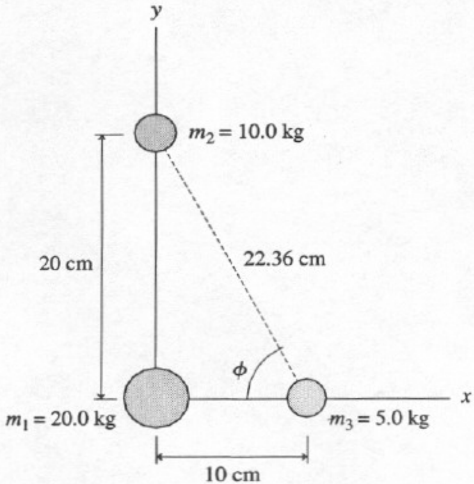
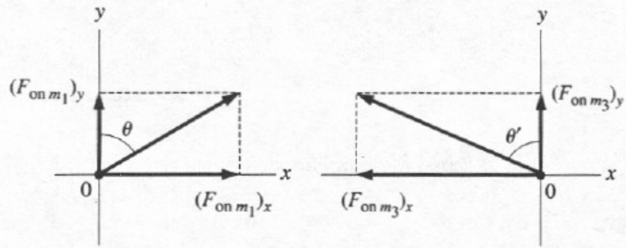
12.34. Visualize: We placed the origin of the coordinate system on the 20.0 kg mass (m_1) so that the 5.0 kg mass (m_3) is on the x-axis and the 10.0 kg mass (m_2) is on the y-axis.





Solve: **(a)** The forces acting on the 20 kg mass (m_1) are

$$\vec{F}_{m_2 \text{ on } m_1} = \frac{Gm_1m_2}{r_{12}^2}\hat{j} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(20.0\ \text{kg})(10.0\ \text{kg})}{(0.20\ \text{m})^2}\hat{j} = 3.335\times10^{-7}\hat{j}\ \text{N}$$

$$\vec{F}_{m_3 \text{ on } m_1} = \frac{Gm_1m_3}{r_{13}^2}\hat{i} = \frac{(6.67\times10^{-7}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(20.0\ \text{kg})(5.0\ \text{kg})}{(0.10\ \text{m})^2}\hat{i} = 6.67\times10^{-7}\hat{i}\ \text{N}$$

$$\vec{F}_{\text{on } m_1} = 6.67\times10^{-7}\hat{i}\ \text{N} + 3.335\times10^{-7}\hat{j}\ \text{N} \Rightarrow F_{\text{on } m_1} = 7.46\times10^{-7}\ \text{N}$$

$$\theta = \tan^{-1}\left(\frac{F_{m_3 \text{ on } m_1}}{F_{m_2 \text{ on } m_1}}\right) = \tan^{-1}\left(\frac{6.67\times10^{-7}\ \text{N}}{3.335\times10^{-7}\ \text{N}}\right) = 63.4^\circ$$

Thus the force is $\vec{F}_{\text{on } m_1} = \left(7.46\times10^{-7}\ \text{N},\ 63.4^\circ \text{ cw from the } y\text{-axis}\right)$.

(b) The forces acting on the 5 kg mass (m_3) are

$$\vec{F}_{m_1 \text{ on } m_3} = -\vec{F}_{m_3 \text{ on } m_1} = -6.67\times10^{-7}\hat{i}\text{ N}$$

$$F_{m_2 \text{ on } m_3} = \frac{Gm_2m_3}{r_{23}^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(10.0\text{ kg})(5.0\text{ kg})}{[(0.20)^2+(0.10)^2]\text{ m}} = 6.67\times10^{-8}\text{ N}$$

$$\vec{F}_{m_2 \text{ on } m_3} = -(6.67\times10^{-8}\text{ N})\cos\phi\hat{i} + (6.67\times10^{-8}\text{ N})\sin\phi\hat{j}$$

$$= -(6.67\times10^{-8}\text{ N})\left(\frac{10\text{ cm}}{22.36\text{ cm}}\right)\hat{i} + (6.67\times10^{-8}\text{ N})\left(\frac{20\text{ cm}}{22.36\text{ cm}}\right)\hat{j}$$

$$= -(2.983\times10^{-8}\text{ N})\hat{i} + (5.966\times10^{-8}\text{ N})\hat{j}$$

$$\vec{F}_{\text{on } m_3} = -6.968\times10^{-7}\hat{i}\text{ N} + 5.966\times10^{-8}\hat{j}\text{ N}$$

$$F_{\text{on } m_3} = \sqrt{(-6.968\times10^{-7}\text{ N})^2 + (5.966\times10^{-8}\text{ N})^2} = 6.99\times10^{-7}\text{ N}$$

$$\theta' = \tan^{-1}\left(\frac{6.968\times10^{-7}\text{ N}}{5.966\times10^{-8}\text{ N}}\right) = 85.1^\circ$$

Thus $\vec{F}_{\text{on } m_3} = (6.99\times10^{-7}\text{ N, } 85.1^\circ\text{ ccw from the } y\text{-axis})$.

(b) In the flat-earth approximation, $U_g = mgy$. The energy conservation equation thus becomes

$$\frac{1}{2}m_o v_2^2 + m_o g y_2 = \frac{1}{2}m_o v_1^2 + m_o g y_1$$

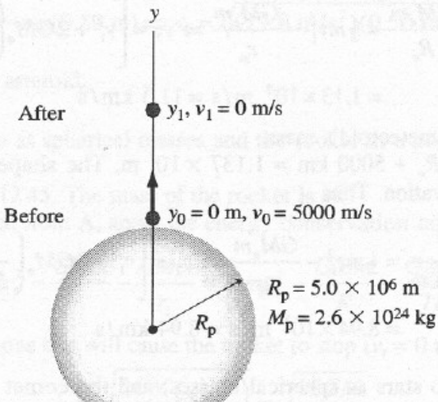
$$\Rightarrow v_2 = \sqrt{v_1^2 + 2g(y_1 - y_2)} = \sqrt{2(9.80\ \text{m/s}^2)(5\times10^5\ \text{m} - 0\ \text{m})} = 3130\ \text{m/s}$$

(c) The percent error in the flat-earth calculation is

$$\frac{3130\ \text{m/s} - 3020\ \text{m/s}}{3020\ \text{m/s}} \approx 3.6\%$$

(d) No, because the hammer will have a forward motion such that while "falling" it will continue to circle the earth.

12.41. Model: Model the planet (p) as a spherical mass and the projectile as a point mass. This is an isolated system, so mechanical energy is conserved.
Visualize: The projectile of mass m was launched on the surface of the planet with an initial velocity v_0.



Solve: **(a)** The energy conservation equation $K_1 + U_1 = K_0 + U_0$ is

$$\frac{1}{2}mv_1^2 - \frac{GM_p m}{R_p + y_1} = \frac{1}{2}mv_0^2 - \frac{GM_p m}{R_p}$$

$$\Rightarrow y_1 = \left[\frac{1}{R_p} - \frac{v_0^2}{2GM_p}\right]^{-1} - R_e = 2.82\times10^6\ \text{m}$$

(b) Using the energy conservation equation $K_1 + U_1 = K_0 + U_0$ with $y_1 = 1000\ \text{km} = 100\times10^6\ \text{m}$:

$$\frac{1}{2}mv_1^2 - \frac{GM_p m}{R_p + y_1} = \frac{1}{2}mv_0^2 - \frac{GM_p m}{R_p}$$

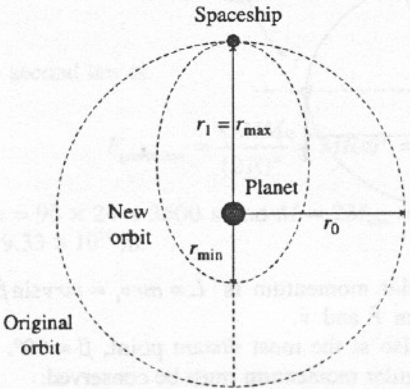
$$\Rightarrow v_1 = \left[v_0^2 + 2GM_p\left(\frac{1}{R_p + y_1} - \frac{1}{R_p}\right)\right]^{1/2}$$

$$= \left[(5000\ \text{m/s})^2 + 2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.6\times10^{24}\ \text{kg})\left(\frac{1}{(5.0\times10^6\ \text{m} + 1.0\times10^6\ \text{m})} - \frac{1}{5.0\times10^6\ \text{m}}\right)\right]^{1/2}$$

$$= 3670\ \text{m/s}$$

12.66. Model: Model the planet (p) as a spherical mass and the spaceship (s) as a point mass.

Visualize:



Solve: (a) For the circular motion of the spaceship around the planet,

$$\frac{GM_p m_s}{r_0^2} = \frac{mv_0^2}{r_0} \Rightarrow v_0 = \sqrt{\frac{GM_p}{r_0}}$$

Immediately after the rockets were fired $v_1 = v_0/2$ and $r_1 = r_0$. Therefore,

$$v_1 = \frac{1}{2}\sqrt{\frac{GM_p}{r_0}}$$

(b) The spaceship's maximum distance is $r_{max} = r_0$. Its minimum distance occurs at the other end of the ellipse. The energy at the firing point is equal to the energy at the other end of the elliptical trajectory. That is,

$$\frac{1}{2}m_s v_1^2 - \frac{GM_p m_s}{r_1} = \frac{1}{2}m_s v_2^2 - \frac{GM_p m_s}{r_2}$$

Since the angular momentum at these two ends is conserved, we have

$$mv_1 r_1 = mv_2 r_2 \Rightarrow v_2 = v_1(r_1/r_2)$$

With this expression for v_2, the energy equation simplifies to

$$\frac{1}{2}v_1^2 - \frac{GM_p}{r_1} = \frac{1}{2}v_1^2(r_1/r_2)^2 - \frac{GM_p}{r_2}$$

Using $r_1 = r_0$ and $v_1 = v_0/2 = \frac{1}{2}\sqrt{\frac{GM_p}{r_0}}$,

$$\frac{1}{2}\left(\frac{1}{4}\frac{GM_p}{r_0}\right) - \frac{GM_p}{r_0} = \frac{1}{2}\left(\frac{1}{4}\frac{GM_p}{r_0}\right)\frac{r_0^2}{r_2^2} - \frac{GM_p}{r_2} \Rightarrow \frac{1}{8r_0} - \frac{1}{r_0} = \frac{r_0}{8r_2^2} - \frac{1}{r_2}$$

$$\Rightarrow \frac{7}{8r_0} + \frac{r_0}{8r_2^2} - \frac{1}{r_2} = 0 \Rightarrow \left(\frac{7}{8r_0}\right)r_2^2 - r_2 + \frac{r_0}{8} = 0$$

The solutions are $r_2 = r_0$ (the initial distance) and $r_2 = r_0/7$. Thus the minimum distance is $r_{min} = r_0/7$.

12.67. Visualize: