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REVIEW #2: TRIGONOMETRY

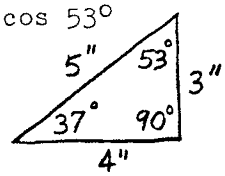
1 A review and drill for students in beginning college physics whose sines and cosines have slipped. The material covered is only that which is necessary for solving elementary physics problems. Turn to the next page, and stay in the top strip.

7

$\sin\theta = \frac{5\ \cancel{cm}}{10\ \cancel{cm}} = \frac{5\ \cancel{miles}}{10\ \cancel{miles}}$

$= \frac{5}{10}$. The units cancel. $\sin\theta$ is a dimensionless (no units) number.

8 The side adjacent to an angle, divided by the hypotenuse is the cosine of the angle. Find cos 37° and cos 53°



$\cos 37° = \boxed{}$

$\cos 53° = \boxed{}$

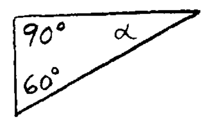
14

$\theta = 90° - \phi$

$\sin\phi = \cos(90° - \phi)$

15

Two angles whose sum is 90° are complementary. One is the complement of the other.



The complement of α is $\boxed{}$°

21

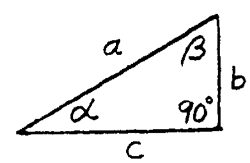
90°

$\cos\beta$ $(90° - \beta)$

22 In terms of the sides, a, b and c,

$\cot\alpha = \boxed{}$ $\sec\alpha = \boxed{}$

$\cos\beta = \boxed{}$



28

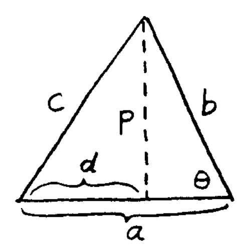
$p = b \sin\theta$

29

Now can you write that

$d = a - \boxed{}$

where you put in the box b and a trig. function of θ, such as sinθ, cosθ etc.



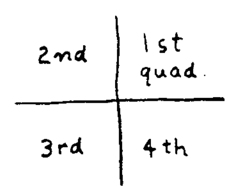
35

Pos: 270° to 90°, Quad 4 and 1

Neg: 90° to 270° Quad 2 and 3

36

In what quadrants will sinθ be negative?



Which ones ?

Useful Relations: (I don't mean you should memorize them. Just remember they are here.)

$\cos\theta = \sin(90° - \theta)$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$\csc\theta = \sec(90° - \theta)$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

$\cot\theta = \tan(90° - \theta)$

1

We move along from one page to the next, at the same level; we do not move down the page.

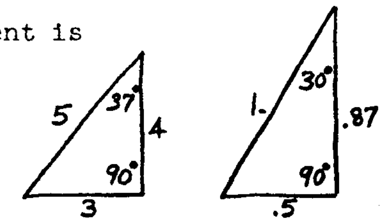
2 Don't make hard work of this; it is not an exam. It is a painless way by which you can improve your speed and accuracy in simple trigonometry. Answer or do what is asked <u>with pencil</u>; don't just think the answer. Then turn to the next page, staying at the same level. The correct answer will be given in the space at the left, numbered to correspond to the question. Now turn the page.

8

$\cos 37^\circ = \frac{4}{5}$

$\cos 53^\circ = \frac{3}{5}$

9 The tangent is $\dfrac{\text{opposite side}}{\text{adjacent side}}$



$\tan 37^\circ = \square$

$\tan 30^\circ = \square$

15

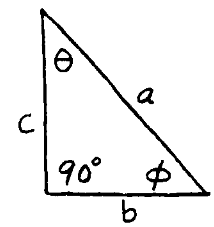
60°

16 In terms of a, b and c,

$\sin\theta = \square$, $\cos\theta = \square$, $\tan\theta = \square$

Use the above to show that

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$



22

$\frac{c}{b}$

$\frac{b}{a}$ $\frac{a}{c}$

23 If $\sec 60^\circ = 2$, then $\csc \square^\circ = 2$

If $\cot 60^\circ = \dfrac{1}{1.73}$ then $\tan 30^\circ = \square$

29

$d = a - b\cos\theta$

30 To summarize, we have:

$c^2 = p^2 + d^2$

$p = b\sin\theta$

$d = a - b\cos\theta$

By substituting among the equations at the left, try to get an expression in the box which does <u>not</u> contain p or d.

$c^2 = \square$

36

3rd and 4th

37 Same question about $\tan\theta$

(It is b/c in fig. 1a.)

In what quadrants will it be negative?

More useful relations:

$\sin\theta = \sqrt{1-\cos^2\theta}$

$\cos\theta = \sqrt{1-\sin^2\theta}$

$\sin 2\theta = 2\sin\theta\cos\theta$

Relations that would be useful:

$\$2 = \$1 \times \cot 5^\circ$!

$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$\sin(\theta-\phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$

$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

$\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

2

The small square is for your score.

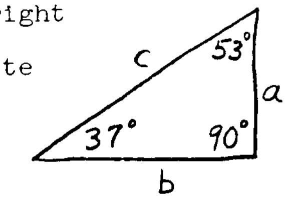
0 if you miss
2 if correct
1 if in between

Now proceed

SCORE HERE

3

Hypotenuses are found only in right triangles and in tropical Africa. The hypotenuse is the side opposite the right angle. Which side is the hypotenuse in this triangle?



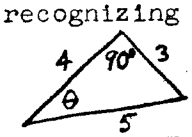
9

$\tan 37^\circ = \frac{3}{4}$

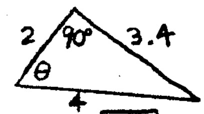
$\tan 30^\circ = \frac{.5}{.87}$

10

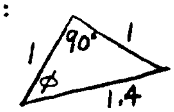
This and the next two frames are drills in quickly recognizing the sin, cos and tan. Fill in:



$\sin\theta = \boxed{}$



$\cos\theta = \boxed{}$



$\tan\phi = \boxed{}$

16

$\sin\theta = \frac{b}{a}$; $\cos\theta = \frac{c}{a}$; $\tan\theta = \frac{b}{c}$

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{b}{\cancel{a}}}{\frac{c}{\cancel{a}}} = \frac{b}{c}$$

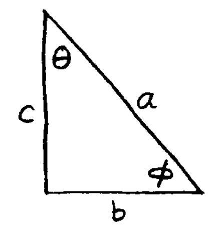
and $\frac{b}{c}$ is $\tan\theta$ QED!

17

Using the relations you see at the left, can you write tanθ in terms of sinø and cosø?

$\tan\theta = \frac{b}{c} =$

Remember, use sinø and cosø, not sinθ and cosθ.



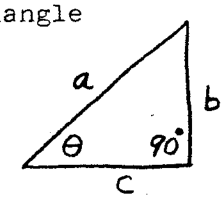
23

30°

$\frac{1}{1.73}$

24

In this triangle



$b = a\ \boxed{}$

$c = a\ \boxed{}$

30

This is the famous Cosine Law

$c^2 = b^2 + a^2 - 2ab\cos\theta$

(See footnote # 1, bottom of this page.)

31

In <u>any</u> triangle (not just a right triangle) if two sides <u>and</u> the included angle are given, you can use the Cosine Law to solve for the third side, which is opposite the known angle. It is extremely useful.

(No ques.)

37

2nd and 4th

It may help to think of

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

38

In fig. 2a we see that the tip of the "clock-hand", r, traces a circle. The part of the circle *a* is the <u>arc</u>. When $\theta = 90^\circ$, $\frac{a}{r} = \boxed{}$

<u>Footnote # 1</u>:

$$\begin{aligned} c^2 &= (b\sin\theta)^2 + (a - b\cos\theta)^2 \\ &= b^2\sin^2\theta + a^2 - 2ab\cos\theta + b^2\cos^2\theta \\ &= b^2(\sin^2\theta + \cos^2\theta) + a^2 - 2ab\cos\theta \end{aligned}$$

but $(\sin^2\theta + \cos^2\theta) = 1$

IMPORTANT: C IS THE SIDE <u>OPPOSITE θ</u>

Answers from Appendix

0	.5	.6	.7	.8	.87	1
1	.87	.8	.7	.6	.5	0
0	.58	.75	1	1.33	1.73	∞
∞	1.73	1.33	1	.75	.58	0
1	1.15	1.25	1.4	1.67	2	∞
∞	2	1.67	1.4	1.25	1.15	1

3

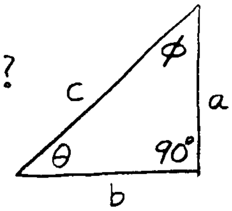
Side c

4

The side opposite θ is

a, b or c ?

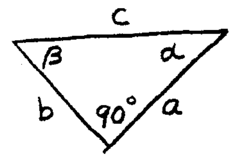
The side opposite $\varnothing$ is

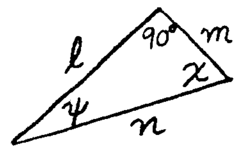


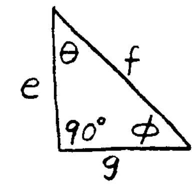
10

$\frac{3}{5}$ $\frac{2}{4}$ 1

11







$\tan\alpha =$ $\frac{m}{n} =$ $\frac{g}{f} =$

17

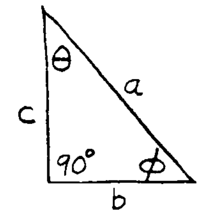
$$\frac{\cos\phi}{\sin\phi} = \frac{b/\cancel{a}}{c/\cancel{a}} = \frac{b}{c} = \tan\theta$$

or, you could have used: $\cos\varnothing = \sin\theta$ and $\sin\varnothing = \cos\theta$

18

Rewrite $\tan\varnothing$ in terms of $\sin\varnothing$ and $\cos\varnothing$. $\tan\varnothing =$ Using this and the answers at the left, can you express $\tan\theta$ in terms of $\tan\varnothing$?

$\tan\theta =$



24

$b = a\sin\theta$

$c = a\cos\theta$

25

From this answer (at left) write

$b^2 + c^2 = a^2$ (? ? ?).

(Remember that the square of $a\sin\theta$ is written $a^2\sin^2\theta$)

31

32

From here on we shall refer to the diagrams on the sheet labelled appendix at the end. You may find it best to tear it off and keep it before you.

In fig. 1a we are going to keep r fixed in length, and let θ vary. (The hypotenuse, r, will move like the hand of a clock.) What will $\cos\theta$ be when θ reaches 90°?

38

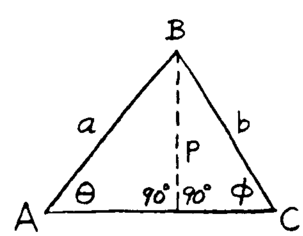
$$\frac{a}{r} = \frac{\frac{1}{4} \times 2\pi\cancel{r}}{\cancel{r}} = \frac{\pi}{2}$$

39

$\frac{a}{r}$ is used as a measure of the angle. The unit is radian. In the previous example, $\theta = \frac{\pi}{2}$ radians. That is, $\theta = 90^\circ = \frac{\pi}{2}$ radians.

How many radians make 360°?

Another useful relation: In the triangle ABC which is not a right triangle, $a\sin\theta = b\sin\varnothing$. This is easy to show, because both $a\sin\theta$ and $b\sin\varnothing$ are equal to the perpendicular, p, so they are equal to each other. This is the Law of Sines.



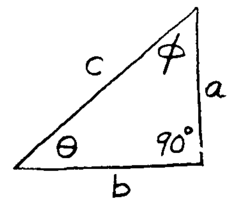
4

a

b

5

The side which is neither the hypotenuse nor the opposite side is the adjacent side. The side adjacent to Ø is $\boxed{}$



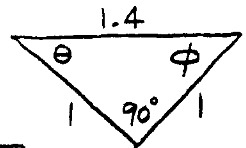
11

$\tan\alpha = \frac{b}{a}$

$\frac{m}{n} = \sin\psi$ or, $\cos\chi$

$\frac{g}{f} = \sin\theta$ or, $\cos\phi$

12



$\frac{1}{\tan\theta} = \boxed{}$ $\frac{1}{\cos\phi} = \boxed{}$ $\frac{1}{\sin\theta} = \boxed{}$

18

$\tan\phi = \frac{\sin\phi}{\cos\phi}$

$\tan\theta = \frac{1}{\tan\phi}$

19

Notice that complementary begins with co. The cosine is the sine of the complementary angle. There is also a cotangent (cot), and the cotangent is the tangent of the complementary angle.

(Remember in frame 15 we said that two complementary angles are those whose sum is 90°.)

No ques. →

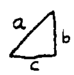
25

$b^2 + c^2 = a^2 (\sin^2\theta + \cos^2\theta)$

(See footnote # 2, at bottom of this page.)

26

Now recall that we started with the right triangle (shown below) of hippopotamus a, in which of course $a^2 = b^2 + c^2$. So if you apply this fact to the answer you just got, at the left, and do a little cancelling, you will have proved that:



$\sin^2\theta + \cos^2\theta = \boxed{}$

32

0

33

In the same figure (1a in appendix) compare the length c when θ is 80° and when θ is 100°. If you find no difference in length, is there any other difference, such as a matter of plus or minus direction?

39

$\frac{a}{r} = \frac{2\pi \cancel{r}}{\cancel{r}} = 2\pi$ radians

(a = $2\pi r$ or the full circumference)

40

Fig. 3 is just the same as fig. 2b, except that θ is much smaller. The interesting thing, when we make θ very small is that a and b become very nearly the same length. Since θ (in radians) is a/r and sinθ is b/r, you can see that as the angle gets smaller and smaller, θ and sinθ get more indistinguishable. In other words you can write θ instead of sinθ, if you know θ is small.

Footnote # 2: When writing the square of sinθ, we write $\sin^2\theta$ rather than $(\sin\theta)^2$. The latter would be correct, but would involve more symbols. We never write $\sin\theta^2$, as someone might think it meant the sin of the square of θ, which it does not. It is the square of the sin of θ we want, and that is correctly indicated by $\sin^2\theta$. This may seem like nit-picking, but there are some kinds of trouble it will keep us out of, believe me!

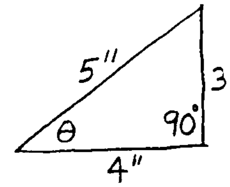
5

a

6

The side opposite θ divided by the hypotenuse is sinθ (see footnote)

In this right triangle, sinθ = $\boxed{}$



12

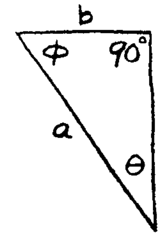
1 1.4 1.4

13

In this triangle,

sinθ = $\boxed{}$ and cosØ = $\boxed{}$

Therefore sin$\boxed{}$ = cos$\boxed{}$



19

Neat eh? There are two more members of the family of terms:

secant (sec) and cosecant (csc)

20

Secant (sec) is the name for the reciprocal of cosine. Sec is almost cos spelled backwards which helps some. I suppose soc was considered and found to be too flippant. Anyway, secθ = $\frac{1}{\cos\theta}$ and also, the <u>co</u>secant is the secant of the <u>co</u>mplementary angle.

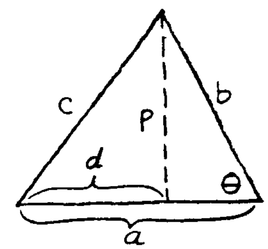
26

$$\sin^2\theta + \cos^2\theta = \frac{b^2 + c^2}{a^2} = 1$$

(Because $b^2 + c^2 = a^2$)

27

This is not a right triangle, but the dotted line p divides it into two right triangles. Can you write c^2 in terms of p^2 and d^2 ?



33

It has the same magnitude but it is positive when θ = 80° and negative when θ = 110°.

34

For what ranges of angles and in what quadrants will c be negative?

40

If you didn't see any possible use for that last proposition, you will change your mind when you get to centripetal acceleration!

41

Thinking in a similar way (θ still being very small) and looking at the same figures, can you infer what one might, with very little inaccuracy, substitute for cosθ and for tanθ?

cosθ ≙ tanθ ≙

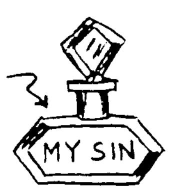
(the ≙ means "approximately equal")

<u>Footnote</u>: The pronunciation of sin is unexpected, when you come to think of it.

Thisaway

(Like <u>sign</u>)

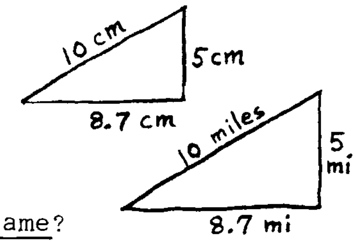
Not thisaway



6

$\frac{3''}{5''}$ or $\frac{3}{5}$

7 Sinθ is exactly the same in these two triangles. How can that be, when the triangles are of such different size (cm and miles)? In the first,



$\sin\theta = \frac{5\,cm}{10\,cm}$; in the second,

$\sin\theta =$? Why are they the <u>same</u>?

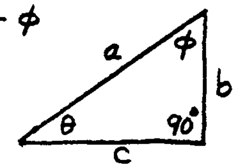
13

$\sin\theta = \frac{b}{a}$; $\cos\phi = \frac{b}{a}$

$\sin\theta = \cos\phi$

or, $\sin\phi = \cos\theta$

14 You were reminded in Review # 1 that the sum of the three angles inside a triangle is always 180 degrees. In this triangle, then, $\theta = \square\,deg - \phi$



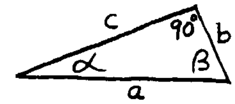
Combining this with the relation you just got for the answer to frame 13,

$\sin\phi = \cos(\square\,deg - \phi)$

20 I'm sure you like tidiness as much as I do. The co rule works fine. Too bad the other relations aren't as orderly. See footnote, bottom of this page.

21 Time for a few more frames of drill. Fill in the blanks

$\alpha + \beta = \square^\circ$



$\sin\alpha = \square\,\beta$

$\frac{1}{\tan\beta} = \tan(90^\circ - \square^\circ)$

27

$c^2 = p^2 + d^2$

28 Now write p in terms of b and a trig. function of θ. (a trig. "function" of θ is, for example, sinθ, or cosθ.)

34

90° to 270°

2nd and 3rd quadrants

35 You recall that cosθ is c/r . (r is of course always positive.) Now answer the following about cosθ.

	Range of angle	Quadrant
cosθ is positive:		
cosθ is negative:		

41

$\cos\theta \overset{a}{=} 1$

$\tan\theta \overset{a}{=} \theta$ in radians

42 This finishes the review. I suggest you keep these sheets, for the useful relations they contain. The table you filled out in the appendix will be especially useful, inasmuch as 95% of all physics problems are made with angles of 0, 30, 37, 45, 53, 60 and 90 degrees. (Makers of problems have little imagination.)

WHAT WAS YOUR SCORE? $\square$ Maximum possible: 70

<u>Footnote to 20</u>: Other relations among the six "functions" are:

$\sec\theta = \frac{1}{\cos\theta}$ (we had this) $\csc\theta = \frac{1}{\sin\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

They are useful, but not essential. You can always avoid using them, for example you can write 1/sinθ instead of writing cscθ. It is the same thing. So if you don't have time to learn everything, let some of these go for a while.

APPENDIX

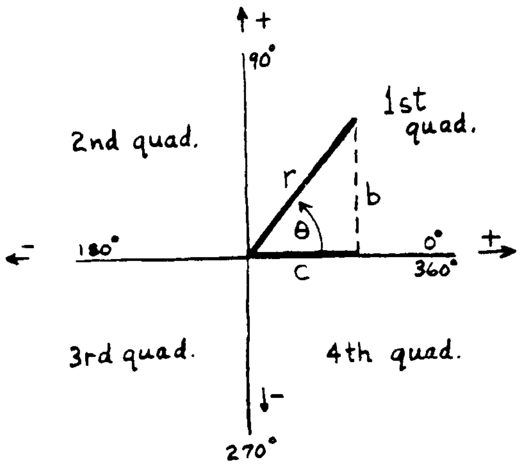


Fig. 1a

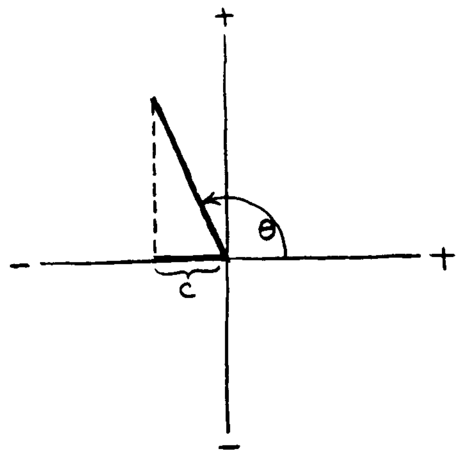


Fig. 1b

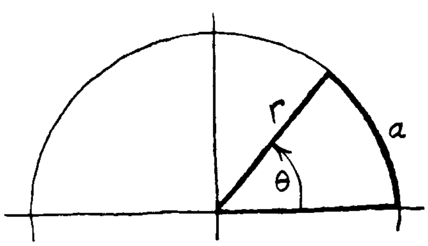


Fig. 2a

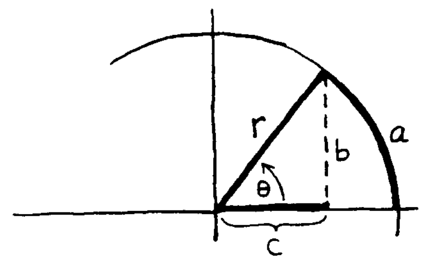


Fig. 2b

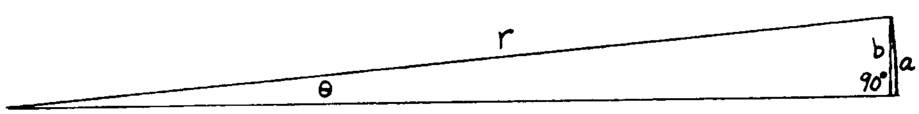


Fig. 3

Do-it-yourself table for the angles most used in physics problems. Working from the values given in the first line and the relations at the bottom of the first page, you can fill in the rest of the table. To check, see bottom of third page.

	0°	30°	37°	45°	53°	60°	90°
sin	0	.5	.6	.7	.8	.87	1.
cos							
tan							
cot							
sec							
csc							