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REVIEW #1: ANGLES AND TRIANGLES

1 A review and drill for students beginning college physics who are a little rusty on angles and triangles. The material covered is only that which is necessary for solving elementary physics problems. Turn to the next page, <u>and stay in the top strip</u>.

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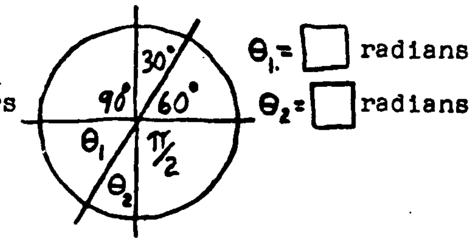
8 No answer here unless you want to give your own circumference.
We'll try you on some radians. (Remember that 2π radians = 360°.)

9 Here's a pie with some cuts through the center. Some angles are given in degrees. Give the others in radians. Example: 90° is $\frac{1}{2}\pi$ radians.



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16 The first and the last

17 Given two similar triangles, if we are allowed to magnify or reduce the size of one (without changing its shape) and to slide it around on the page in any way we please, we should be able to make it fit exactly on the other. This would be a "mind's eye" test for similarity.

No question; Turn page

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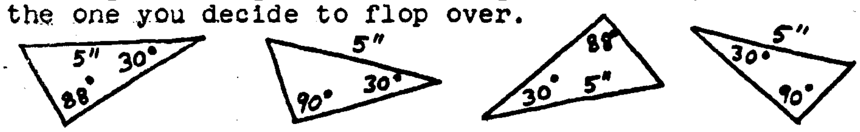
24 The second and fourth (You had to fill in the missing angles.)

25 Here you can make two of these fit each other by sliding them around. To make the other two fit each other you will have to flop one upside down (like turning a postage stamp over, glue side up). Shade the one you decide to flop over.



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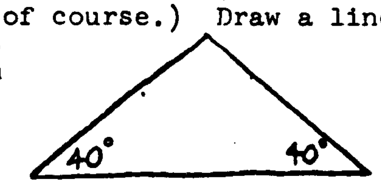
32 $\frac{.866}{1}$

33 In this triangle two angles are the same. The term is <u>isosceles</u>. (A Greek word of course.) Draw a line so as to split this isosceles triangle into two parts which are mirror images of each other.



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40 Oppositely similar

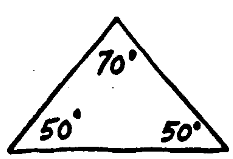
41 One of these angles is 45° and one is 20°. How sharp is your eye? Pick them out.

---

48 $x^2 = 49 - 25$

$a = \sqrt{b^2 + c^2}$

49 Is there anything wrong with this triangle?



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1

We move along from one page to the next, at the same level; we do not move down the page.

2

Take it easy; this is not an exam. It should be as psycho-therapeutical as a cross word puzzle, and twice as beneficial to your grade in physics.

Answer or do what is asked with pencil; don't just do it in your head. Then turn to the next page, staying at the same level. The correct answer will be given in the space at the left, numbered to correspond to the question. Now turn the page. →

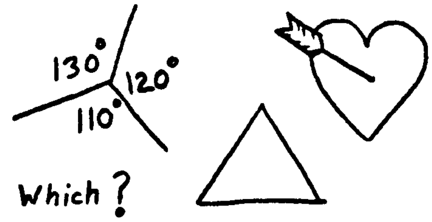
9

$\theta_1 = \frac{\pi}{3}$ radians

$\theta_2 = \frac{\pi}{6}$ radians

10

We now leave angles and shift to triangles. 999 chances in 1000 you know a triangle when you see it. But I would like to have you prove you are not the 1 in 1000.

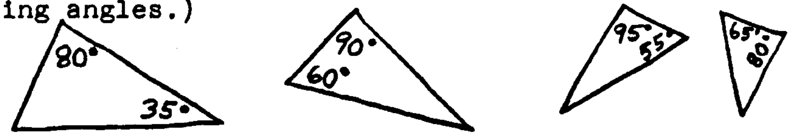


17

This was a free one!

18

By magnifying the size and skidding these triangles around on the page, which two can be made to fit one another exactly? (First you may have to fill in some missing angles.)

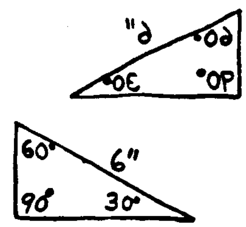


25

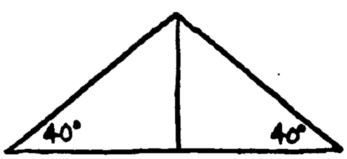
The second or the fourth

26

Instead of turning a figure upside down, you can get the same result by holding the paper up before a mirror and viewing the image in the mirror. Naturally, what you see is called a MIRROR IMAGE. Are these two figures mirror images of each other?

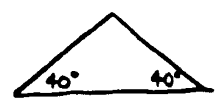


33



34

This illustrates (and it can easily be proven in more formal fashion) that in an isosceles triangle not only are two angles equal but also two sides are equal. The 3rd side is the base. Label the two equal sides and the base:



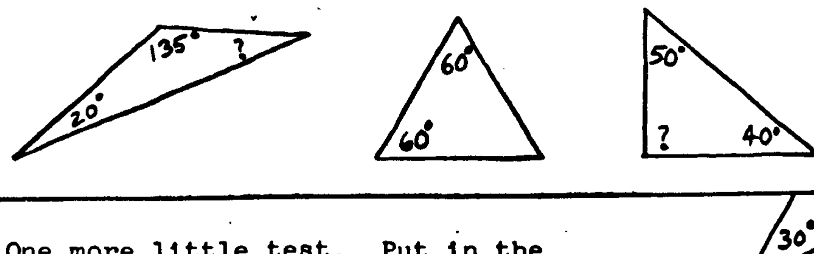
41

The first is 45°

The fourth is 20°

42

Write the missing angle in each:



49

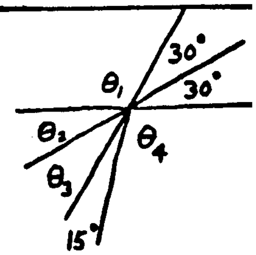
Impossible, because the angles add up to only 170 degrees.

50

One more little test. Put in the missing angles, in radians. That is, π times something, in each case.

$\theta_1 =$ $\theta_2 =$

$\theta_3 =$ $\theta_4 =$



2

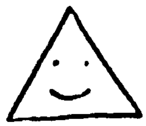
The small square is for your score.
0 if you miss completely
2 if correct
1 if in between

Now proceed → HERE

3

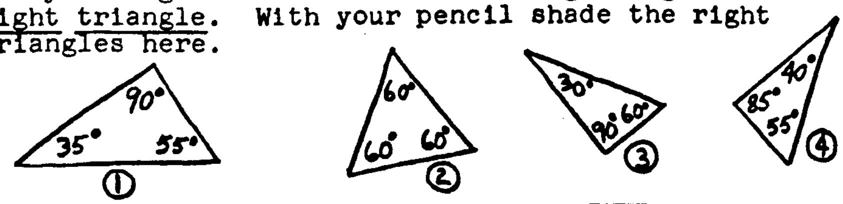
Just to be sure we both are talking about the same thing, mark, or shade, the figure that is an angle. Always mark or write the answer; don't just think it.

10



11

Any triangle which contains one right angle is a right triangle. With your pencil shade the right triangles here.

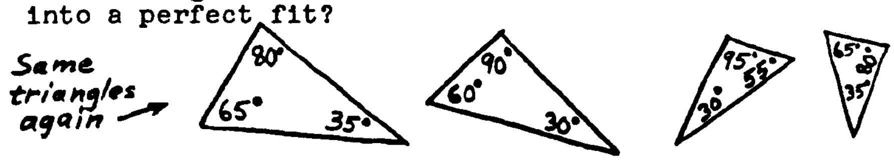


18

The first and the fourth

19

How can you be certain that no amount of magnifying and sliding around would ever bring any other two into a perfect fit?



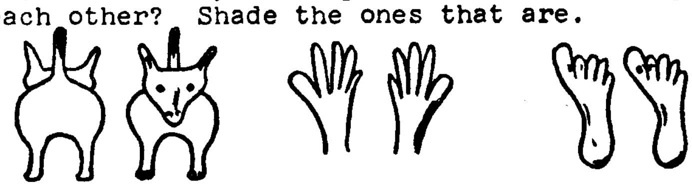
26

Yes.

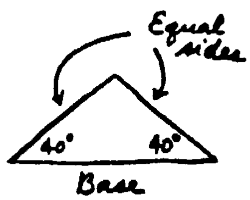
(The numbers got reversed too, but I couldn't help that!)

27

Just for fun, which pairs are mirror images of each other? Shade the ones that are.

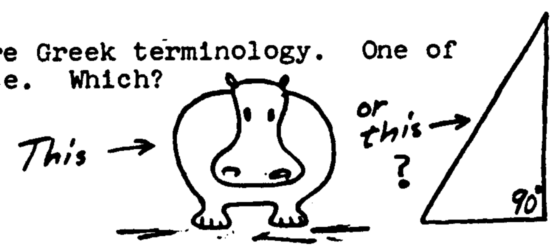


34



35

Let's try some more Greek terminology. One of these is a hypotenuse. Which?



42

25°, 60° and 90°

43

A 70°-70°-40° triangle would be an

________________ triangle.

Two triangles whose angles (going around clockwise) are 40°-50°-90° and 50°-40°-90° are certainly

________________ ______________________

50

$\theta_1 = \frac{2}{3}\pi \qquad \theta_2 = \frac{1}{6}\pi$

$\theta_3 = \frac{1}{6}\pi \qquad \theta_4 = \frac{7}{12}\pi$

radians

51

That is all I have for you. Now you are an expert on angles, triangles and hippopotomusses. The maximum score you could have gotten is 88. Put your score here ☐

GOOD LUCK ON YOUR PHYSICS TEST!

3

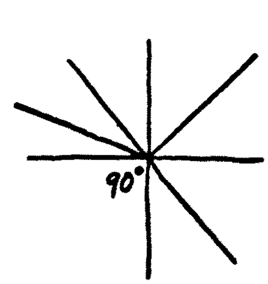
Yes, you have talent.

We can proceed.

2?

4

Here are angles of 20, 30, 40, 45, 50 and 90 degrees. The 90° is labelled for you. See if you can label the others. (Sight along the lines to see which pass through the center without bending. If you need a little further help, see footnote #1 below.)

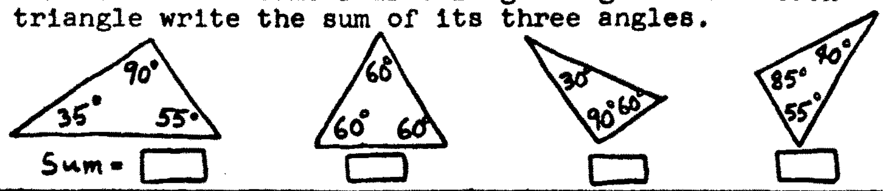


11

The first and third

12

Here are the same four triangles again. For each triangle write the sum of its three angles.



19

The angles are different, and angles cannot be changed by magnifying or sliding the triangle around.

20

Try a word rule for similarity: "Two triangles are similar if the three angles in one are the

same as the corresponding ______ ________ _____

____ _____________"

27

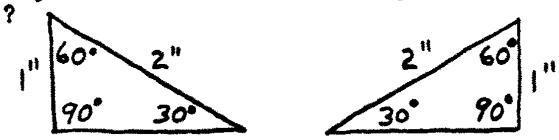
The hands.

MOM is its own mirror image!

If you think Fido is, try him in a mirror!

28

Now would you say that these two triangles would be called congruent?



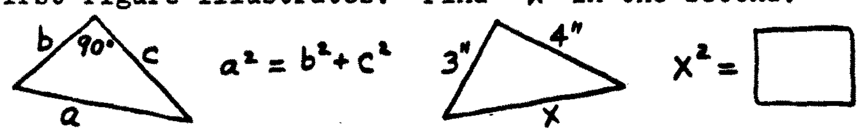
I won't put you on the spot, because I didn't know the answer myself. Proceed.

35

The one on the right--on the right triangle, that is!

36

The Greek philosopher Pythagoras found that in any right triangle the square of the hypotenuse equals the sum of the squares of the other two sides. The first figure illustrates. Find X^2 in the second.

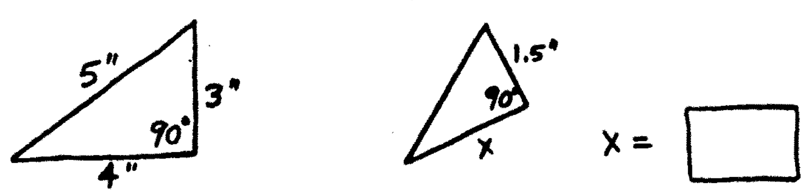


43

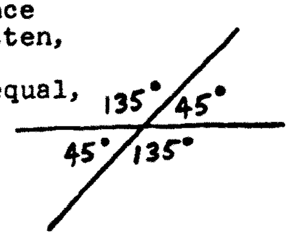
Isosceles
Oppositely similar

44

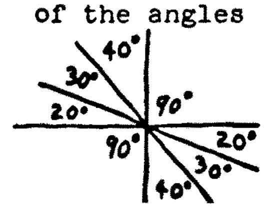
In these two similar triangles



Footnote # 1. I'm sure you once knew, but possibly have forgotten, that when two straight lines cross, the angles formed are equal, in pairs, like this:

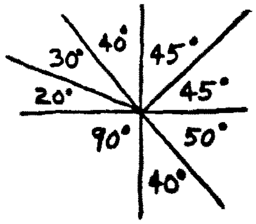


Also, when any number of lines intersect at a point, the sum of the angles is 360°, thus:



Sum is 360°. Add if you don't believe it.

4



5

It's important to be able to estimate angles. Below are angles of 15, 25, 45, 60, 65 and 70 deg. See if you can label them. (See also footnote # 2 at bottom of this page.)

12

All 180°

13

In this picture shade the right triangle

20

Three angles in the other

21

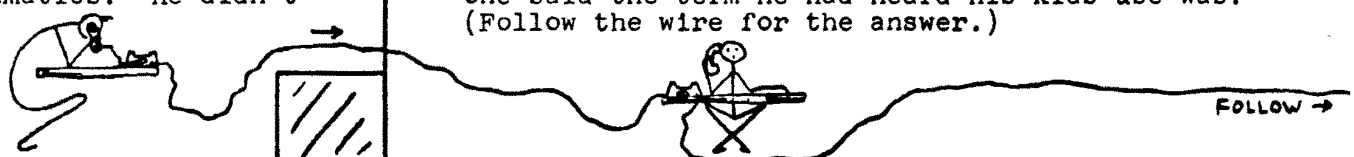
I have already reminded you that the sum of the angles inside of every triangle is → [ ]°
(This by the way isn't a rule just for right triangles; it holds for <u>all</u> triangles.)

28

I phoned a Professor of Mathematics. He didn't know.

29

So he phoned another Professor of Mathematics. This one said the term he had heard his kids use was: (Follow the wire for the answer.)



36

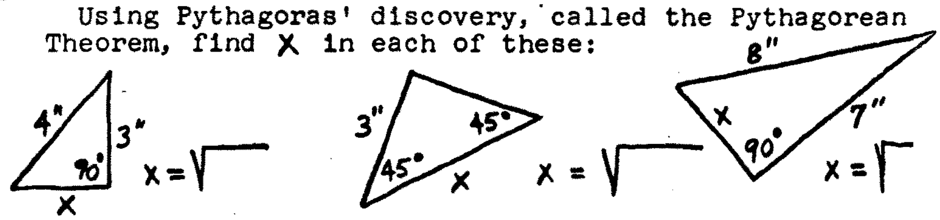
$$x^2 = 3^2 + 4^2$$

$$\therefore\ x^2 = 25$$

and $x = 5$ inches

37

Using Pythagoras' discovery, called the Pythagorean Theorem, find X in each of these:

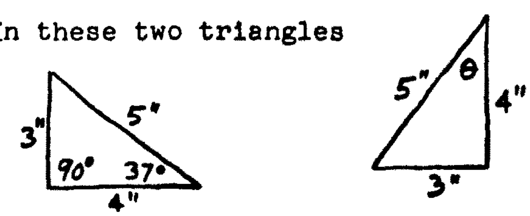


44

X = 2"

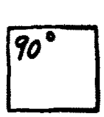
45

In these two triangles



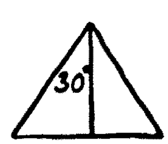
θ = [ ]

Footnote # 2. If you keep these four pictures in mind you will be able to estimate the more common angles.

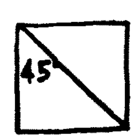


A square, for 90°

A triangle of 3 equal sides for 60°



Half of same for 30°



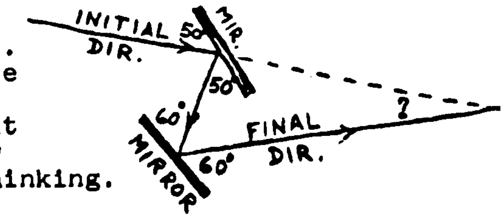
Square with diagonal for 45°

5

45°, 65°, 25°, 60°, 70°, 15°

6

This is a light ray, reflected by two mirrors. We want to know the angle between the initial and final directions. Try it your way; then I'll show you a way that avoids thinking.



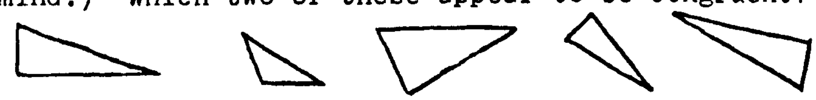
13

Only the second one.

(I hope you used the 180° test. The three angles inside any triangle add up to 180°.)

14

If by sliding one triangle around on the page you can make it fit exactly on another, the two are said to be congruent. (You have to move it in your mind.) Which two of these appear to be congruent?



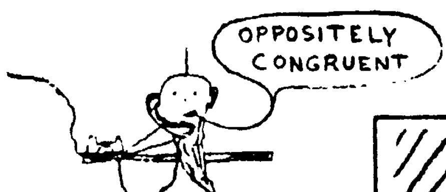
21

180°

22

So let's re-state the rule for similarity; using no more angles than necessary. "Two triangles are similar if ________ ________ in one are the same as the corresponding ________ ________ in the other."

29



30

Following the same idea, what name can you dream up for a pair of triangles that have the same three angles but in reverse order, and that are also of different size?

________ ________

(We'll spare the mathematician the pain of this one!)

37

$x = \sqrt{7}"$ $x = \sqrt{18}"$ $x = \sqrt{15}"$

(In the second triangle the side not labelled is of course 3" in length. It is isosceles.)

38

We have covered enough ground. We will end with a few practice problems in mixed-up order, to increase your speed of recognition. What kind of triangle is each?

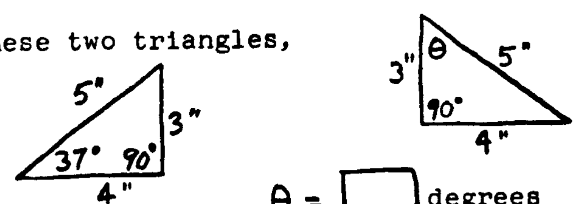
#1 ________ #2 ________

45

37°

46

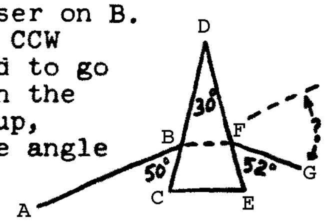
Given these two triangles,



Find θ. θ = ☐ degrees

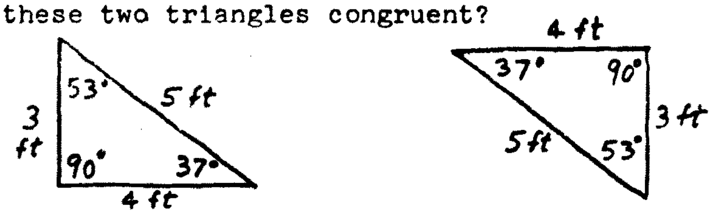
6
20°, counterclockwise.
The way that eliminates thinking is in footnote # 3 (this page).

7
Try one more. A light ray goes through a prism. Lay pencil along AB, eraser on B. Turn counterclockwise (CCW) to CD; CCW to DE; CCW to FG. But now you need to go clockwise 180° to get the eraser on the right end again (+180). Add them up, with the correct + or - signs. The angle between AB and FG is: ☐°



14
The first and fifth

15
Are these two triangles congruent?

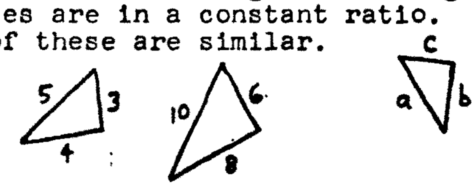


22
Two angles
Two angles

23
Now when we are dealing with right triangles we can make an even simpler definition of similarity. "Two right triangles are similar if ________ angle (other than the ________ angle) in one is the same as the corresponding angle in the other".

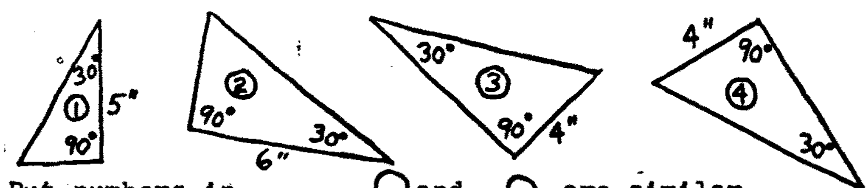
30
"Oppositely similar" would be a logical name.

31
In similar triangles the lengths of the respective sides are in a constant ratio. To illustrate: All 3 of these are similar.



$\frac{a}{b}$ = ☐

38
1. Isosceles
2. Right

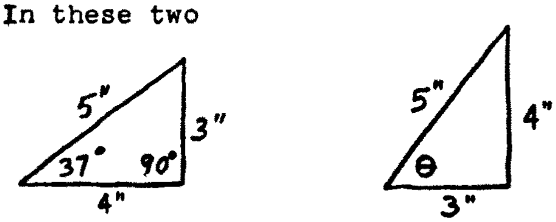
39


Put numbers in the circles

○ and ○ are similar
○ and ○ are congruent

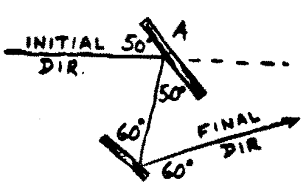
46
$\theta = 53°$

47
In these two



θ = ☐

Footnote # 3.



1. Hold your pencil parallel to the initial direction; eraser to the right.
2. Rotate pencil clockwise, till parallel with mirror A. Write down +50° (That's the amount you rotated; plus indicates clockwise.)
3. Clockwise another 50 deg., writing down + 50°.
4. Counterclockwise 60 deg., becoming parallel to mirror B. Write down - 60°.
5. Counterclockwise another 60 deg., becoming parallel with the final direction. Write down - 60 deg. again.
6. Add up all your figures. - 20°. (The overall result was counterclockwise.) Dull? Just bookkeeping? Yes, but look Ma, no thought!

7

$$\begin{array}{r} -50^\circ \\ -30^\circ \\ -52^\circ \\ +180^\circ \\ \hline +48^\circ \end{array}$$

8

Enough of that. What is the difference between pie and π? Both give circumference: pie when eaten and π when multiplied.

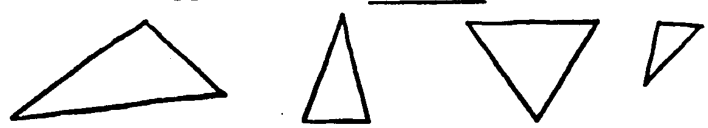
How many π radians are there in 360°? If hazy, see footnote # 4 (this page).

15

Yes. If you could skid them around you could make one cover the other exactly.

16

Two triangles that are of the same shape but which are not the same size are said to be similar. Shade the two that appear to be similar.



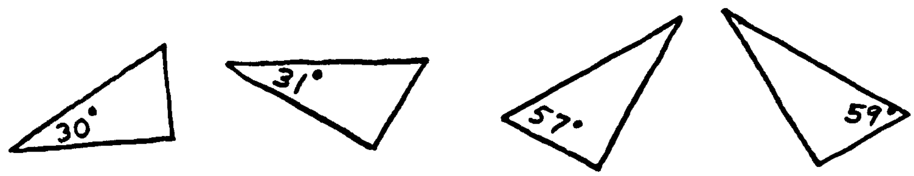
23

one

right

24

These are right triangles. Shade the similar ones.



31

$\frac{a}{b} = \frac{10}{8}$ or $\frac{5}{4}$

32

Here are "oppositely similar" right triangles:

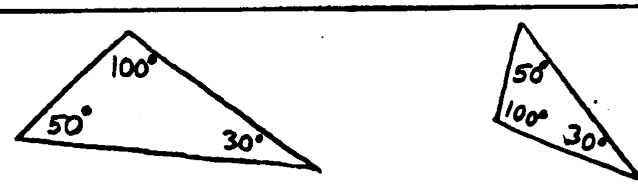
$\frac{x}{.6} =$ ☐

39

① and ②

③ and ④

40



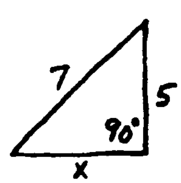
These triangles are ____________ ____________

47

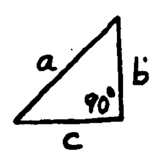
53°
(Remember that the sum of the angles = 180°.)

48

In these two right triangles

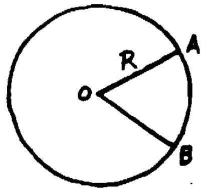


$x^2 =$ ☐



$a = \sqrt{}$

Footnote # 4.



In this circular pie the arc AB is the distance measured along the rim or edge, from A to B. It is just the part of the circumference between A and B. The angle, between OA and OB can be given in degrees, but it can also be given in radians. The number of radians is the arc divided by the radius. It is

$\frac{\text{arcAB}}{R}$ radians.

2π radians and 360 degrees are the same thing. For a full circle the arc is the circumference, and it is $2\pi R$. So

$$\frac{\text{the circumference}}{R} = \frac{2\pi \cancel{R}}{\cancel{R}} = 2\pi \text{ radians}$$