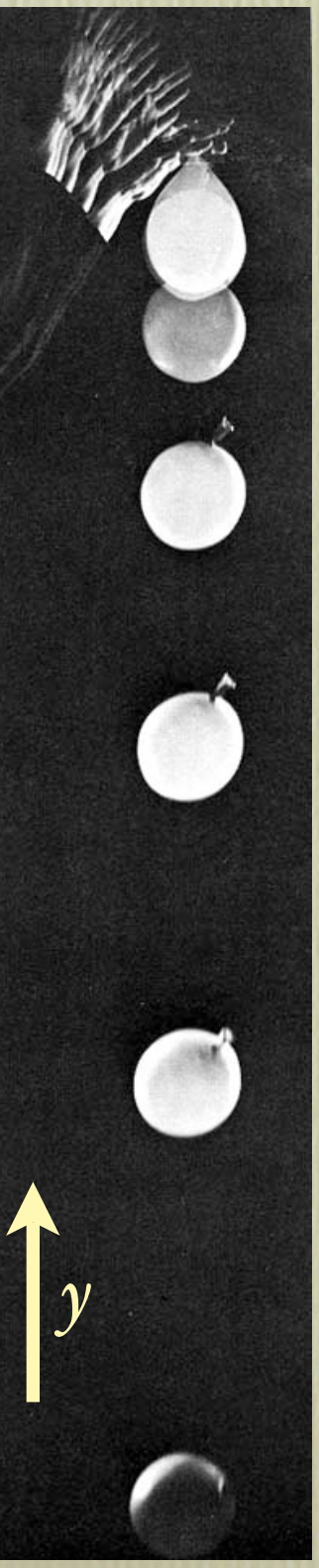
Mechanical Energy

The currency of the universe.

Free Fall



$$F = ma$$

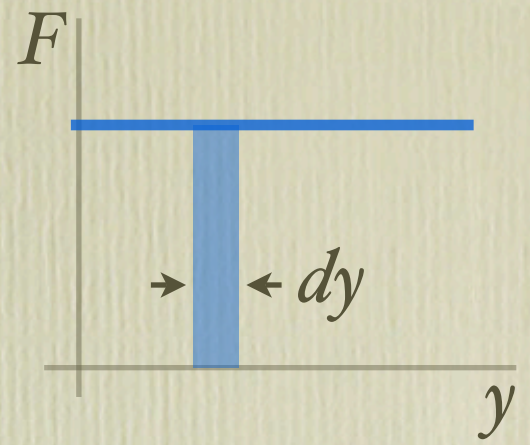
$$F = m(dv/dt)$$

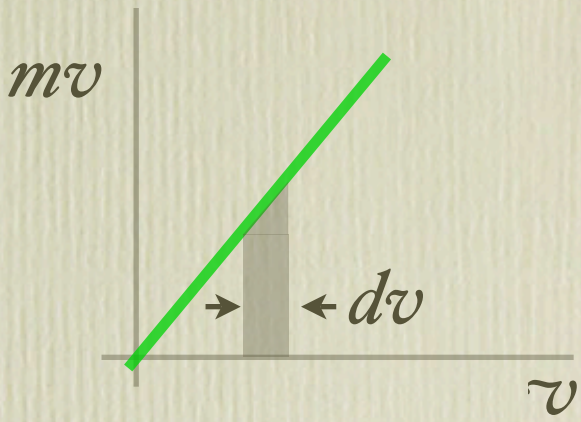
$$Fdt = mdv$$

$$(Fdt)v = (mdv)v$$

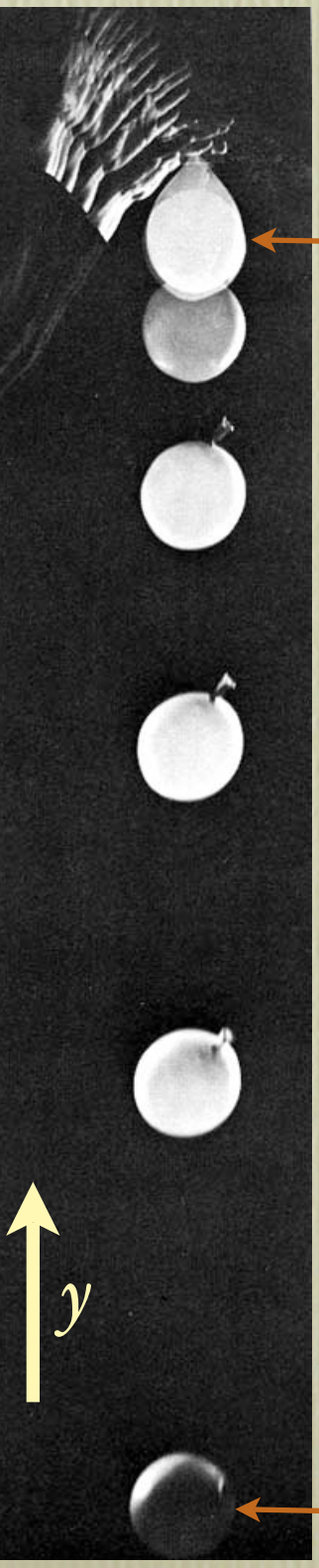
$$Fdt\frac{dy}{dt} = mvdv$$

$$\boxed{Fdy = (mv)dv}$$



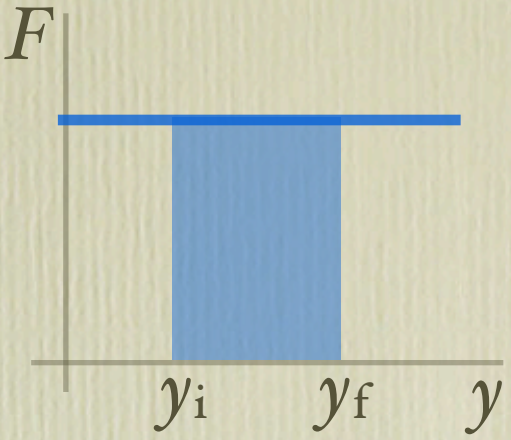


Free Fall

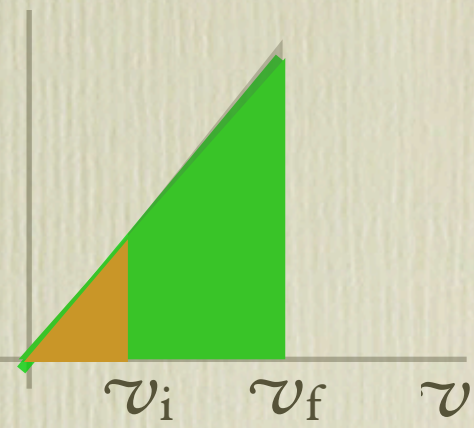


$$F dy = (mv) dv$$

$$\int_{y_i}^{y_f} F dy = \int_{v_i}^{v_f} mv dv$$



$$\int_{y_i}^{y_f} F dy = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$



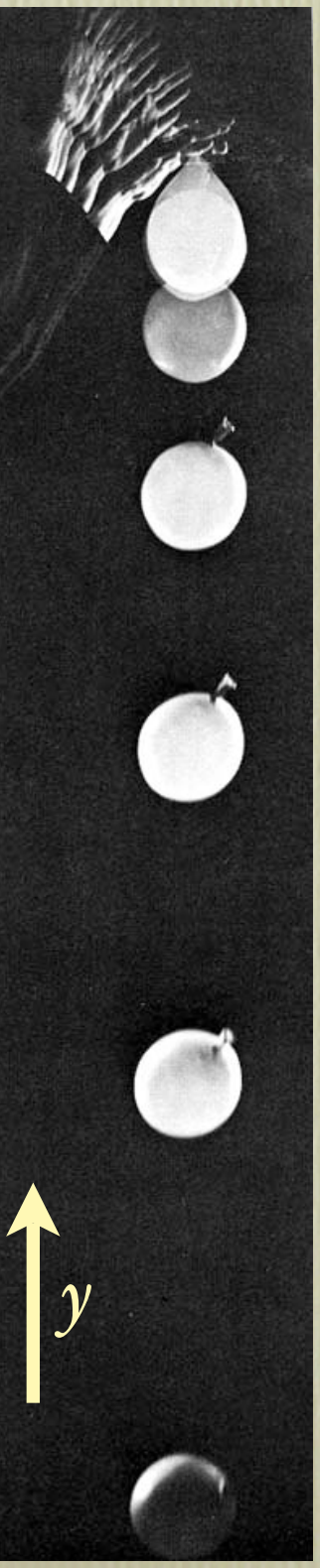
The above is true in general, for any $F(y)$.

For free fall, where $F = -mg$ is constant:

$$-mgy_f + mgy_i = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

gravitational potential energy, U_g

kinetic energy, K



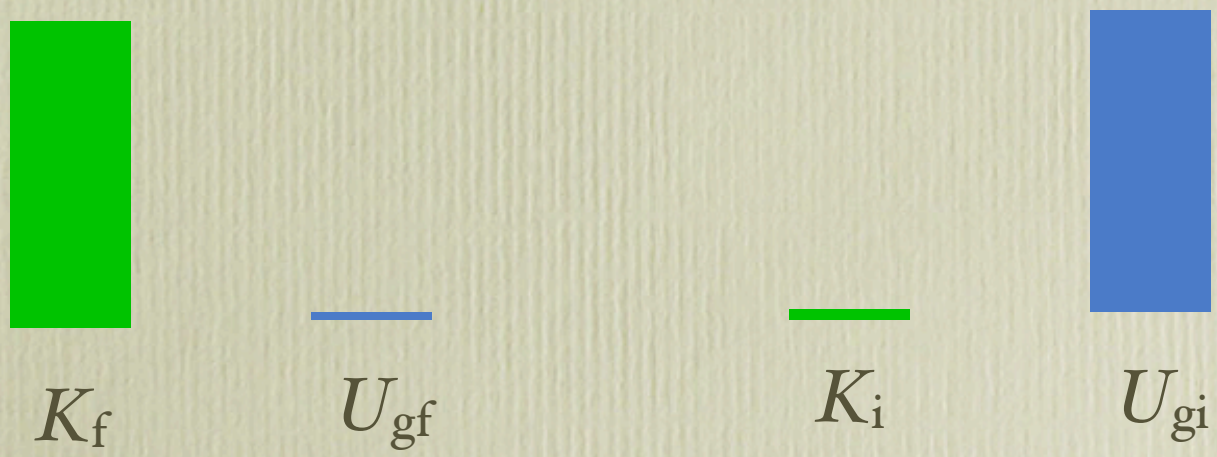
Free Fall

$$-mgy_f + mgy_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

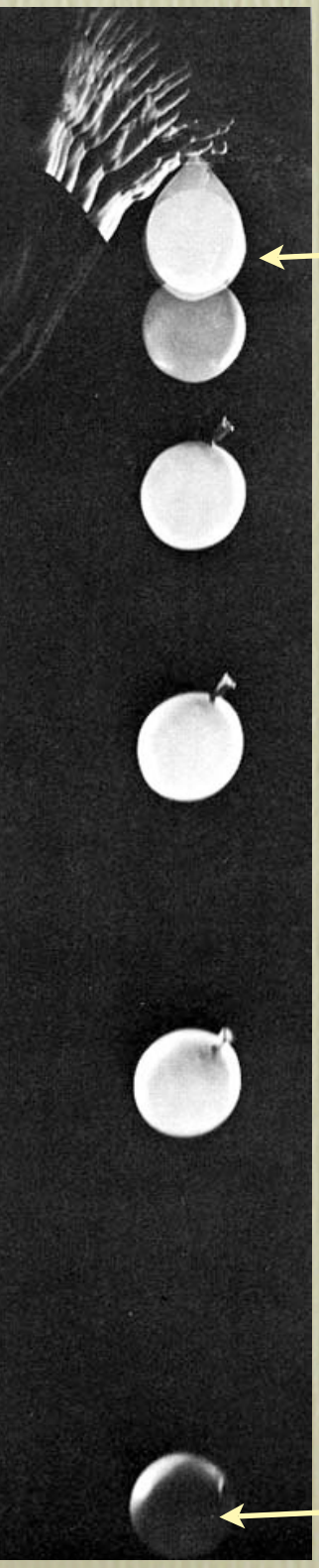
$$(\Delta U_g = -\Delta K)$$

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

Final Mechanical Energy = Initial Mechanical Energy



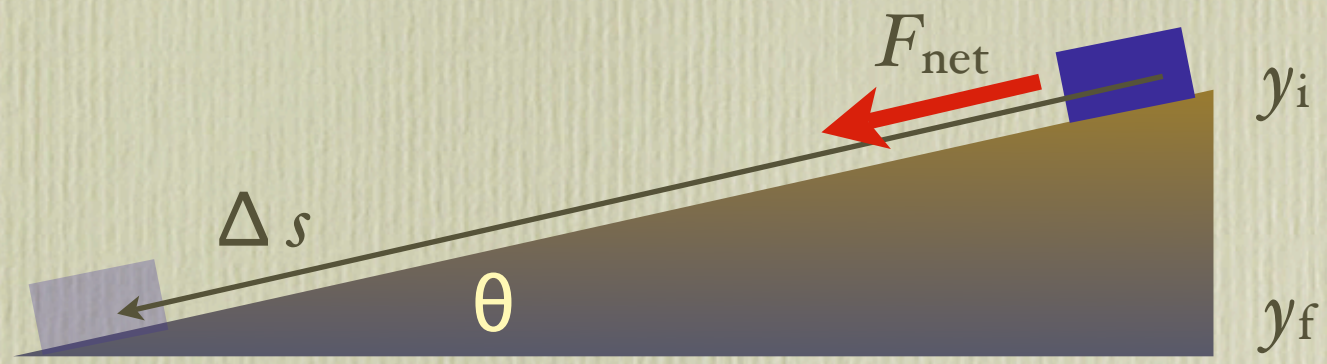
At any position during the fall, $K + U_g$ is the same.



The Zero Level

- The value of y that corresponds to $U_g = 0$ is arbitrary.
- All physical quantities of interest involvea differences in U_g.
- Therefore a constant value added has no effect.
- Once a zero level is established, it should not be changed through the complete analysis of a problem.
- Choose a zero level that is convenient.

Sliding down a ramp



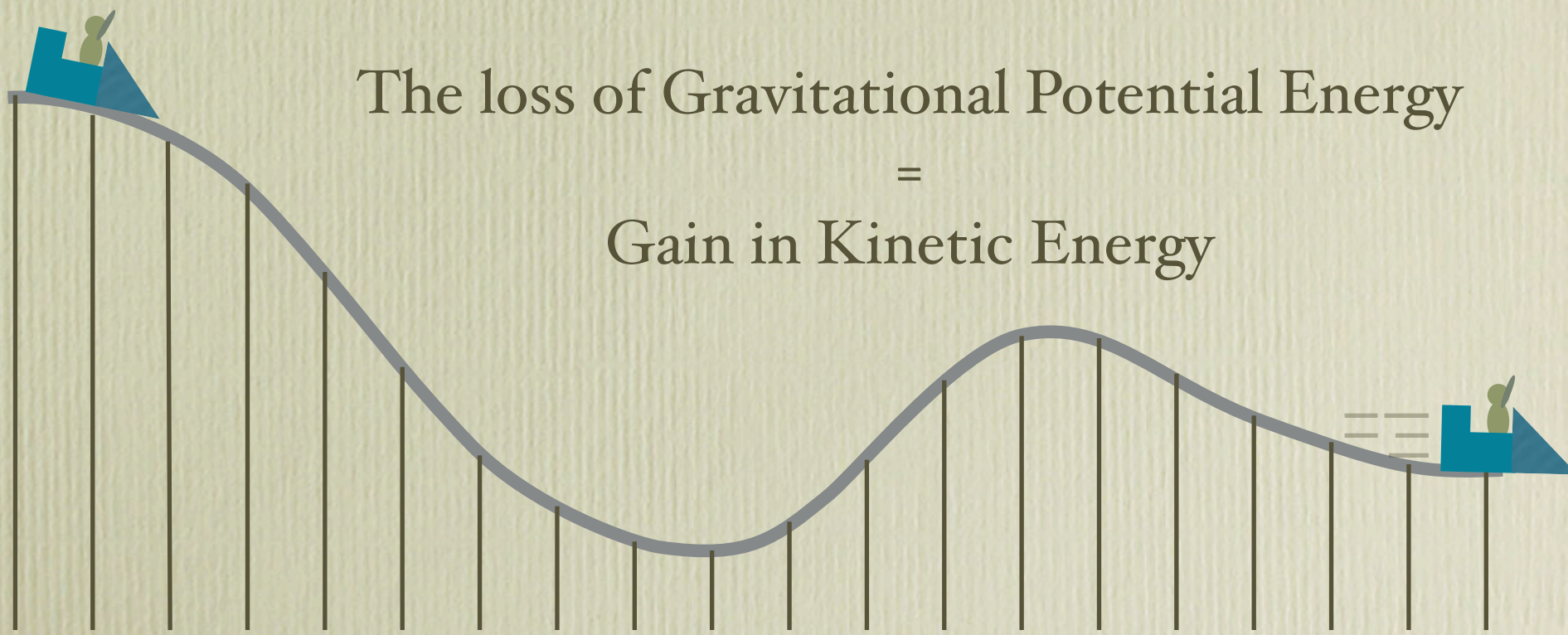
$$\Delta s = \frac{\Delta y}{\sin\theta}$$

$$F_{\text{net}} = mg\sin\theta$$

$$\int_{s_i}^{s_f} F_{\text{net}}\,ds = \int_{y_i}^{y_f} (-mg)\sin\theta\,\frac{dy}{\sin\theta} = -mgy_f + mgy_i$$

Any descent path, without friction

$$-mgy_f + mgy_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$



The loss of Gravitational Potential Energy

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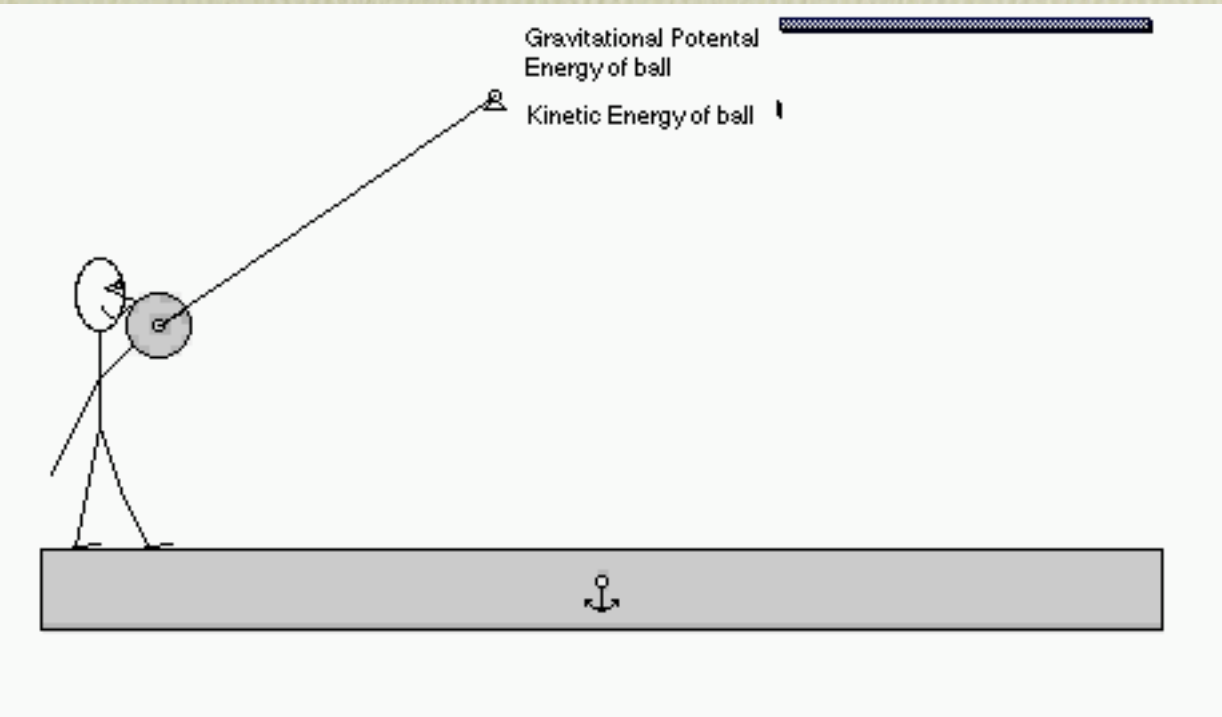
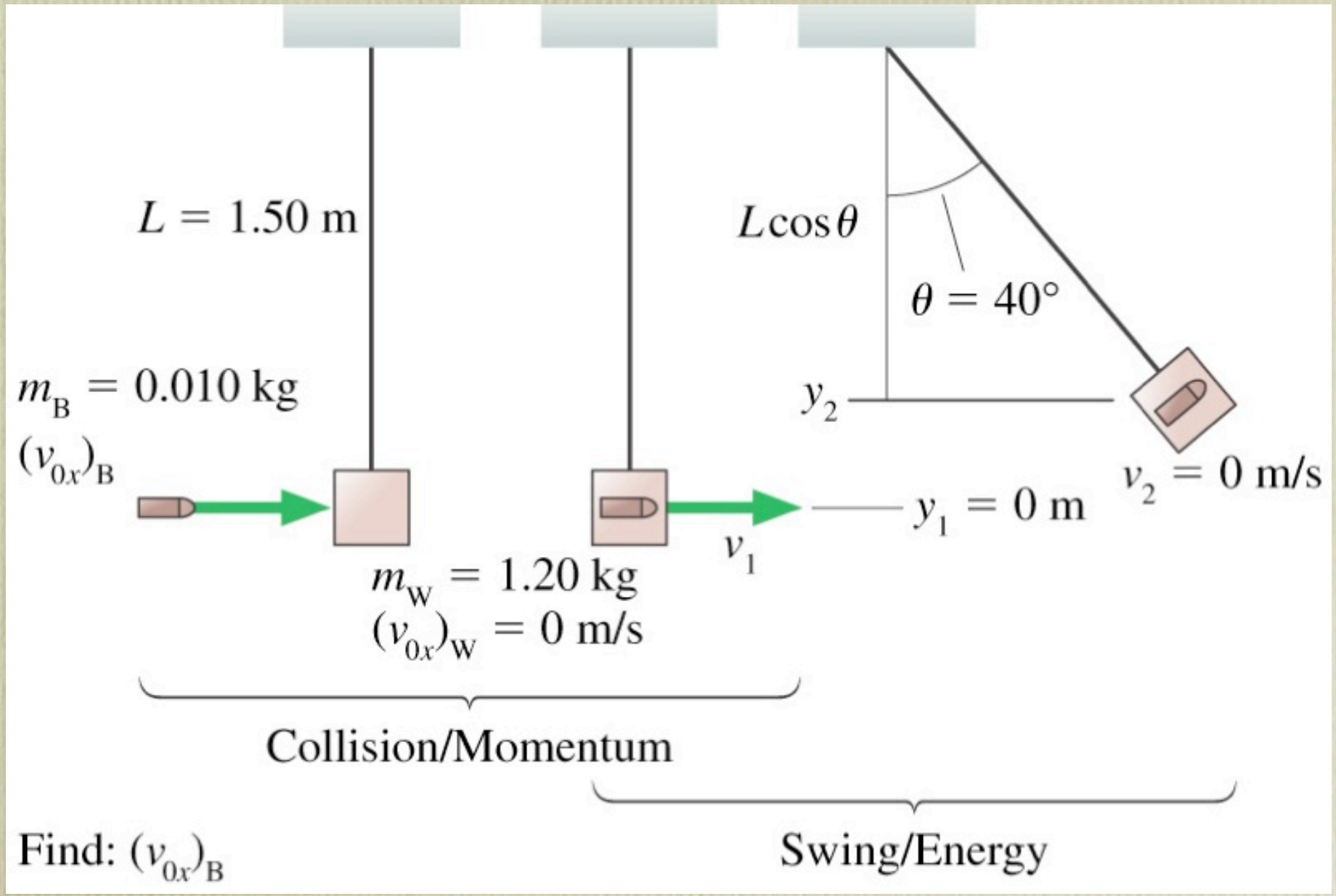
Gain in Kinetic Energy

Suicide Pendulum



Ballistic Pendulum



317 m/s