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1

REVIEW # 4: POWERS OF TEN

The mass of an electron is
.000000000000000000000000000009 kg.
After doing this painless review you will write it 9×10^{-31} kg., and save yourself mistakes on exams too! Move horizontally. Turn to next page, frame 2.

7

10^{21}
(This number, I am told, is a sextillion. Our aim is to make such words useless!)

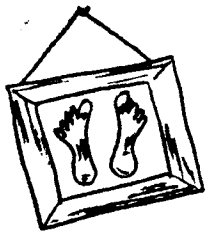
8

Now multiply:

$10^{5} \times 10^{-2} = 10^{\square}$

and

$10^{2} \times 10^{-2} = 10^{\square}$



At left: mankind's first digital computer.

(10^{8} B.C.)

14

1.5×10^{-18}

The point took 18 steps to get there.

15

It works in the other direction too. The number below is one and a quarter quintillion. Walk the point until it is to the right of the 1, and count off as you go. The power of ten will be the number of steps you took.

$1250000000000000000. = 1.25 \times 10^{\square}$

21

$3 \times 10^{-4} - .2 \times 10^{-4}$

which is

2.8×10^{-4}

22

Re-do this column, so it can be added.

1.5×10^{-6}	$\times 10^{-6}$
1.8×10^{-4}	$\times 10^{-6}$
1.1×10^{-3}	$\times 10^{-6}$
2.5×10^{-5}	$\times 10^{-6}$

28

2×10^{-13}

29

Negative exponents are handled in a similar way. Give this one the reducing treatment.

$$\frac{4 \times 10^{3} \times 2.2 \times 10^{-2}}{4.4 \times 10^{-6}}$$

35

$10^{7} \times \sqrt{25}$

$= 5 \times 10^{7}$

36

To take cube root we divide the exponent by 3.

Example: $\sqrt[3]{10^{9}} = 10^{3}$. Can you do $\sqrt[3]{10^{10}}$?

Hint: First write it as $\sqrt[3]{10^{9} \times 10}$

42

(a) 10^{-1}

(b) 10^{-12}

Quite different!

43

If you raise 2×10^{4} to the third power, that is, $(2 \times 10^{4})^{3}$ both the 2 and the 10^{4} get raised to the third power.

Answer is 8×10^{12}. Do this: $(4 \times 10^{2})^{\frac{1}{2}} = ?$

1

The correct answer will be in this space.

Give yourself a mark in the little square

2 if right
0 if wrong
1 if in between

HERE

2

In 10^3, the 3 is the exponent and it says that we have three tens, all multiplied together; that is, $10^3 = 10 \times 10 \times 10$.

Or, $10^3 = 1000$. Finish this: $10^5 =$

8

10^3 and 10^0

9

We haven't run into 10^0 before. (Remember where it came from: $10^2 \times 10^{-2}$ which is 100 x .01 or just one.) 10^0 is 1. We won't do anything more with 10^0 now; just tuck it away in your cranium. Next, let's express 5,000,000 as $\square \times 10^{\square}$

15

1.25×10^{18}

16

One more bit of drill. Multiply these:

$10^{-5} \times 800 \times 10^8$

$= \square \times 10^{\square}$

Lo! the lowly amoeba! To multiply he must divide!

22

1.5×10^{-6}
$180. \times 10^{-6}$
$1100. \times 10^{-6}$
$25. \times 10^{-6}$

23

Add them, and express the answer so that there is only one figure to the left of the decimal.

Could you have worked this problem as well by first converting them all to something times 10^{-3} ?

29

2×10^7

30

What can you do about this one?

$$\frac{10^2 + 10^3}{10^2}$$

Watch out! You can't cancel that 10^2 against the other 10^2. No! Not when there is another term added. Peek at the answer if you need to.

36

It can further be rewritten as

$\sqrt[3]{10^9} \times \sqrt[3]{10.}$

Then you get

$10^3 \times \sqrt[3]{10}$

37

So you see there was one 10 we had to leave under a $\sqrt[3]{\ }$ sign. Of course we can get it out by looking up $\sqrt[3]{10}$, which is 2.154, so our ans. would be 2.154×10^3. Try $\sqrt[3]{2.7 \times 10^{13}}$. (Keep in mind that 27 is the cube of something.)

43

2 x 10 or 20

(It is $\sqrt{4} \times \sqrt{100}$)

44

We have no more new tricks to introduce, so let's exercise our talents a little. Exercise is the glue that sticks things in your mind! Step the decimal point and count off.

$1650000000000000. = 1.65 \times 10^{\square}$

$35000000. = 3.5 \times 10^{\square}$

2

10x10x10x10x10

or, 100,000

3

A negative exponent, such as 10^{-3} means the reciprocal of 10^3. Example:

$10^{-3} = \frac{1}{10^3}$. Similarly, $10^3 = \frac{1}{10^{-3}}$.

Finish this. (Put exponent in square) $\frac{1}{10^{-2}} = 10^{\square}$

9

5×10^6

10

5,500,000 would be expressed as 5.5×10^6. (It could be written as 55×10^5, but <u>by custom</u> we do it so there is only one figure to the left of the decimal point.

5,525,000 would be $\square \times 10^6$

16

800×10^3 is ok, but a better form would be

8×10^5

17

Our next project is to learn to <u>add</u> and <u>subtract</u> numbers like 2.5×10^5 and 1.2×10^4. The catch is that the two have different powers of ten. We can add or subtract <u>if</u> we first make them have the <u>same</u> powers of ten.

Turn page →

23

1.3065×10^{-3}

Yes you could have.

24

Now let's go back to multiplication for a moment. When we want to multiply 1.5×10^3 by 2×10^2 we do it by multiplying the 1.5×2 to get 3 and the $10^3 \times 10^2$ to get 10^5, making 3×10^5. Try $3 \times 10^7 \times 4 \times 10^5$.

30

11

31

How did we get it? Either by writing $\frac{10^2 + 10^3}{10^2}$ as $\frac{10^2(1+10)}{10^2}$ or as $\frac{1.1 \times 10^3}{10^0}$ and cancelling as shown. Now you do: $\frac{10^5 + 10^2}{10^2} = ?$

37

3×10^4

CHANGE TO SMOGS! ONLY $\frac{1}{\sqrt{10^{-4}}}$ % AS MUCH NICOTINE

(By laboratory test.)

38

Now let me tell you <u>another way of writing</u> square roots and cube roots.

$\sqrt{100}$ is $100^{\frac{1}{2}}$ and $\sqrt[3]{100}$ is $100^{\frac{1}{3}}$

$\sqrt[3]{10^3}$ would be $(10^3)^{\frac{1}{3}}$. When a power is raised to a power, we multiply the exponents together, e.g., $(10^3)^{\frac{1}{3}}$ is 10^1. What is $(10^6)^{\frac{1}{2}}$?

44

1.65×10^{15}

3.5×10^7

45

Add these:

5.2×10^{27}

2.0×10^{25}

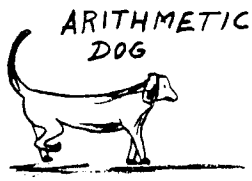
5.4×10^{26}

$= \square \times 10^{\square}$

3

10^{2}

4



(Puts down 3 and carries 1 !)

Do these:

$10^{-2} = \frac{1}{10^{\square}}$

$10^{3} = \frac{1}{10^{\square}}$

10

5.525×10^{6}

11

Before we go further, a little drill:

Complete these:

$\frac{1}{10^{-5}} = 10^{\square}$; $\frac{1}{5 \times 10^{8}} = \square \times 10^{\square}$

17

Here is the problem again:

$2.5 \times 10^{5} + 1.2 \times 10^{4}$

= ?

18

If we move the decimal in the second number by one step to the left, we have $2.5 \times 10^{5} + .12 \times 10^{5}$. Now we can add and get 2.62×10^{5} . Try rewriting the following so that they both will have the same powers of 10.

2×10^{3} and 3.5×10^{2} .

24

12×10^{12} , but we prefer to write

1.2×10^{13}

25

Do these:

$5 \times 10^{3} \times 6 \times 10^{-3} =$

$3 \times 10^{3} \times 1.2 \times 10^{-10} =$

31

1001

or 1.001×10^{3}

32

We haven't mentioned division, as such, but I guess you know how to do it, from what we have done with fractions. If you want to divide 10^{6} by 10^{4} , all you have to do is write it $\frac{10^{6}}{10^{4}} = 10^{2}$. Or you can simply subtract exponents.

38

10^{3}

39 The same goes for integral powers as for fractional powers. Study these examples, and note especially how + and - exponents are treated.

$(10^{2})^{3} = 10^{6}$ $(10^{2})^{-3} = 10^{-6}$ $(10^{-2})^{-2} = 10^{4}$

$(10^{-2})^{\frac{1}{2}} = 10^{-1}$ $(10^{-2})^{-\frac{1}{2}} = 10^{1}$ $10^{-\frac{1}{2}} = \frac{1}{\sqrt{10}}$

45

5.76×10^{27}

46

What is the cube (not cube root) of 4×10^{-7} ? Write the answer with one digit to the left of the decimal point.

4

$\frac{1}{10^2}$

$\frac{1}{10^{-3}}$

5

0.1 is, of course $\frac{1}{10^1}$ or 10^{-1}

Complete this: $.001 = 10^{\square}$

11

10^5, and

$\frac{1}{5} \times 10^{-8}$ or 2×10^{-9}

12

Put these into exponential form:

$.0000000000001 = 10^{\square}$

$11000000. = \square \times 10^{\square}$

18

2×10^3 and $.35 \times 10^3$

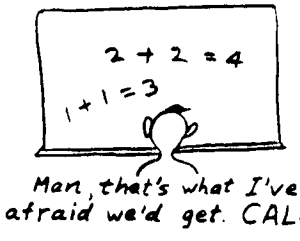
(20×10^2 and 3.5×10^2 would be all right also.)

19

Now see if you can add these two numbers together.

1.5×10^4 and

2×10^2



Man, that's what I've been afraid we'd get. CALCULUS!

25

30

3.6×10^{-7}

26

If you have $\frac{5 \times 10^3}{2 \times 10^3}$ you can follow the old rule of cancelling like factors in numerator and denominator. What do you have left?

32

Now we come to the last operation and the hardest one: Roots and fractional powers.

33 The square root of 10^4 is 10^2. You can see this from the reverse operation, namely that 10^2 multiplied by itself is 10^4. You do this one: $\sqrt{10^{12}} = ?$ (The question is, what, when multiplied by itself gives 10^{12} ?)

39

And what about $(10^5)^{\frac{1}{2}}$? This gives $10^{2.5}$, which is $10^2 \times 10^{\frac{1}{2}}$.

40

Try these yourself

$(10^{10})^{\frac{1}{2}} = 10^{\square}$ $\quad$ $(10^9)^{\frac{1}{2}} = 10^{\square}$

46

6.4×10^{-20}

47

Do some cancelling and check this one:

$$\frac{202 \times 10^{16} \times 2 \times \sqrt{10^{14}}}{4040000} = 10^{\square}$$

5

10^{-3}

6

Notice that in numbers like $10000 = 10^4$ the exponent is the same as the number of zeros.

But with negative exponents, e.g., $.0001 = 10^{-4}$, the exponent is one greater than the number of zeros. It would tax our memory less if the two cases were not different. But we can't argue with arithmetic! Complete this one $.0000000001 = 10^{\square}$

12

10^{-13}

1.1×10^7

13

Put these into exponential form, so that there is one figure to the left of the decimal.

$0.12 =$ $.0033 =$

19

$1.5 \times 10^4 + .02 \times 10^4$, which makes 1.52×10^4.

20

Here's one where a negative exponent is involved. Add these

$$1.5 \times 10^2 + 4 \times 10^{-1} = ?$$

26

$\frac{5}{2}$ or 2.5

27

If you have $\frac{5 \times 10^3}{2 \times 10^5}$, remember that 10^5 is $10^2 \times 10^3$, so cancel what you can. You have left:

33

10^6

34

Now comes the problem: what if the exponent is not an even number?

For instance $\sqrt{10^7}$? We can rewrite it $\sqrt{10^6 \times 10}$ which is $\sqrt{10^6} \times \sqrt{10}$, which is $10^3 \times \sqrt{10}$. How would you handle $\sqrt{3 \times 10^7}$?

40

10^5 / $10^{\frac{9}{2}}$ or $10^{4.5}$

41

Try these:

$(10^8)^{-\frac{1}{2}} = 10^{\square}$ $(10^9)^{-\frac{1}{2}} = 10^{\square}$

$(10^{-2})^{-6} = 10^{\square}$ $(10^{-\frac{1}{2}})^{-6} = 10^{\square}$

47

10^{19}

48

Cancel and check the answer here, all at one shot, without first removing the square root signs.

$$\frac{10^2 \times \sqrt{10^3} \times \sqrt{10^6}}{\sqrt{10^{11}}} = 10^1$$

(or, plain 10)

6

10^{-10}

If you forget whether it is one more or one less than the number of zeros, think of $.1 = 10^{-1}$, as a check.

7

To multiply, we simply add the exponents. For example, $10^2 \times 10^3 = 10^5$. This is reasonable, because $10^2 \times 10^3 =$ 10 x10 x 10x10x10 which is five tens multiplied together.

Multiply this: $10^6 \times 10^{15} = 10^{\square}$

13

1.2×10^{-1} and 3.3×10^{-3}

14

Did you notice that the exponent you had to use was just the number of places you had to step the decimal point over, to get it to the right of the first figure which was not zero. Do this one, and count as you walk the point along.

$.000000000000000015 = 1.5 \times 10^{\square}$

20

150.4 or

1.504×10^2

21

Here's another: (A subtraction, and both have negative exponents besides. Watch out!)

$3 \times 10^{-4} - 2 \times 10^{-5}$

27

$\frac{5}{2 \times 10^2}$ which is

2.5×10^{-2}

28

Shake what you can out of this fellow. That is, reduce him to his simplest form.

$$\frac{4 \times 10^3 \times 2.2 \times 10^2}{4.4 \times 10^{18}}$$

34

Rewrite it

$\sqrt{30 \times 10^6}$ and

the ans. is $10^3 \times \sqrt{30}$

35

You can work this one the same way, only you can end up without any $\sqrt{\ }$ at all.

$\sqrt{2.5 \times 10^{15}} =$

41

10^{-4} $10^{-4.5}$

10^{12} 10^3

42

Be sure you distinguish between these two operations: (a) Multiplying 10^{-4} by 10^3, and (b) raising 10^{-4} to the 3rd power.

(a) $10^{-4} \times 10^3 = 10^{\square}$ (b) $(10^{-4})^3 = 10^{\square}$

48

If you checked it give yourself 2.

Incidentally, if we could pay you for doing this review would you like $\$2^{(2^3)}$ or $\$2^{(3^2)}$?

49

Verify this and you get your diploma.

$$\frac{10^5 \times 10^{-\frac{1}{2}} \times 9 \times 10^8 \times \frac{10^6}{\sqrt{10^3}} \times 10^{-20}}{10^6 \times 10^{-19} \times 6 \times 10 \times \sqrt{10000}} = 1.5 \times 10^7$$

THE END. How much did you make? (Maximum: 88) $\square$