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1

REVIEW #3: VECTORS

A booster shot for students in beginning college physics who need quick help on vectors. It goes no further than composition and resolution of vectors in a plane, with some applications to typical textbook physics problems. Now turn to the next page, and stay in the top strip. Go →

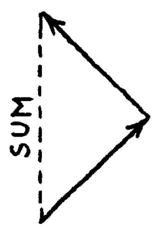
9

No. Some things don't have direction. They are called scalar quantities. See footnote #2, bottom of page 2.

10

We add vectors by stringing them end to end, tail to head. To do this, we can slide them around on the paper, as long as we preserve the directions and lengths. Try stringing these two. Connect the head of one to the tail of the other.

18



19

Knowing that each of the original vectors makes 45° with the horizontal and has magnitude 2 mi., find the magnitude of the sum vector and its direction. Simple geometry will do this for you. (It is a right-angled triangle.)

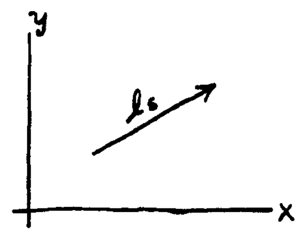
Magnitude: Direction:

27

$y_s = y_1 + y_2$

28

How do we find θ_s, the direction of the sum vector with respect to the x-axis? Here is the sum vector. Using it as a hypotenuse, make a right triangle under it. Label the sides ℓ_s, x_s and y_s, and designate the angle θ_s.



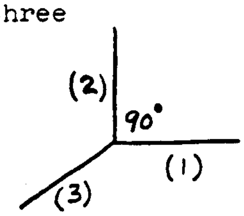
36

standard,

time-honored,

37

time-worn, moth-eaten example of all: three strings pulling from a tie point. Forces: (1) 87 lb., horizontal toward the right; (2) 50 lb. vertically upward. Represented as vectors, these forces are as shown in next frame. Turn.



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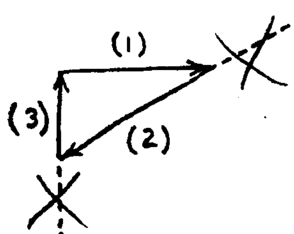


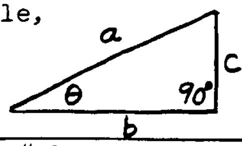
46

If the information about a vector that is not given is the direction, we can, as mentioned, draw it with a dot in the middle that indicates a pivot it can spin around, so:

Re-do the previous problem, except this time let the directions of (2) and (3) be the two unknown quantities. (no ques.)

Footnote # 1: Useful trig. relations (thrown in without extra charge).

Definitions: In a right triangle,



$\sin\theta = \frac{c}{a}$ $\cos\theta = \frac{b}{a}$ $\tan\theta = \frac{c}{b}$

$\sin 30^\circ = .5 = \cos 60^\circ$

$\sin 45^\circ = .7 = \cos 45^\circ$

$\sin 60^\circ = .87 = \cos 30^\circ$

$\tan 30^\circ = .58$

$\tan 45^\circ = 1.0$

$\tan 60^\circ = 1.7$

1

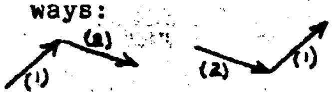
We move along from one page to the next, at the same level; we do not move down the page.

2

Don't make hard work of this; it's not an exam. If you follow instructions, your facility at handling vectors cannot help but improve. Do what is asked <u>with pencil</u>; don't just think your answers. Then turn <u>to the next</u> page, staying at the same level. The correct answer will be given in the space at the left, numbered to correspond to the question. Now turn the page, staying in the top strip.

10

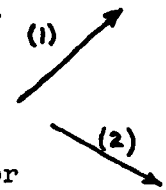
You had two possible ways:



Easy? Just like occupational therapy!

11

Let's call such a string a vector <u>chain</u>. The <u>sum</u> is the <u>single</u> <u>vector</u> we can draw which has its tail at the tail of the chain and its head at the head of the chain. String these, connecting head of (1) to tail of (2). Draw the vector which is the sum.

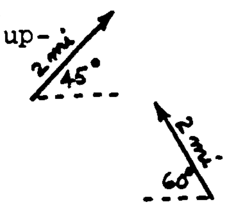


19

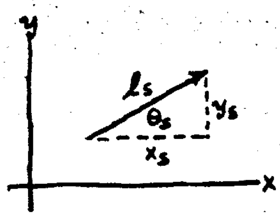
Magnitude: $2\sqrt{2}$ or 2.8 mi.
Direction: up, 90° to horiz.

20

That was a Mickeymouse. But a slight change in one angle will upset this happy state of affairs. String these and draw the sum (don't solve numerically yet).



28



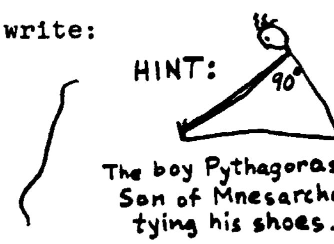
29

Looking at this right triangle, write: ℓ_s in terms of x_s and y_s.

$\ell_s =$

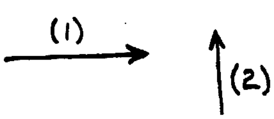
Also, write $\tan\theta_s$ in terms of x_s and y_s.

$\tan\theta_s =$



The boy Pythagoras, Son of Mnesarchus, tying his shoes.

37

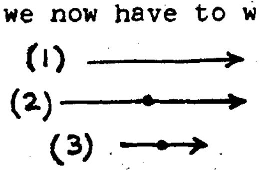


38

Now hook the two vectors at the left into a chain, and <u>invent</u> a third vector to add to the two, which will make the sum of all three <u>zero</u>. That means that after you have added the third, the beginning and end points of the chain of three will be on top of each other.

46

Here are the vectors we now have to work with:



47

Assemble the vectors you have at the left into a closed figure, rotating (2) and (3) in any way necessary.

<u>Footnote #2</u>

Examples of things that have both direction and magnitude (<u>vectors</u>).

Velocity
Acceleration
Displacement
Current
Electric Field
Force

Examples of things that have magnitude only (<u>scalars</u>).

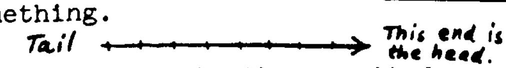
Time
Density
Temperature
Mass
Energy
Power

2

The small square is for your score.
0 if you miss
2 if correct
1 if in between

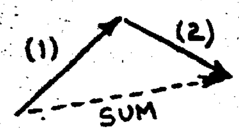
Now proceed HERE

3 By drawing an arrow, we can represent the direction and magnitude of something.



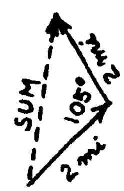
The length of the arrow represents the magnitude, and of course the direction of the arrow represents the direction. Such an arrow is a vector.

11



12 An important point to get is that the order in which the vectors are strung together doesn't count for anything. Add the same two in the other order and see if the sum is any different.

20



21 Now solving for the magnitude and direction of the SUM, at left, presents no great problem. It is the familiar case of the triangle in which two sides and the included angle are known. If you have done my TRIG REVIEW you will know that if you will apply the LAW OF COSINES you will get the answer in a jiffy--or a few jiffies. No ques.

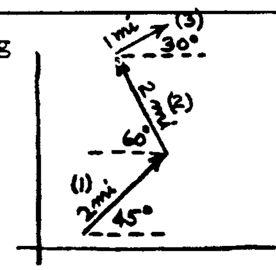
Turn →

29

$$\ell_s = \sqrt{x_s^2 + y_s^2}$$

$$\tan \theta_s = y_s / x_s$$

30 Try working with three. Here are the three we had in frame 22.



Complete this table of components.

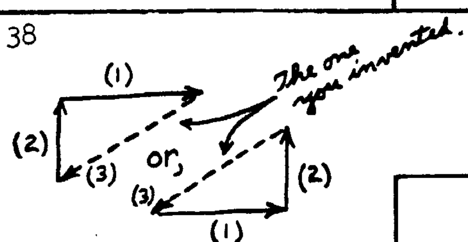
$x_1 = 2\cos 45^\circ$ $y_1 =$

$x_2 =$ $y_2 = 2\sin 60^\circ$

$x_3 =$ $y_3 =$

Watch for the one that is minus!

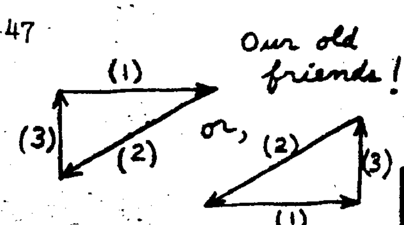
38



39 The dotted vector can be none other than the force in the third string, which is necessary for equilibrium to exist. The vector sum is now zero, which is the condition for equilibrium.

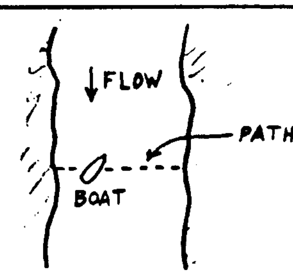
Keep a clear distinction between (a) the sum of several vectors, and (b) the vector you have to add to these vectors to make the sum zero. (a) and (b) are not the same vector, but they are related. How?

47



48 We have not covered all of the possibilities, but I don't want to bore you. As a matter of fact, this type of problem (the assembly of a batch of vectors into a closed figure) can be solved if any two pieces of information are withheld as unknowns. (no ques.)

(Cont'd from p. 9.) Time for one more? This time we'll do velocities. A river 1 mi. wide is flowing at 2 miles per hour north to south. A man is in a rowboat and his rowing speed is 4 mph. He wishes to cross the river and land at a point exactly opposite his starting point. In what direction should his boat be pointed? How long will it take him to cross?



cont'd. →

3

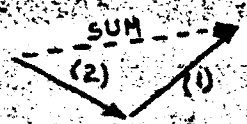
You can fix this term in mind by remembering the title of the tune the Michigan Band plays to give direction and magnitude to the boys on the field.

4

Suppose a dog is chasing a cat, both moving left to right, the dog going 30 mi/hr and the cat 20 mi/hr.

Try representing the motions of the two animals by drawing vectors, making each vector 1/4 inch in length for each 10 mi/hr speed of the animal.

12



The sum is the same

13

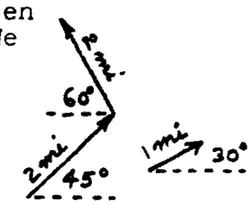
Suppose friend Joe leaves home, goes to the office, which is 2 mi NE of home; then to the BOARD MEETING (name of a saloon) which is 1.4 mi S of the office; then home which is 1.4 mi W of BOARD MEETING. Draw these displacements, in the order mentioned.

21

If we were never going to have more than two vectors to add, we could solve the problem we have just talked about, with a flourish and then quit.

22

But physics isn't that simple. We often have to add up more than two vectors. We need a more versatile method than the Law of Cosines. So on we go! To the two vectors we have (shown at right), tack on a third, which is magnitude 1 mi and is 30° to the horizontal. Draw the SUM of the three.



30

$x_1 = 2\cos 45^\circ$ $y_1 = 2\sin 45^\circ$

$x_2 = -2\cos 60^\circ$ $y_2 = 2\sin 60^\circ$

$x_3 = 1\cos 30^\circ$ $y_3 = 1\sin 30^\circ$

31

Write the sum of the x-components and the sum of the y-components, numerically. Refer to the trig table in the footnote on the first page if necessary but try to do it from your head.

$x_s =$

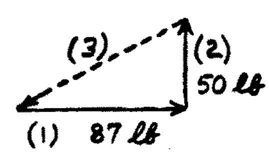
$y_s =$

39

One of them would be the negative of the other. (Head on opposite end.)

40

Making use of the fact that (2) is ⊥ to (1), thus forming a right triangle, write down the magnitude of (3). Don't bother to extract the square-root.

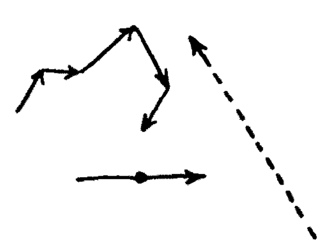


48

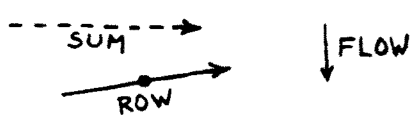
Often we have more than three vectors, to be assembled into a closed figure.

49

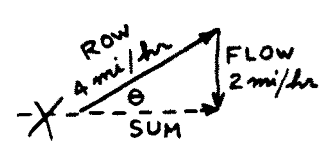
Try assembling these:
Here the two unknowns are one of the directions and one of the magnitudes, as indicated. Try fitting all of these vectors into a closed figure (right here)



(Cont'd from p. 3.)
The vector representing his actual velocity must be horizontal. It is the sum of the other two. The three vectors are:



They fit together this way. θ is seen to be 30° and the magnitude of the sum is 4 cos30° or 3.48. The time to cross is the width of the river (1 mi) divided by the velocity (3.48 mi/hr). It is .29 hours.

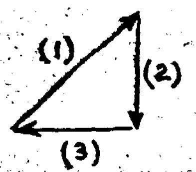


Continue →

4 You probably drew them this way:

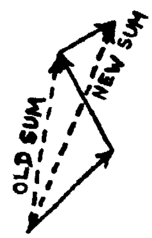
5 But would the vectors have been just as correct if drawn this way?

13



14 If, (as I planned it) the head of (3) falls right at the tail of (1), what would you say is the total displacement of Joe for the day? Let me caution you: the total displacement is like the sum; it is the distance and direction from the starting point of the chain to the ending point of the chain. Not how far Joe walked.

22



23 We want to find the magnitude of the new SUM vector, and also the angle it makes with the horizontal. The way we will go about it is by the method of components. Actually, this is just elementary analytical geometry. We start with a set of coordinate axes, the x and y axes, as shown in the next frame. Turn page; no ques.

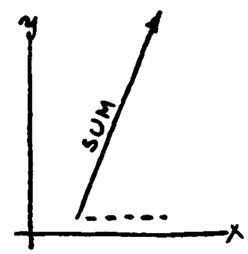
31

x_s = 1.27 mi

y_s = 3.64 mi

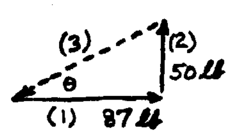
32 You now have the values of x_s and y_s, so you can draw a right triangle under the sum vector, as before. Do so. Label Θ_s. Write in the numerical values of x_s and y_s.

What is, tan Θ_s numerically? $\tan\theta_s = \frac{?}{?}$



40

$\sqrt{87^2 + 50^2}$



41 Again looking at the vector figure as a right triangle, what is the numerical value of the tangent of the angle between sides (1) and (3)?

tan Θ = ☐

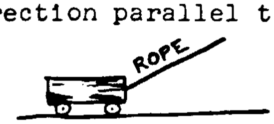
49

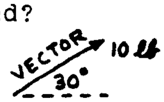
You got this? Good!

50 You will find a whole class of text-book problems in which (a) the physics of the situation says that the sum of the forces, or velocities, or displacements, etc. should be zero and (b) all but two pieces of information are given. These are "THE CASE OF THE TWO MISSING PIECES OF DATA." (For the complete file of them telephone Della Street, on her private line.)

Onward →

There is an important class of problems which involve the finding of a component of a vector quantity. Example: A rope pulls with a force of 10 lb on a car. The rope is at 30° to the road. What is the component of force exerted in the direction parallel to the road?





Here is a general, iron-clad, never-failing rule: Multiply the magnitude of the vector by the cosine of the angle between the vector and the direction in which you want the component. What do you get for the problem of the rope and car at the left? Do it.

Horizontal comp. = ? lb.

go →

5

Yes. Because the vectors represent magnitude (mi/hr in this case) and direction only.

6

The information as to who is chasing whom is not contained in a vector. (There are schemes by which the placement of the arrow on the page or picture can be used to convey other types of information, and I will show some at the end of this review. But forget that for now; we are going to stick to the strict meaning of a vector, which is magnitude and direction.

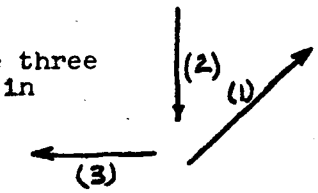
Go →

14

Zero. At the end of the day he is right where he started.

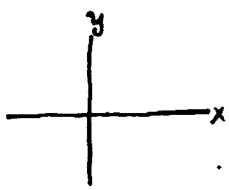
15

String the same three vectors together in another order.



These are the three.

23

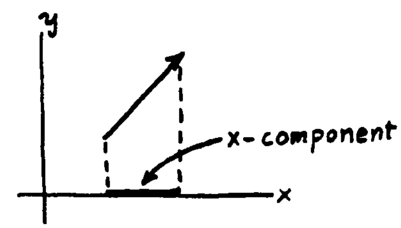


Coordinate axes

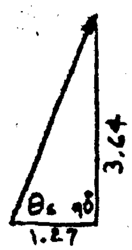
24

Any vector will have an x-projection, or x-component, so:

(Like its shadow on the ground at mid-day.)



32



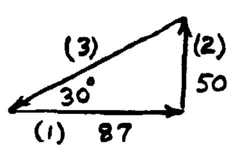
$\tan \theta_s = \frac{3.64}{1.27}$

33

Next, using the Pythagorean theorem, write the magnitude of the sum vector. (Leave it as a square root.)

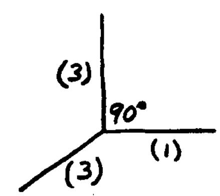
41

Tan θ = 50/87



42

In case you have now forgotten what we were trying to find, it was the magnitude and direction of the force in string (3). Both have now been found.



(No ques.)

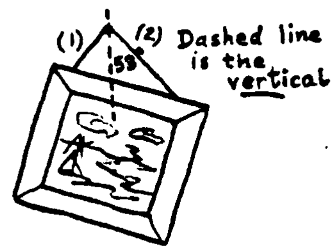
50

Della's file contains among others, "The Case of the Cockeyed Picture."

Since you are now tired after 50 frames I'll do most of the work for you as dessert.

51

The data: weight, acting downward of course, 10 lb; tension in string (2), 6 lb; direction of (2), 53° from vertical. The direction and the tension of string (1) are the two unknowns.

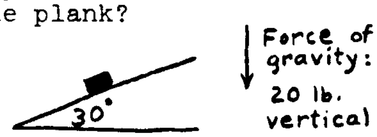


Ans. to last prob:

8.7 lb.

o? 1? 2?

Try another. A block sits on a sloping plank. The block weighs 20 lb and the plank is at 30° to the horizontal. What is the component of force normal to (⊥ to) the plank?



Find it.

Solution: The direction of the component we want here is ⊥ to the plank, or at 30° to the force vector (gravity). So the component is:

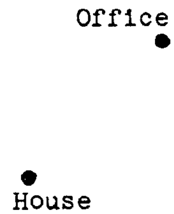
$20\cos 30^\circ = 17$ lb.

OK?

6

So we can't help the cat's chances of escaping by moving the vectors around!

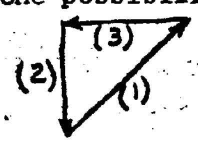
7



Now suppose Joe Handlebars walks from his house to his office, which is 2 mi from his house and exactly N.E. of his house. Can you represent (at right) his change in position (displacement) by a vector?

Draw here:

15 One possibility:

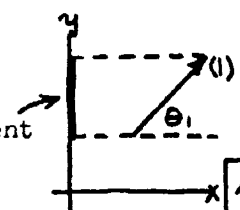


Order makes no diff. Sum is again zero.

16 Of course Joe would have missed the office, or the BOARD MEETING or both, if he had made his displacements in any other order. What a day he would have had, tramping through the fields! But regardless of the order, can you guarantee that he got back home at the end?

24

And a y-component



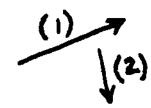
25 Let's refer to the vector (at left) as (1) and adopt the following notation:
ℓ_1, its length.
θ_1, the angle it makes with the horizontal.
x_1, its x-component.
y_1, its y-component.
Assemble a right triangle having sides ℓ_1, x_1 and y_1.

33

$\sqrt{3.64^2 + 1.27^2}$

So we have solved the problem of frame 22, for direction and magnitude.

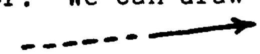
34 One thing we haven't said much about: subtraction. If you can add you can subtract. Remember grade school? "To subtract, change the sign and add." How change the sign of a vector? Move its head to the other end! Try it. Subtract (1) from (2).



42

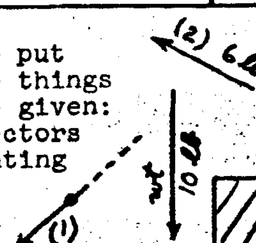
Notice that in the previous problem there were two unknown quantities: magnitude and direction of force (3).

43 Sometimes we are given the <u>direction</u> <u>but</u> <u>not</u> <u>the</u> <u>magnitude</u> of a certain vector. We can draw it with an indefinite length, so:

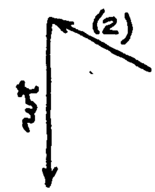


If we are given the <u>magnitude</u> <u>but</u> <u>not</u> <u>the</u> <u>direction</u> of a vector, we can draw it as if it has a pivot around which can turn, so:

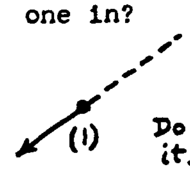
51 First we put down the things that are given: these vectors representing forces.



52 How do we compose a closed triangle from these? We can change the length and direction of (1) to suit. (They are our two unknowns).



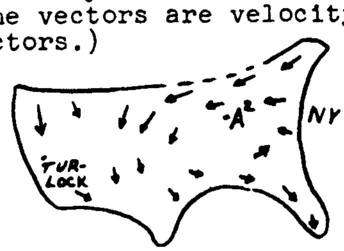
Where can we fit this one in?



Do it.

As a parting shot, I want to show you some ways you can draw vectors to convey more information than magnitude and direction. You will have to be the judge as to whether I was a liar in the first few frames! (I will of course claim I was not.)

What do you make of this? (The vectors are velocity vectors.)



It is the wind direction and speed, on Nov. 4. The placement of each vector on the map tells you what locality registered that wind velocity.

7

The vector points NE (45^o from the horizontal) and it is 2 units in length. (Each unit represents a mile.)

8

Notice that the vector doesn't give any information as to where Joe's home, or office, is. It tells only in what direction he moved and how many miles. Can you draw a vector to represent Joe's return journey?

16

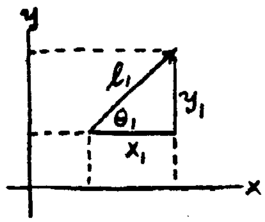
Of course. The sum of his displacements is zero, regardless of the order in which they are added. His total displacement is still zero.

17

Let's continue with adding vectors, but now let's not only draw the sum vector graphically but let's find its direction and magnitude, in terms of an angle and a number. No question this time.

onward →

25



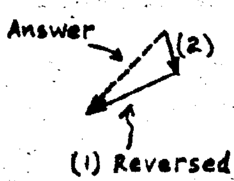
26

Now write an expression for x_1 and y_1 in terms of ℓ_1 and the sin or cos of Θ_1.

x_1 =

y_1 =

34



35

So far we have seen what a vector represents (and what it does not represent) and we have seen how to add or subtract any number of vectors. Now let's apply this to some situations that are more like physics problems.

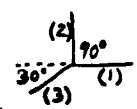
(No ques.) Go →

43

The unknowns in a problem might be, for example, the direction of one vector and the magnitude of another.

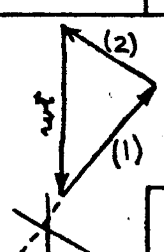
44

Suppose, in the problem of the three strings I had given the values of all the quantities except the magnitude of (2) and the magnitude of (3). Now draw (not hooked in a chain) the three vectors representing the given information. (All directions are known; two will have dashed tails.)



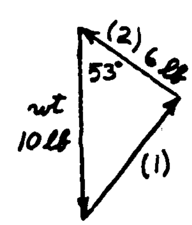
52

We have to swing (1) clear around on its pivot and then trim off the excess.

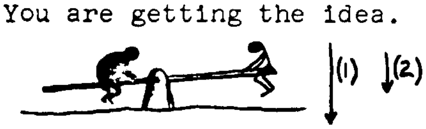


53

The length and direction of (1) have now been fixed, in the drawing at least. They were the two unknowns. To push it to the bitter end let's find the quantities numerically. We put in the angle we were given. (53°)

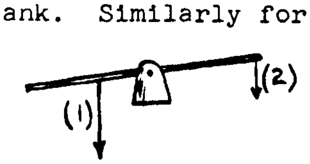


You are getting the idea.



Vector (1) is the downward force exerted by the fat man; (2) by the thin man.

Here we place (1) so as to tell where the fat man's weight is exerted on the plank. Similarly for (2).



One last one.

Here each tells the speed and direction a car had, as it passed two points on the road, A and B.

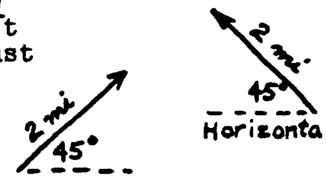
8

9 We hope we have it settled that a vector means the same wherever we draw it so long as length and direction are preserved. The suit Joe is wearing cost $50. Can you represent that by a vector?

Go back to first page, second row.

17 So dust off your trig! (See footnote #1 at the bottom of page 1.)

18 First, string these two vectors and draw the sum vector (don't solve numerically just yet.)

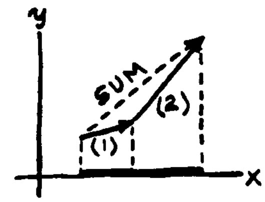


26

$$x_1 = \ell_1 \cos \theta_1$$

$$y_1 = \ell_1 \sin \theta_1$$

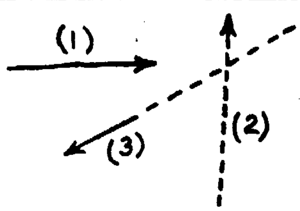
27 When there are two vectors to be added, it's easy to see that x_s (the x-component of the sum vector) is the same as $x_1 + x_2$, the sum of the x-components of the two vectors. What does y_s = ?



35 One of the first physics applications of vectors concerns forces in strings.

36 The text book says that if three strings are tied together at one point, and if there is some tension in each string, and if the whole thing is at equilibrium, that is, if it has no tendency to move, then the vector sum of the forces acting on the tie point is zero. Let's see how this works out, by taking the most

44

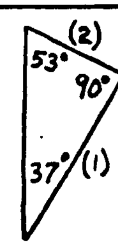


45 Now the only problem is where to trim off the lengths of (2) and (3). Perhaps when you try to fit them into a closed figure the question will answer itself. Try it and see.

53 We have a triangle with two sides and the included angle known. At worst we could find the other side and the other angle by the Law of Cosines.

54 If this is a textbook problem, we expect a neat solution. We look for some simple relation. Aha! Our old friend the 3-4-5 triangle! Therefore tension (1):tension (2) as 4:3, or force (1) is 8 lb. The angle of string (1) is of course 37° to the vertical. QED.

NOW GO TO THE BOTTOM FRAME OF THE 3rd PAGE.



We can go further: to return to the dog chasing the cat (frame 4) we not only can place the dog-velocity vector behind the cat-velocity vector; we can color all dog-vectors black and all cat-vectors pink. So, many gimmicks can be used to convey information in addition to direction and magnitude, which are the properties of the vectors themselves. Well, did you decide I misled you in the beginning?

Now, as they say on TV,

"THANKS FOR BEING WITH US."
And let me add my own hope, that you will still be with us after the Mid-term! I hope I have improved your chances.

How well did you do? (Maximum possible was 72 points.)