speed

(a) Walking stride

The speed of the circular motion is the walking speed.

(b) Forces in the stride

The $x$-axis points down, toward the center of the circle.

The radius of the circular motion is the length of the leg from the foot to the hip.

The circular motion requires a force directed toward the center of the circle.

$$v_{\text{max}} = \sqrt{gr}$$
Loop-the-Loop

A roller coaster car goes through a vertical loop at a constant speed. For positions A to E, rank order the:

- centripetal acceleration
- normal force
- apparent weight

Answer:
Centripetal acceleration: same for all
Normal force: A & E, D & B, C
Apparent weight: A & E, D & B, C
Over the Top

A handful of professional skaters have taken a skateboard through an inverted loop in a full pipe. For a typical pipe with diameter 14 ft, what is the minimum speed the skater must have at the very top of the loop?
Orbital Motion

Phobos is one of two small moons that orbit Mars. Phobos is a very small moon, and has correspondingly small gravity—it varies, but a typical value is about 6 mm/s$^2$. Phobos isn’t quite round, but it has an average radius of about 11 km. What would be the orbital speed around Phobos, assuming it was round with gravity and radius as noted?
Newton’s law of gravity  If two objects with masses $m_1$ and $m_2$ are a distance $r$ apart, the objects exert attractive forces on each other of magnitude

$$F_{1\text{on}2} = F_{2\text{on}1} = \frac{Gm_1m_2}{r^2}$$  \hspace{2cm} (6.21)$$

The forces are directed along the line joining the two objects.

The constant $G$ is called the gravitational constant. In the SI system of units, $G$ has the value

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$$
A typical bowling ball is spherical, weighs 7 kg, and has a diameter of 22 cm. Suppose two bowling balls are right next to each other in the rack. What is the gravitational force between the two—magnitude and direction?

For a spherical mass, the force of gravity outside is as if all the mass were concentrated at its centre:

\[ F_{1\text{on}2} = G \frac{m_1 m_2}{d^2} \]

\[ F_{1\text{on}2} = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \times \frac{7\text{ kg} \times 7\text{ kg}}{(0.22\text{ m})^2} \]

\[ = 6.76 \times 10^{-8} \text{ N} \]
Example

Three bowling balls are touching each other on a flat surface. What is the net gravitational force of the two of them on the third?

\[ \vec{F}_{\text{net on3}} = \vec{F}_{1\text{on3}} + \vec{F}_{2\text{on3}} \]
Example

Three bowling balls are touching each other on a flat surface. What is the net gravitational force of the two of them on the third?

\[ \vec{F}_{\text{net on 3}} = \vec{F}_{1\text{on 3}} + \vec{F}_{2\text{on 3}} \]
Gravity on Other Worlds

A 60 kg person stands on each of the following planets. In which case is her weight highest?

A. 

B. 

C. 

Answer: A, B, C
Gravity and Orbits

A spacecraft is orbiting the moon in an orbit very close to the surface—possible because of the moon’s lack of atmosphere. What is the craft’s speed? The period of its orbit?

Phobos is the closer of Mars’ two small moons, orbiting at 9400 km from the center of Mars, a planet of mass $6.4 \times 10^{23}$ kg. What is Phobos’ orbital period? How does this compare to the length of the Martian day, which is just shy of 25 hours?
A satellite orbits the earth. A Space Shuttle crew is sent to boost the satellite into a higher orbit. Which of these quantities increases?

A. Speed
B. Angular speed
C. Period
D. Centripetal acceleration
E. Gravitational force of the earth

Answer: C
A coin sits on a rotating turntable.

1. At the time shown in the figure, which arrow gives the direction of the coin’s velocity?

Answer: A
A coin sits on a rotating turntable.

2. At the time shown in the figure, which arrow gives the direction of the frictional force on the coin?

Answer: D
A coin sits on a rotating turntable.

3. At the instant shown, suppose the frictional force disappeared. In what direction would the coin move?

Answer: A
Additional Examples

At Talladega, a NASCAR track, the turns have a 370 m radius and are banked at 33°. At what speed can a car go around this corner with no assistance from friction?

The Globe of Death is a spherical cage in which motorcyclists ride in circular paths at high speeds. One outfit claims that riders achieve a speed of 60 mph in a 16 ft diameter sphere.

What would be the period for this motion?

What would be the apparent weight of a 60 kg rider at the bottom of the sphere?

Given these two pieces of information, does this high speed in this small sphere seem possible?